CBESSJ

PURPOSE

Compute the real or complex component of the Bessel function of the first kind and order v for a complex argument where v is a non-negative real number.

DESCRIPTION

The Bessel function of the first kind with order v (v is a non-negative real number) can be defined as:

$$J_v(z) = \left(\frac{z}{2}\right)^v \sum_{k=0}^{\infty} \frac{\left(\frac{-z^2}{4}\right)^k}{k!\Gamma(v+k+1)} \qquad \textbf{(EQ Aux-54)}$$

where z is a complex number, Γ is the Gamma function and ! is the factorial function.

The imaginary part of the input argument must be less than the logarithm of the largest single precision number on the given computer. In addition, the input number times its complex conjugate cannot be zero. The order is restricted to values between 0 and 100.

SYNTAX 1

LET <y2> = CBESSJR(<r1>,<i1>,<v>) <SUBSET/EXCEPT/FOR qualification>
where <r1> is the real component of a number, variable or parameter;
<i1> is the complex component of a number, variable or parameter;
<v> is a non-negative number, variable, or parameter that specifies the order of the Bessel function;
<y2> is a variable or a parameter (depending on what <r1> and <i1> are) where the computed Bessel value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the real component.

SYNTAX 2

LET <y2> = CBESSJI(<r1>,<i1>,<v>) <SUBSET/EXCEPT/FOR qualification>
where <r1> is the real component of a number, variable or parameter;
<i1> is the complex component of a number, variable or parameter;
<v> is a non-negative number, variable, or parameter that specifies the order of the Bessel function;
<y2> is a variable or a parameter (depending on what <r1> and <i1> are) where the computed Bessel value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the complex component.

EXAMPLES

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LET XR = CBESSJR(2,1,2)
LET XC = CBESSJI(2,1,2)
LET AR = CBESSJR(R1,C1,3)
LET AC = CBESSJI(R1,C1,3)
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NOTE 1

DATAPLOT uses the routine BESJCF from the BESPAK library. This library was written by David Sagin (Sookne), Computer Center, Tel Aviv University.

NOTE 2

Although DATAPLOT does not allow negative orders, negative orders can be calculated with the following relation:

$$J_{-v}(z) = \cos(\pi v)\, J_v(z) - \sin(v\pi)\, Y_v(z) \qquad \textbf{(EQ Aux-55)}$$

where z is a complex number and Y_v is the Bessel function of the second kind. The functions CBESSJR, CBESSJI, CBESSYR, and CBESSYI can be used to compute the relevant Bessel functions.

NOTE 3

The Kelvin functions of the first kind are defined as follows:

$$\text{ber}_v(x) + i\ \text{bei}_v(x) = J_v\left(xe^{\frac{3\pi i}{4}}\right) \quad \textbf{(EQ Aux-56)}$$

The CEXP and CEXPI functions can be used to compute the arguments to the CBESSJR and CBESSJI functions. The PROGRAM example below demonstrates this for Kelvin functions of order 0 and 1.

NOTE 4

The Hankel functions of the first and second kind are defined as:

$$H_v^{(1)}(z) = J_v(z) + i\ Y_v(z) \quad \textbf{(EQ Aux-57)}$$

$$H_v^{(2)}(z) = J_v(z) - i\ Y_v(z) \quad \textbf{(EQ Aux-58)}$$

where J_V is the Bessel function of the first kind and Y_V is the Bessel function of the second kind. The DATAPLOT functions CBESSJR, CBESSJI, CBESSYR, and CBESSYI can be used to compute the various components of the Hankel functions. The COMPLEX ADDITION and COMPLEX SUBTRACTION commands can be used for complex addition and subtraction to combine terms.

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

BESSJN	=	Compute the Bessel function of the first kind, order N, and real argument.
CBESSYR	=	Compute the real component of the Bessel function of the second kind, order N, and complex argument.
CBESSYI	=	Compute the complex component of the Bessel function of the second kind, order N, and complex argument.
CBESSIR	=	Compute the real component of the modified Bessel function of order N and complex argument.
CBESSII	=	Compute the complex component of the modified Bessel function of order N and complex argument.
CBESSKR	=	Compute the real component of the modified Bessel function of the third kind, order N, and complex argument.
CBESSKI	=	Compute the complex component of the modified Bessel function of the third kind, order N, and complex argument.

REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (pages 355-433).

"Note on Backward Recurrence Algorithms," Olver and Sookne, Mathematics of Computation, Volume 26, October 1972.

"Recurrence Techniques for the Calculation of Bessel Functions," Goldstein and Thaler, Mathematics of Computation, Volume 13, April 1959.

"Bessel Functions of Complex Argument and Integer Order," Sookne, Journal of Research of the National Bureau of Standards, Series B, Volume 77A, July-December, 1973.

APPLICATIONS

Special Functions

IMPLEMENTATION DATE

94/9

PROGRAM

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. Compute Kelvin functions of type 1 and order 0 and 1
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LET X = SEQUENCE 0.2 0.2 10
LET N = SIZE X
LET XI = 0 FOR I = 1 1 N
LET CONST = 3*PI/4
LET AR = CEXP(0,CONST)
LET AI = CEXPI(0,CONST)
LET CR = AR FOR I = 1 1 N
LET CI = AI FOR I = 1 1 N
LET YR YC = COMPLEX MULTIPLICATION X XI CR CI
LET ORDER = 0
LET BER0 = CBESSJR(YR,YC,ORDER)
LET BEI0 = CBESSJI(YR,YC,ORDER)
LET ORDER = 1
LET BER1 = CBESSJR(YR,YC,ORDER)
LET BEI1 = CBESSJI(YR,YC,ORDER)
LINE SOLID DASH DOT DOT
TITLE KELVIN FUNCTIONS OF ORDER 0 AND 1
PLOT BER0 BEI0 BER1 BEI1 VS X
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