EXACT RATIONAL FIT

PURPOSE

Carries out an exact rational function fit.

DESCRIPTION

An exact rational fit is a data analysis technique for determining good starting values for the parameters in rational function models (a type of nonlinear model). This command is usually followed by the FIT command. This command can be used only under certain specialized conditions. If

n =number of observations to be fitted;
n1=desired degree of numerator;
n2=desired degree of denominator;

then

n1 + n2 must = n -1.

n1 and n2 are integers from 0 to 10. Typical values of n1 and n2 are from 0 to 3. In practice, this technique is rarely used on the original data (because n is usually much larger than 10 or 20). This technique is usually applied to a selected subset of the original data, with the selection done so that the subset mimics quite closely the original data. An exact rational fit to the subset thus provides starting values which may be used for a least squares fit to the full data set.

SYNTAX 1

EXACT <n1>/<n2> RATIONAL FIT <y1> <x1> <SUBSET/EXCEPT/FOR qualification>

where <n1> is an integer number or parameter from 0 to 10 that is the desired degree of the numerator;
<n2> is an integer number or parameter from 0 to 10 that is the desired degree of the denominator;
<y1> is the response (= vertical axis) variable;
<x1> is the independent (= horizontal axis) variable;

and where the <SUBSET/EXCEPT/FOR qualification> is optional and rarely used in this context.

As discussed above, y1 and x1 usually only have a relatively small number of observations.

SYNTAX 2

EXACT <n1>/<n2> RATIONAL FIT <y1> <x1> <y2> <x2> <SUBSET/EXCEPT/FOR qualification>

where <n1> is an integer number or parameter from 0 to 10 that is the desired degree of the numerator;
<n2> is an integer number or parameter from 0 to 10 that is the desired degree of the denominator;
<y1> is the response (= vertical axis) variable;
<x1> is the independent (= horizontal axis) variable;
<y2> is the response (= vertical axis) variable;
<x2> is the independent (= horizontal axis) variable;

and where the <SUBSET/EXCEPT/FOR qualification> is optional and rarely used in this context.

In this syntax, y1 and x1 usually have only a few observations, while y2 and x2 are usually the original data with many observations. The exact rational fit is carried out on y1 and x1, and then predicted values (in PRED) and residuals (in RES) are computed by applying the exact-fit coefficients to the original data set (x2,y2).

EXAMPLES

EXACT 1/1 RATIONAL FIT Y2 X2
EXACT 2/3 RATIONAL FIT Y2 X2 Y X

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

FIT	=	Carries out a least squares fit.
PRE-FIT	=	Carries out a least squares pre-fit.
SPLINE FIT	=	Carries out a spline fit.
SMOOTH	=	Carries out a smoothing.

ANOVA	=	Carries out an ANOVA.
MEDIAN POLISH	=	Carries out a median polish.
PRED	=	A variable where predicted values are stored.
RES	=	A variable where residuals are stored.
RESSD	=	A parameter where the residual standard deviation is stored.
RESDF	=	A parameter where the residual degrees of freedom is stored.
REPSD	=	A parameter where the replication standard deviation is stored.
REPDF	=	A parameter where the replication degrees of freedom is stored.
LOFCDF	=	A parameter where the lack of fit cdf is stored.
PLOT	=	Generates a data or function plot.

APPLICATIONS

Fitting

IMPLEMENTATION DATE

Pre-1987

PROGRAM

```
. EXAMPLE -- TOM HAHN COPPER THERMAL EXPANSION STUDY
.              DATA FILE IN DATAPLOT REFERNCE CATALOG
. MODEL   -- QUADRATIC/QUADRATIC AND CUBIC/CUBIC TECHNIQUE FOR
.            UPDATING AND IMPROVING RATIONAL FIT MODEL.  A SECOND
.            FIT IS RESTRAINED TO GO THROUGH THE ORGIN.
READ HAHN1.DAT X Y
MULTIPLOT CORNER COORDINATES 0 0 100 100; MULTIPLOT 2 2
CHARACTER O; CHARACTER SIZE 1.0
LINE BLANK
TITLE RAW DATA
Y1LABEL COEFFICIENT OF THERMAL EXPANSION (COPPER)
X1LABEL TEMPERATURE (DEGREES KELVIN)
PLOT Y X
.
LET X2 = DATA 10 50 120 200 800
LET Y2 = DATA 0 5 12 15 20
EXACT 2/2 FIT Y2 X2 Y X
.
TITLE EXACT QUADRATIC/QUADRATIC FIT
PLOT Y PRED VS X
.
LET X2 = DATA 40 30 FOR I = 6 1 7
LET Y2 = DATA 3 2 FOR I = 6 1 7
EXACT 3/3 RATIONAL FIT Y2 X2 Y X
.
TITLE EXACT CUBIC/CUBIC RATIONAL FIT
PLOT Y Y PRED VS X
END OF MULTIPLOT
```

