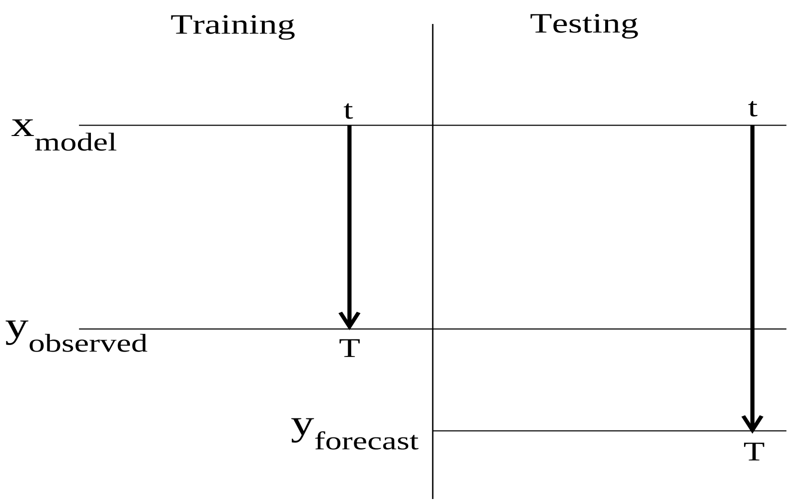
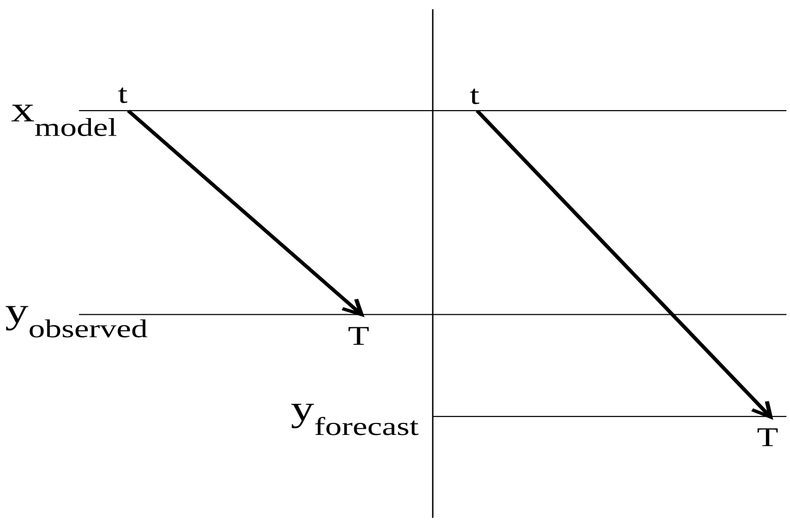
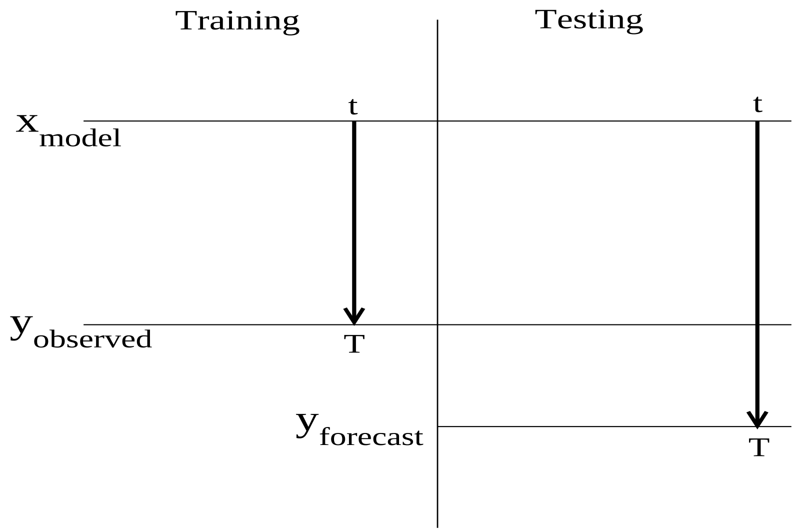
More on MOS, Perfect Prog, and More

