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Bootstrap Estimators of Discard Rates Using Domestic Sea Sampling Data

by

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ABSTRACT---Two bootstrap estimators of discard rates are examined: the ratio of total discards to total time spent fishing for a given set of tows (aggregate ratio) and the average discard per tow (average rate). The estimators are applied to tow data collected by the NEFC Domestic Sea Sampling Program in 1989 for the purpose of estimating discard and catch rates for Cod (*Gadus morhua*), Haddock (*Melanogrammus aeglefinus*), Yellowtail flounder (*Limanda ferruginea*) in the large mesh otter trawl fishery operating in the Gulf of Maine. Comparisons of the estimators indicate that the aggregate ratio estimator should outperform the average rate estimator. When applied to the Sea sampling data however, both estimators underestimate total catches of Cod and Yellowtail. The magnitude of the underestimates suggests that the estimates of total discard for Cod and Yellowtail are probably conservative and may indicate that the Sea sampling data are not representative of fishing effort directed at these species. In contrast, quarterly discard estimates for Haddock appear reasonable based on the concordance between actual and estimated catches and on the relatively small mean squared errors for the Haddock discard rate. Overall, it is recommended that the Sea sampling data be used cautiously to estimate discard rates on a quarterly basis.

INTRODUCTION

It is necessary to account for the amount of fish discarded by fishermen in order to assess the effect of fishing effort on future recruitment of commercially exploited species. Discard estimates obtained in dockside interviews depend on the captain's recollection and may be unreliable due to fatigue, poor memory, lack of interest in the discarded catches, and so on. Discard estimates obtained by directly observing fishing operations at sea (as is done in the Domestic Sea Sampling Program) are likely to be very accurate for the individual fishing trip. Yet since relatively few fishing trips can be sampled by observers, data collected by observers may not be representative of the discarding practices of the entire fleet.

In general, considerable variation in fishing vessel performance can be expected due to differences in vessel size, fishing gear, fishing area, crew experience, and luck. It is desirable to reduce variability in vessel performance as much as possible for the purpose of estimating discard rates. This can be accomplished by organizing data collected from the fishing fleet into more or less homogeneous units since observer records can be categorized according to gear type, area of capture, and time of capture. In this study, discard rates of three species, Cod (*Gadus morhua*), Haddock (*Melanogrammus aeglefinus*), and Yellowtail flounder (*Limanda ferruginea*), are estimated for one fishery, the Gulf of Maine large mesh (mesh $\geq$ 5.5 inches) otter trawl fishery of 1989.

The estimation of a discard rate requires that a time interval be specified for which the rate will apply. In this study, quarterly samples of observed tows for the Gulf of Maine large mesh otter trawl fishery for 1989 are treated as independent and identically distributed draws from the set of all tows within this fishery. The assumption that tows are independent requires that the autocorrelation between tows within a trip should be similar in magnitude and sign to the crosscorrelation between tows across different trips and vessels.

Nonparametric estimation methods can be applied to estimate target parameters (e.g. discard rates of Cod, Haddock, and Yellowtail) when the joint distribution of the these parameters is unknown. In this study, a nonparametric method called the "bootstrap" is used to estimate discard rates as well as to estimate the bias and variance of these rates (Efron 1982).

METHODS

It is usually necessary to estimate parameters to describe the distribution a random variable within a given population. In this study, the (vector-valued) random variable of interest is the weight of Cod, Haddock, and Yellowtail, discarded per quarter in the Gulf of Maine large mesh otter trawl fishery, the population is the set of all tows made by vessels in this fishery, but the distribution of the random variable is unknown. To begin, assume that all tows are independent and identically distributed. That is, $T_1, T_2, ..., T_n$ are iid draws from F, where F is an unspecified distribution and T_j denotes a single tow ($j=1,...,n$).
Additional assumptions are unnecessary since the samples are drawn from an unspecified distribution (cf. Efron 1982).

Suppose that $D(\underline{T},F)$ denotes the random variable of interest that depends on the (unordered) set of tows $\underline{T}$. For example, D might be the amount (pounds) of Cod discarded in a fishery. Based on an observed sample of tows $\underline{t}$, one estimates a parameter (or set of parameters) to describe the distribution of D. One might want to know the expectation of D,

(i) What is $E_F[D(\underline{T},F)]$?

Alternatively, one might want to know the probability that D is very large, say greater than 100000,

(ii) What is $Pr_F\{ D(\underline{T},F) > 100000 \}$?