The following output is generated.

```
 EXACT RATIONAL FUNCTION FIT
       NUMBER OF POINTS IN FIRST SET    =        5
       DEGREE OF NUMERATOR              =        2
       DEGREE OF DENOMINATOR            =        2

 NUMERATOR  --A0  A1  A2          =      -0.301E+01       0.369E+00      -0.683E-02
 DENOMINATOR--B0  B1  B2          =       0.100E+01      -0.112E-01      -0.306E-03

       APPLICATION OF EXACT-FIT COEFFICIENTS
       TO SECOND PAIR OF VARIABLES--

       NUMBER OF POINTS IN SECOND SET           =      236
       NUMBER OF ESTIMATED COEFFICIENTS         =        5
       RESIDUAL DEGREES OF FREEDOM              =      231

       RESIDUAL SUM OF SQUARES                  =  0.68722278E+03
       RESIDUAL STANDARD DEVIATION (DENOM=N-P)  =  0.17248161E+01
       AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)    =  0.82943726E+00
       LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL =  0.27050836E+01
       LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL = -0.11428773E+02
       LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL =  0.11428773E+02


 EXACT RATIONAL FUNCTION FIT
```

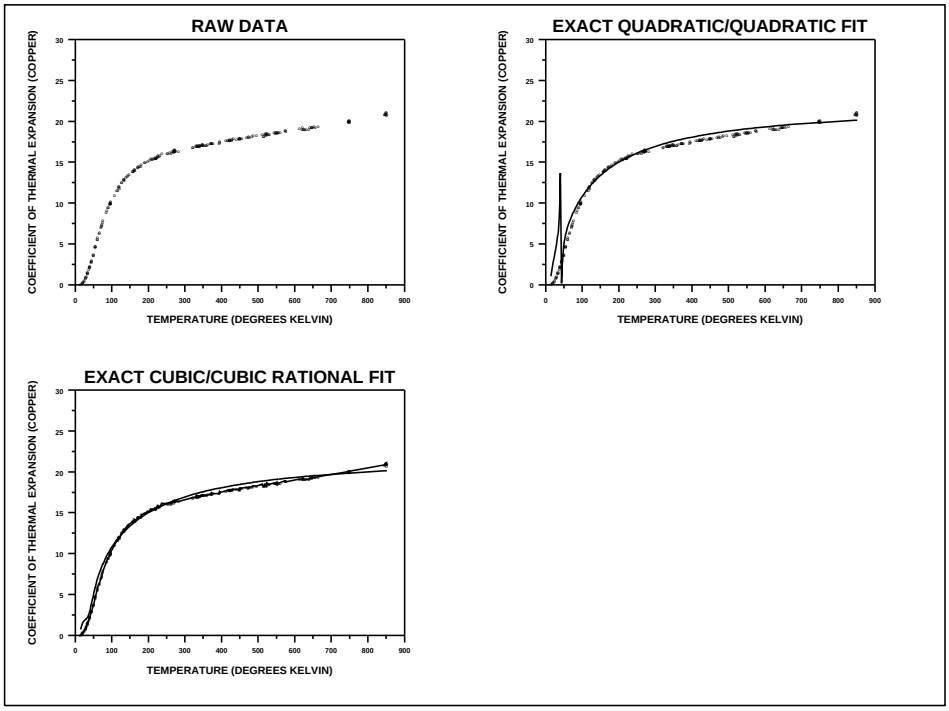
```
       NUMBER OF POINTS IN FIRST SET    =        7
       DEGREE OF NUMERATOR              =        3
       DEGREE OF DENOMINATOR            =        3

 NUMERATOR  --A0  A1  A2  A3      = -0.2322993E+01  0.3528976E+00 -0.1382551E-01
0.1765684E-03
 DENOMINATOR--B0  B1  B2  B3      =  0.1000000E+01 -0.3394208E-01  0.1099545E-03
0.7905308E-05

       APPLICATION OF EXACT-FIT COEFFICIENTS
       TO SECOND PAIR OF VARIABLES--

       NUMBER OF POINTS IN SECOND SET          =      236
       NUMBER OF ESTIMATED COEFFICIENTS        =        7
       RESIDUAL DEGREES OF FREEDOM             =      229

       RESIDUAL SUM OF SQUARES                 =  0.78246452E+02
       RESIDUAL STANDARD DEVIATION (DENOM=N-P)  =  0.58454049E+00
       AVERAGE ABSOLUTE RESIDUAL   (DENOM=N)    =  0.46998626E+00
       LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL =  0.95733070E+00
       LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL = -0.13497944E+01
       LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL =  0.13497944E+01
```



```
.
MULTIPLOT 2 2
FIT Y = (A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
TITLE (A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
X2LABEL RESIDUAL STANDARD DEVIATION = ^RESSD
PLOT Y PRED VS X
TITLE
Y1LABEL RESIDUALS
PLOT RES X
X1LABEL
NORMAL PROBABILITY PLOT RES
END OF MULTIPLOT
```

