PHASE PLANE DIAGRAM

PURPOSE
Generates a phase plane diagram.

DESCRIPTION
A first order differential equation is one of the form:

$$y'=F(y,t)$$

where t is an independent variable (usually time), y is a dependent variable, and y' is the derivative of y. A second order differential equation is one of the form:

$$y''=F(y,y',t)$$

where y'' is the second derivative of y.

When the functional form of the differential equation is known (e.g., $y'' = x^2 + y^2 - y'$), there are two common ways to graphically show the solution for the differential equation. The first method is to plot y_i (and y_i' for the second order case) against t_i where t_i, y_i, and y'_i represent the values at time i. A variation is a 3d-plot of (t_i, y_i, y_i') The second method is to plot the phase diagram of the solution curves. Phase space is defined to be the coordinate system consisting of the dependent variable (y in our case) and each of its derivatives. The independent variable (t in our case) is only plotted implicitly. For the second order case, the phase diagram is a plot of (y_i, y'_i). Phase diagrams are not normally drawn for the first order case since there is only one axis (the first method is used instead). They can be extended to third order differential equations (plot (y_i, y'_i, y''_i)) by drawing a 3d-plot. Although the idea of phase space extends to higher dimensions, there is no obvious way to graph the phase diagram.

If the functional equation is known for a second order differential equation, the RUNGE KUTTA command can be used to find numerical estimates for y and y'. Then the PLOT command can be used to plot the solution using either method. For example, enter PLOT YPRIME Y VS T for the first method and PLOT YPRIME VS Y for the phase diagram.

The PHASE PLANE DIAGRAM is used for the case where the functional form of the differential equation is unknown. It works on a set of data points (i.e., values for y and optionally for time) to give a graphical estimate of the phase diagram. It uses the fact that y'=dy/dx (i.e., the change in y divided by the change in x). The one variable case (i.e., y only) plots the following:

Vertical   axis =   Y(i+1) - Y(i);
Horizontal axis =   Y(i).

The two variable case plots the following:

Vertical   axis   = (Y(i+1) - Y(i))/(X(i+1)-X(i));
Horizontal axis   = Y(i).

For the one variable case, the X values are assumed to be equally spaced and equal to 1 (that is, dx=1) and Y(i+1) - Y(i) is an estimate of dy. For the two variable case, X(i+1) - X(i) is an estimate of dx and Y(i+1) - Y(i) is an estimate of dy. In either case, the vertical axis is an estimate of the derivative of Y.

SYNTAX 1
PHASE PLANE DIAGRAM <y>   <SUBSET/EXCEPT/FOR qualification>
where <y> is a response variable;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

SYNTAX 2
PHASE PLANE DIAGRAM <y> <x>   <SUBSET/EXCEPT/FOR qualification>
where <y> is a response variable;
      <x> is the independent variable (usually time);
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

EXAMPLES
PHASE PLANE DIAGRAM Y1 X
PHASE PLANE DIAGRAM Y1
PHASE PLANE DIAGRAM Y1 Y2 SUBSET TAG > 3

NOTE 1
A related technique is to plot y(t+1) versus y(t). This can be done in DATAPLOT with the LAG PLOT command.

NOTE 2

Most differential equations textbooks give a slightly different derivation for the phase diagram. They use the fact that second (and higher order) differential equations can be rewritten as a system of first order differential equations. For example, the differential equation

$$y1''+y1'=t^2 + y1^2$$

can be transformed into the two equations

$$y1'=y2$$
$$y2'=t^2+y1^2-y2$$

Then y1 and y2 are used as the coordinate system for the phase diagram. Since y2 = y1', the phase diagram is equivalent to what was derived earlier.

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

LINES	=	Sets the types for plot lines.
CHARACTER	=	Sets the types for plot characters.
LAG PLOT	=	Generates a lag plot.
PLOT	=	Generates a data or function plot.
MULTIPLOT	=	Allows multiple plots per page
RUNGE KUTTA	=	Numerically solve a first or second order differential equation.

REFERENCE

"Differential Equations, Dynamical Systems, and Linear Algebra," Hirsch and Smale, Academic Press, 1974 (pp. 2-4).

"Chaos," James Gleick, Penguin Press, 1987 (pp. 264-266).

APPLICATIONS

Differential Equations

IMPLEMENTATION DATE

88/9

PROGRAM
. Y = DEFLECTION FROM HORIZONTAL (IN INCHES)
. X = DISTANCE OUT ONTO BEAM (X = 0, 2, 4, 6, ..., 100 INCHES)
. P = LOAD (IN POUNDS) AT FREE END (HERE = 64)
. L = LENGTH (IN INCHES) OF BEAM (HERE = 100)
. E = YOUNG'S MODULUS (= 6 * 10**6)
. I = MOMENT OF INERTIA OF CROSS SECTION (HERE = 0.128 INCHES**4)
. -----START POINT----------------------------------
.
. STEP 1--DEFINE THE PHYSICAL PARAMETERS OF THE BEAM
LET P = 64
LET L = 100
LET E = 6*10**6
LET I = 0.128
. STEP 2--DEFINE THE RIGHT-SIDE FUNCTION OF Y''(X) = F
LET FUNCTION F1 = P*(L-X)/(E*I)
LET FUNCTION F2 = (1+YP**2)**(3/2)
LET FUNCTION F = F1*F2
. STEP 3--DEFINE INITIAL CONDITIONS AND DEFINE THE DESIRED SEQUENCE
. OF POINTS AT WHICH TO COMPUTE THE SOLUTION CURVE.
LET Y(1) = 0
LET YP(1) = 0
LET X = SEQUENCE 0 2 100
. STEP 4--SOLVE THE DIFFERENTIAL EQUATION
LET Y YP = RUNGA-KUTTA F X
.
X1LABEL Y
Y1LABEL Y' DERIVED FROM DATA
TITLE AUTOMATIC
PHASE PLANE DIAGRAM Y X