Caren Marzban
Eugenia Kalnay
Scott Sandgathe

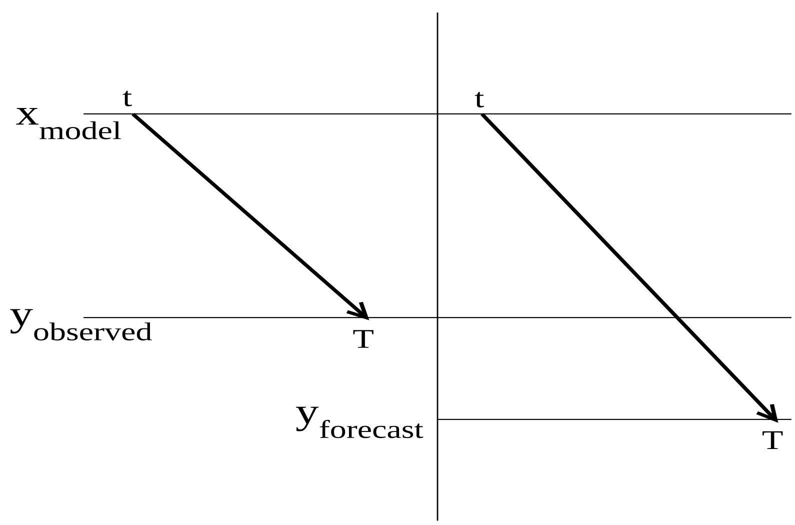
Question: Let x=700mb, y=T2m . Which MOS is better?







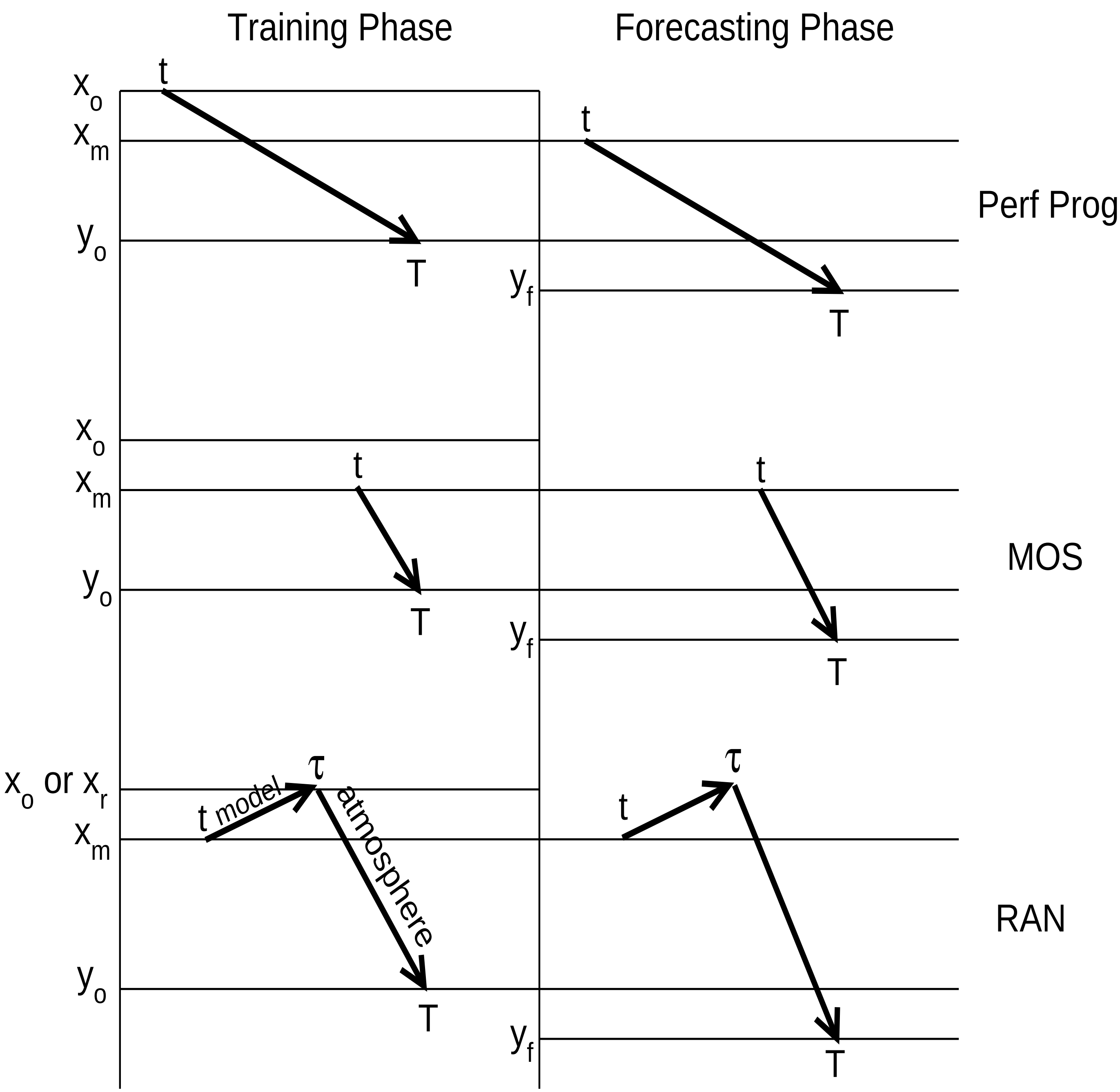
Better if NWP model looses skill **slower** than atmosphere.