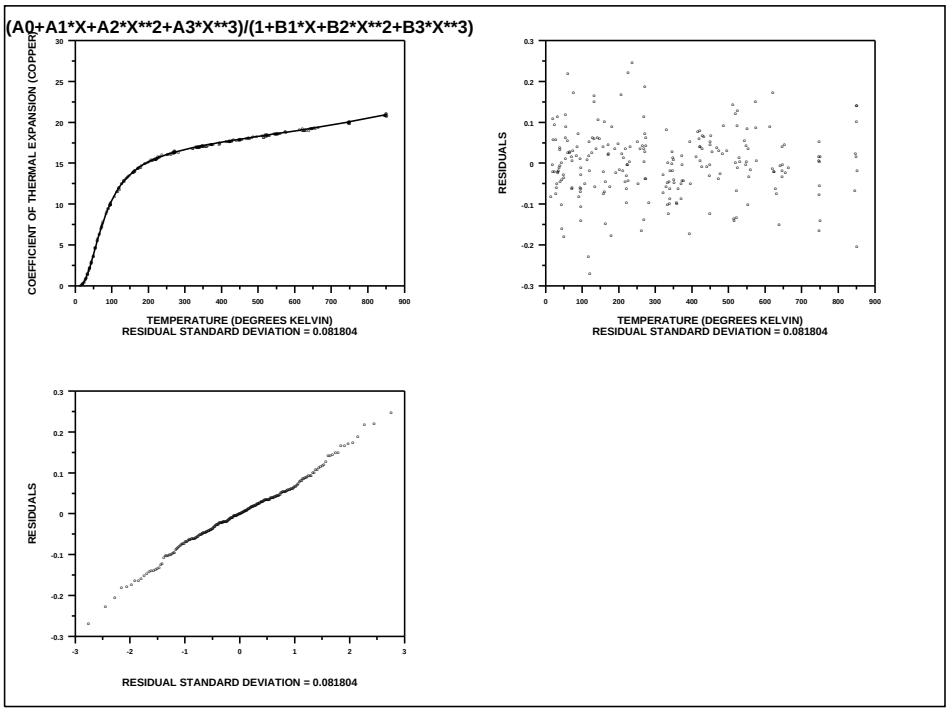
The following output is genberated.

```
 LEAST SQUARES NON-LINEAR FIT
       SAMPLE SIZE N =      236
       MODEL--Y =(A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
       REPLICATION CASE
       REPLICATION STANDARD DEVIATION =     0.8131711930D-01
       REPLICATION DEGREES OF FREEDOM =           1
       NUMBER OF DISTINCT SUBSETS     =         235

 ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
  NUMBER      MEASURE    STANDARD  *  ESTIMATES
                         DEVIATION *
 ----------------------------------*-----------
     1--  0.10000E-01  0.58454E+00 *-0.23230E+01 0.35290E+00-0.13826E-01 0.17657E-03-
0.33942E-01 0.10995E-03
                                   * 0.79053E-05
     2--  0.50000E-02  0.40287E+00 *-0.25300E+01 0.32456E+00-0.12182E-01 0.17700E-03-
0.22379E-01 0.12185E-03
                                   * 0.81249E-05
     3--  0.25000E-02  0.32996E+00 *-0.17333E+01 0.20948E+00-0.77357E-02 0.11611E-03-
0.17614E-01 0.10220E-03
                                   * 0.52967E-05
     4--  0.37500E-02  0.32033E+00 *-0.57350E+00 0.55947E-01-0.14363E-02 0.33719E-04-
0.15187E-01 0.13339E-03
                                   * 0.14423E-05
     5--  0.18750E-02  0.11743E+00 * 0.11902E+01-0.12037E+00 0.36912E-02 0.13591E-05-
0.82885E-02 0.24253E-03
                                   *-0.77788E-08
     6--  0.93750E-03  0.88757E-01 * 0.11666E+01-0.13050E+00 0.42694E-02-0.20343E-05-
0.56409E-02 0.24709E-03
                                   *-0.15139E-06
     7--  0.46875E-03  0.83495E-01 * 0.11117E+01-0.12547E+00 0.41466E-02-0.15771E-05-
0.57279E-02 0.24290E-03
                                   *-0.13046E-06
