SPLINE FIT

PURPOSE

Carries out a B-spline fit.

DESCRIPTION

A spline fit is a data analysis technique for estimating (via the least squares criterion) the parameters in a spline polynomial model. It is commonly used to fit curves that have different shapes in different areas of the horizontal axis variable. Knot points are defined to delineate these different regions. Separate spline polynomials are fit in these different areas. The distinction of spline fits is that the fitted curve will be smooth at the knot points.

The INTERPOLATION LET subcommand is used to perform cubic spline interpolation.

SYNTAX 1

<degree> SPLINE FIT <y> <x> <x2> <SUBSET/EXCEPT/FOR qualification>

where <degree> is the degree of the spline fit:

LINEAR (or 1ST or FIRST)
QUADRATIC (or 2ND or SECOND)
CUBIC (or 3RD or THIRD) (the default)
QUARTIC (or 4TH or FOURTH)
QUINTIC (or 5TH or FIFTH)
SEXTIC (or 6TH or SIXTH)
SEPTIC (or 7TH or SEVENTH)
OCTIC (or 8TH or EIGTH)
NONIC (or 9TH or NINTH)
DEXIC (or 10TH or TENTH);

<y> is the response (vertical axis) variable;
<x> is the independent (horizontal axis) variable;
<x2> is the knots variable;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

SYNTAX 2

<degree> SPLINE FIT <y> <x> <SUBSET/EXCEPT/FOR qualification>

where <degree> is the degree of the spline fit (same choices as for syntax 1);
<y> is the response (vertical axis) variable;
<x> is the independent (horizontal axis) variable;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

With this syntax, the knots variable is specified with the KNOTS command before doing the SPLINE FIT command.

EXAMPLES

```
SPLINE FIT Y X X2
CUBIC SPLINE FIT Y X K
```

NOTE 1

The knots variable contains the values along the X axis which define the end points of sub-domains (a separate spline is fit in each sub-domain). The individual points are “splined” together at these knot points.

NOTE 2

The values for the spline fit are placed in the internal variable PRED. The residuals (the difference between the fitted values and the raw data) are placed in the internal variable RES.

NOTE 3

A maximum of 50 knot points can be defined.

NOTE 4

Cubic splines are the most commonly used. Degrees higher than 3 are rarely used.

DEFAULT

Cubic splines (i.e., degree 3).

SYNONYMS

None

RELATED COMMANDS

KNOTS	=	Specify the knots variable for a spline fit.
INTERPOLATE	=	Perform a cubic spline interpolation.
FIT	=	Compute a least squares fit.
LOWESS	=	Compute a locally weighted least squares.
SMOOTH	=	Perform a smoothing of a variable.

REFERENCE

"Spline Functions in Data Analysis,", S. Wold, Technometrics, 1974 (pp. 1-11).

"Numerical Recipes, The Art of Scientific Computing (FORTRAN Version)," Press, Flannery, Teukolsky, and Vettering, Cambridge Press, 1989 (Chapter 3).

APPLICATIONS

Spline fitting

IMPLEMENTATION DATE

Pre-1987

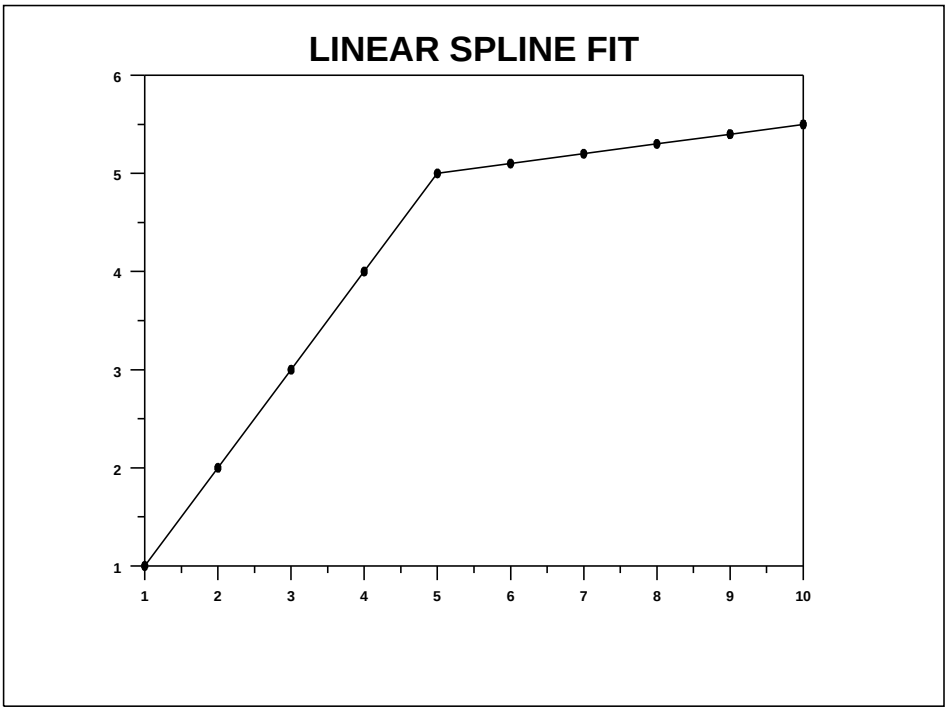
PROGRAM 1

```
LET X = DATA 1 2 3 4 5 6 7 8 9 10
LET Y = DATA 1 2 3 4 5 5.1 5.2 5.3 5.4 5.5
LET KNOT(1) = 5
LINEAR SPLINE FIT Y X KNOT
CHARACTER CIRCLE BLANK
LINE BLANK SOLID
CHARACTER FILL ON
CHARACTER SIZE 1.2
TITLE LINEAR SPLINE FIT
PLOT Y PRED VS X
```