Better if NWP model looses skill **faster** than atmosphere.

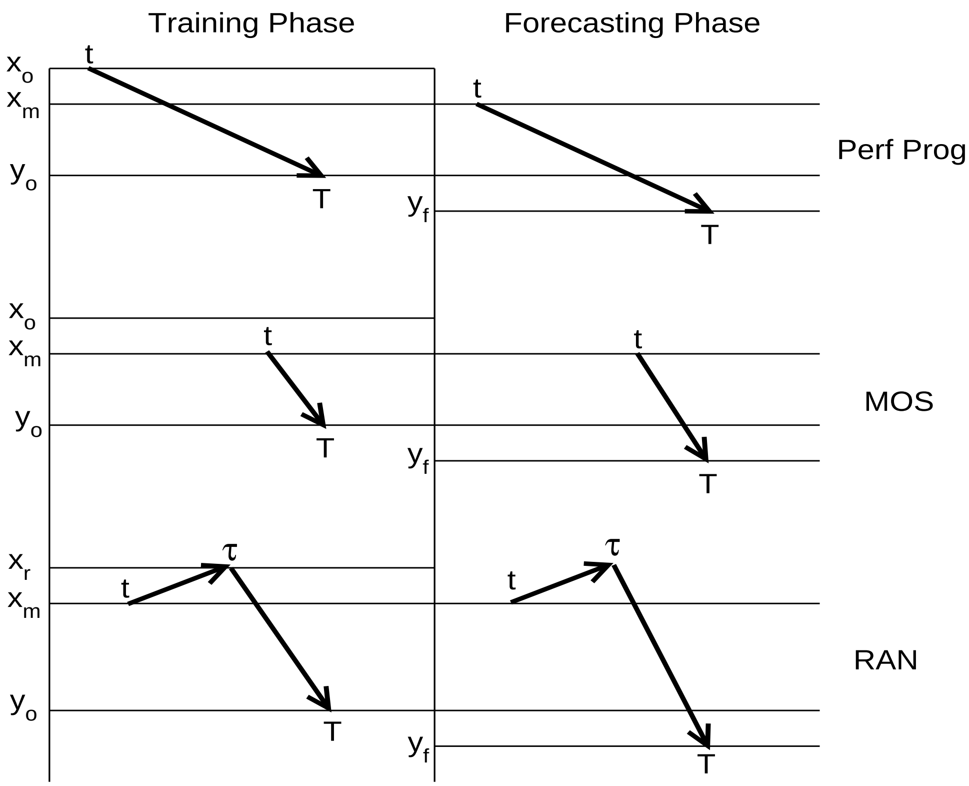
So, 2 new possibilities:

1) Let t be variable, and find optimal value.
2) Separate model and atmosphere losses.