     8--  0.23437E-03  0.81938E-01 * 0.10830E+01-0.12311E+00 0.40951E-02-0.14471E-05-
0.57584E-02 0.24088E-03
                                   *-0.12420E-06
     9--  0.11719E-03  0.81806E-01 * 0.10784E+01-0.12275E+00 0.40877E-02-0.14290E-05-
0.57604E-02 0.24059E-03
                                   *-0.12328E-06
    10--  0.58594E-04  0.81804E-01 * 0.10793E+01-0.12282E+00 0.40886E-02-0.14285E-05-
0.57613E-02 0.24064E-03
                                   *-0.12326E-06
```

```
FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)    T VALUE
1  A0                   1.07913       (0.1710    )         6.3
2  A1                -0.122801        (0.1203E-01)        -10.
3  A2                 0.408837E-02    (0.2252E-03)         18.
4  A3                -0.142848E-05    (0.2610E-06)        -5.5
5  B1                -0.576111E-02    (0.2468E-03)        -23.
6  B2                 0.240629E-03    (0.1060E-04)         23.
7  B3                -0.123254E-06    (0.1217E-07)        -10.

RESIDUAL    STANDARD DEVIATION =         0.0818038210
RESIDUAL    DEGREES OF FREEDOM =         229
REPLICATION STANDARD DEVIATION =         0.0813171193
REPLICATION DEGREES OF FREEDOM =           1
LACK OF FIT F RATIO =       1.0121 = THE  32.1265% POINT OF THE
F DISTRIBUTION WITH    228 AND      1 DEGREES OF FREEDOM
COEF AND SD(COEF) WRITTEN TO FILE DPST1F.DAT
```



```
.
MULTIPLOT 2 2
FIT Y = (A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
TITLE (A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
X2LABEL RESIDUAL STANDARD DEVIATION = ^RESSD
Y1LABEL COEFFICIENT OF THERMAL EXPANSION (COPPER)
X1LABEL TEMPERATURE (DEGREES KELVIN)
PLOT Y PRED VS X
TITLE
Y1LABEL RESIDUALS
PLOT RES X
X1LABEL
NORMAL PROBABILITY PLOT RES
END OF MULTIPLOT
```