The following output is generated.

```
 LEAST SQUARES SPLINE FIT
       SAMPLE SIZE N   =       10
       MODEL--LINEAR SPLINE
       NUMBER OF KNOTS =        1
       NO REPLICATION CASE

       INTERVAL   LOWER         UPPER      NUMBER OF
        NUMBER    KNOT          KNOT      OBSERVATIONS
                                          IN INTERVAL
       -----------------------------------------------
           1   -INFINITY   0.5000000E+01       4
           2 0.5000000E+01   +INFINITY         6

       PARAMETER ESTIMATES
       -----------------------------------------------
       INTERVAL    1--    A10      =   0.9536743E-06
       INTERVAL    1--    A11      =   0.9999995E+00

       INTERVAL    2--    A20      =   0.4499998E+01
```

```
INTERVAL    2--    A21      =   0.1000003E+00

RESIDUAL    STANDARD DEVIATION =         0.0000007390
RESIDUAL    DEGREES OF FREEDOM =           7
```



PROGRAM 2

```
READ SWANSON1.DAT Y X
.
LET KNOT = DATA 70 90 95 110 140 160 190 240
CUBIC SPLINE FIT Y X KNOT
END OF CAPTURE
CHARACTER CIRCLE BLANK
LINE BLANK SOLID
CHARACTER SIZE 1.2
TITLE CUBIC SPLINE FIT
PLOT Y PRED VS X
```

The following output is generated.

```
 LEAST SQUARES SPLINE FIT
       SAMPLE SIZE N   =      198
       MODEL--CUBIC SPLINE
       NUMBER OF KNOTS =        8
       NO REPLICATION CASE

       INTERVAL   LOWER         UPPER      NUMBER OF
        NUMBER    KNOT          KNOT      OBSERVATIONS
                                          IN INTERVAL
       ------------------------------------------------
           1   -INFINITY   0.7000000E+02      17
           2 0.7000000E+02 0.9000000E+02      20
           3 0.9000000E+02 0.9500000E+02       5
           4 0.9500000E+02 0.1100000E+03      15
           5 0.1100000E+03 0.1400000E+03      30
           6 0.1400000E+03 0.1600000E+03      20
           7 0.1600000E+03 0.1900000E+03      30
           8 0.1900000E+03 0.2400000E+03      50
           9 0.2400000E+03   +INFINITY        11

       PARAMETER ESTIMATES
       ------------------------------------------------
       INTERVAL    1--    A10      =   0.1141443E+06
       INTERVAL    1--    A11      =  -0.4559661E+04
       INTERVAL    1--    A12      =   0.6539211E+02
       INTERVAL    1--    A13      =  -0.3225141E+00

       INTERVAL    2--    A20      =  -0.9751449E+05
       INTERVAL    2--    A21      =   0.4511429E+04
       INTERVAL    2--    A22      =  -0.6419489E+02
       INTERVAL    2--    A23      =   0.2945667E+00

       INTERVAL    3--    A30      =   0.1930424E+07
       INTERVAL    3--    A31      =  -0.6308654E+05
       INTERVAL    3--    A32      =   0.6868937E+03
       INTERVAL    3--    A33      =  -0.2487243E+01

       INTERVAL    4--    A40      =  -0.7346048E+06
       INTERVAL    4--    A41      =   0.2107228E+05
       INTERVAL    4--    A42      =  -0.1989887E+03
       INTERVAL    4--    A43      =   0.6211163E+00
```

```
INTERVAL    5--    A50      =   0.1673756E+06
INTERVAL    5--    A51      =  -0.3527184E+04
INTERVAL    5--    A52      =   0.2464284E+02
INTERVAL    5--    A53      =  -0.5655491E-01

INTERVAL    6--    A60      =   0.1445784E+06
INTERVAL    6--    A61      =  -0.3038674E+04
INTERVAL    6--    A62      =   0.2115349E+02
INTERVAL    6--    A63      =  -0.4824698E-01

INTERVAL    7--    A70      =  -0.1545682E+06
INTERVAL    7--    A71      =   0.2570324E+04
INTERVAL    7--    A72      =  -0.1390275E+02
INTERVAL    7--    A73      =   0.2478685E-01

INTERVAL    8--    A80      =   0.3373448E+05
INTERVAL    8--    A81      =  -0.4028755E+03
INTERVAL    8--    A82      =   0.1745670E+01
INTERVAL    8--    A83      =  -0.2666518E-02

INTERVAL    9--    A90      =  -0.2518377E+07
INTERVAL    9--    A91      =   0.3149852E+05
INTERVAL    9--    A92      =  -0.1311768E+03
INTERVAL    9--    A93      =   0.1819480E+00


RESIDUAL    STANDARD DEVIATION =       216.6169891357
RESIDUAL    DEGREES OF FREEDOM =         186
```