Training Phase
Forecasting Phase
x_o
x_m
y_o
t
T
y_f
Perf Prog
MOS
x_o or x_r
t model
τ
atmosphere
RAN

Variable	Definition	Example
$x_o(t)$	Observed predictor	Observed 700mb height
$x_m(t)$	Model predictor	Model forecast of 700 mb height
$x_r(\tau)$	Reanalysis predictor	Reanalysis value of 700 mb height
$y_o(T)$	Observed predictand	Observed 2m temperature
$y_f(t,T)$	Forecast predictand	Forecast 2m temperature



$$
\begin{aligned}
\text{PP:} \quad & y_o(T) = f[x_o(t)] + \text{error}, \qquad && y_f(t,T) = f[x_m(t)]\ . \\
\text{MOS:} \quad & y_o(T) = f[x_m(t)] + \text{error}, \qquad && y_f(t,T) = f[x_m(t)]\ . \\
\text{RAN:} \quad & x_r(\tau) = f[x_m(t)] + \text{error}, && \\
& y_o(T) = g[x_r(\tau)] + \text{error}, \qquad && y_f(t,\tau,T) = g\left[f[x_m(t)]\right].
\end{aligned}
$$

Question: Which is better?

$$\begin{aligned} \text{Linear}\, f(x) &= \alpha\, x + \beta \\ \text{Minimize MSE} &\rightarrow \text{OLS estimates} \\ \text{Evaluate MSE} &= \text{Var} + \text{Bias}^2 \;\; \text{and Uncertainty} \end{aligned}$$

Calculation ... $\rho \sim$ generalized correlation coefficient.

$$\begin{aligned} V_P &= \sigma^2[y_o(T)] + \rho_P^2(t_P, T) - 2\,\rho_P(t_P, T)\,\rho_M(t_P, T) \\ V_M &= \sigma^2[y_o(T)] - \rho_M^2(t_M, T) \\ V_R &= \sigma^2[y_o(T)] + \rho_R^2(t_R, \tau_R) - 2\,\rho_R(t_R, \tau_R, T)\,\rho_M(t_R, T) \end{aligned}$$

$$\begin{aligned} B_P &= \alpha_P(t_P, T)\left[\overline{x_o(t_P)} - \overline{x_m(t_P)}\right] \\ B_M &= 0 \\ B_R &= 0 \end{aligned}$$

$$\begin{aligned} V_P(t_P) - V_M(t_M) &= [\rho_P(t_P) - \rho_M(t_P)]^2 + [\rho_M^2(t_M) - \rho_M^2(t_P)] \\ V_R(t_R, \tau_R) - V_M(t_M) &= [\rho_R(\tau_R, t_R) - \rho_M(t_R)]^2 + [\rho_M^2(t_M) - \rho_M^2(t_R)] \\ V_R(t_R, \tau_R) - V_P(t_P) &= [\rho_R(\tau_R, t_R) - \rho_M(t_R)]^2 \\ &- [\rho_P(t_P) - \rho_M(t_P)]^2 + [\rho_M^2(t_P) - \rho_M^2(t_R)] \end{aligned}$$

MOS outperforms PP and RAN, in terms of Bias and Variance.
Gauss-Markov Theorem.

Forecast uncertainty depends on sample size, N.

$$\sigma_P^2[y_f] \;=\; \frac{\sigma^2[y_o]}{N_P}\;\;\{1 + (\frac{x_m - \overline{x_o}}{\sigma[x_o]})^2\}$$

$$\sigma_M^2[y_f] \;=\; \frac{\sigma^2[y_o]}{N_M}\;\;\{1 + (\frac{x_m - \overline{x_m}}{\sigma[x_m]})^2\}$$

$$\sigma_R^2[y_f] \;=\; \sigma^2[y_o]\,\{\frac{1}{N_R} + (\frac{x_m - \overline{x_m}}{\sigma[x_m]})^2\;(\frac{r^2[x_m, x_r]}{N_M} + \frac{r^2[x_r, y_o]}{N_R})\}$$

$$N_R > N_M \longrightarrow \text{RAN has lower uncertainty than MOS.}$$

50-year reanalysis data!
RAN outperforms MOS in terms of uncertainty.

Conclusion

Among MOS, PP, and RAN:

1) Only MOS and RAN are bias-free.
2) MOS is better than RAN in terms of variance.
3) RAN is better than MOS in terms of uncertainty.

That's all linear.

3.5) Nonlinear MOS is better than MOS.

Next question: Nonlinearly, which is better?

Acknowledgement: COMET