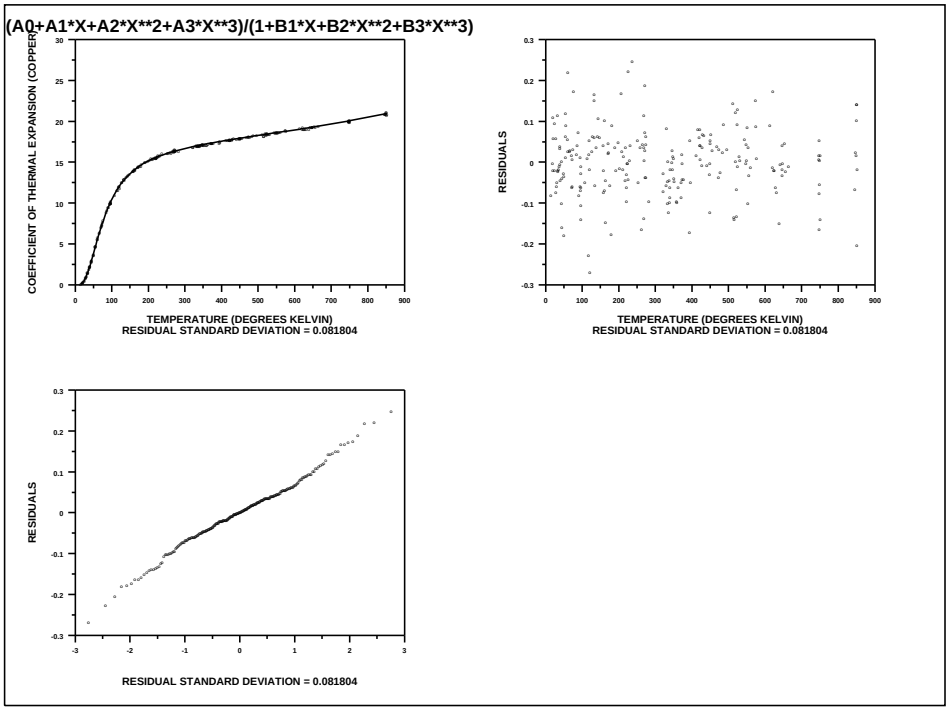
The following output is generated.

```
 LEAST SQUARES NON-LINEAR FIT
       SAMPLE SIZE N =      236
       MODEL--Y =(A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
       REPLICATION CASE
       REPLICATION STANDARD DEVIATION =     0.8131711930D-01
       REPLICATION DEGREES OF FREEDOM =           1
       NUMBER OF DISTINCT SUBSETS     =         235

 ITERATION  CONVERGENCE  RESIDUAL  *  PARAMETER
  NUMBER      MEASURE    STANDARD  *  ESTIMATES
                         DEVIATION *
 ----------------------------------*-----------
     1--  0.10000E-01  0.81804E-01 * 0.10791E+01-0.12280E+00 0.40884E-02-0.14285E-05-
0.57611E-02 0.24063E-03
                                   *-0.12325E-06

       FINAL PARAMETER ESTIMATES           (APPROX. ST. DEV.)    T VALUE
       1  A0                   1.07913       (0.1710    )          6.3
       2  A1                  -0.122803      (0.1202E-01)         -10.
       3  A2                   0.408847E-02  (0.2251E-03)          18.
       4  A3                  -0.142861E-05  (0.2609E-06)         -5.5
       5  B1                  -0.576078E-02  (0.2469E-03)         -23.
       6  B2                   0.240633E-03  (0.1060E-04)          23.
       7  B3                  -0.123259E-06  (0.1216E-07)         -10.

       RESIDUAL    STANDARD DEVIATION =         0.0818038136
       RESIDUAL    DEGREES OF FREEDOM =         229
       REPLICATION STANDARD DEVIATION =         0.0813171193
       REPLICATION DEGREES OF FREEDOM =           1
       LACK OF FIT F RATIO =       1.0121 = THE  32.1265% POINT OF THE
       F DISTRIBUTION WITH    228 AND      1 DEGREES OF FREEDOM
       COEF AND SD(COEF) WRITTEN TO FILE DPST1F.DAT
```


(A0+A1*X+A2*X**2+A3*X**3)/(1+B1*X+B2*X**2+B3*X**3)
COEFFICIENT OF THERMAL EXPANSION (COPPER)
TEMPERATURE (DEGREES KELVIN)
RESIDUAL STANDARD DEVIATION = 0.081804
RESIDUALS
TEMPERATURE (DEGREES KELVIN)
RESIDUAL STANDARD DEVIATION = 0.081804
RESIDUALS
RESIDUAL STANDARD DEVIATION = 0.081804