To understand how the bootstrap can handle either (i) or (ii), it is useful to consider the bootstrap as a computational procedure (algorithm). For question (i), the bootstrap algorithm to estimate $E_F[D(\underline{T},F)]$ based on an observed tow sample $\underline{t}$ of size n consists of 3 steps:

(1) Fit the empirical probability distribution, denoted by F, to the observed tow sample $\underline{t}$. That is, F assigns probability 1/n to t_j for j=1,...,n
(2) Create a bootstrap sample $\underline{T}$ from F of size n, where $T_1, T_2, ..., T_n$ are iid from F. Then compute $D(\underline{T}, F)$ based on this bootstrapped sample.
(3.i) Independently repeat step (2) a large number of times B, obtaining B bootstrapped estimates of D, denoted by $D_1, D_2, ..., D_B$. Then calculate

$$E_F[D] = \frac{1}{B}\sum_{b=1}^{B} D_b$$

The bootstrap algorithm for (ii) is identical to that for (i), except that step (3.i) changes to

(3.ii) Independently repeat step (2) B times and, for each bootstrapped sample, record whether D_b is greater than 100000. Then calculate

$$Pr_F\{D > 100000\} = \frac{\text{Number of times } D_b > 100000}{B}$$

For the bootstrap to work well, the underlying (unknown) distribution F should reflect a sampling unit for which the iid assumption makes sense. For the Gulf of Maine large mesh otter trawl fishery, sampling units were defined by quarter of tow observation. This definition allows quarterly length frequency data to be applied to the estimates of weight discarded.

Defining the estimators

Two estimators that apply tow by tow data to compute discard rates with the bootstrap are examined: the ratio of total discards to total time spent fishing for a given set of tows (aggregate ratio) and the average discard per tow (average rate). Both estimators consider discards per unit time so that potential problems with the ratio of discard to catch can be avoided (i.e. tows with positive discards but no landed catch). If discard rates do not fluctuate widely on a tow by tow basis, the sample average should adequately describe the central tendency of the distribution of discard rates. Alternatively, if discard rates fluctuate substantially, possibly because some tows are stopped prematurely when discard rates are high, it may be better to use the aggregate ratio. Regardless of which estimator is chosen, an estimate of the total discard within a quarter is calculated as the sum of the quarterly estimated discard rate times the total amount of time spent fishing by the entire fleet in that quarter.

To define the estimators, suppose that a bootstrap sample $\underline{T}$ consisting of a total of n tows has been generated, where $\underline{T} = \{T_1, T_2, \ldots, T_n\}$. Let D_j be the discarded weight and L_j be the tow duration in days fished during the jth tow in $\underline{T}$.

Aggregate ratio estimator, R

One way to estimate the weight discarded per unit time is to aggregate the tow samples first by computing the total amount discarded and the total tow duration for the bootstrap sample, and then to compute the discard rate as the ratio of these amounts. This leads to the aggregate ratio estimator, denoted by R,

$$R = \frac{\sum_{i=1}^{n} D_i}{\sum_{j=1}^{n} L_i}$$

The aggregate ratio estimator calculates discard rate as the ratio of the expectation of two random variables: total weight discarded and total days fished. Notice also that if all tows were sampled, the estimate of total discard would be exact.

Average rate estimator, R_{avg}

Another way to estimate the weight of discarded fish per unit time is to use the average of the observed discard per tow duration for the bootstrapped sample. This leads to the average rate estimator, denoted by R_{avg},

$$R_{avg} = \frac{1}{n} \sum_{i=1}^{n} \frac{D_i}{L_i}$$

The average rate estimator computes the discard rate as the expectation of the ratio of discarded weight to days fished. Note that if all tows were sampled, then the estimate of total discard would probably not be exact. Thus, R_{avg} is inferior to R in the sense that R_{avg} is not necessarily a consistent estimator of total discard.

Comparison of the precision of R and R_{avg}

An analytic comparison of the precision of the R and R_{avg} estimators can be made in certain cases. Suppose that, for every tow T_j, the tow duration L_j is observed without error and that the discarded weight is observed with an error e_j, where the e_j are independent and normally distributed with mean 0 and variance v. That is, assume that $D_1 + e_1, D_2 + e_2, \ldots, D_n + e_n$ are observed, where $e_1, e_2, \ldots, e_n$ are iid Normal(0,v) random variables.

It can be shown that the distribution of observational errors, E and E_{avg}, of the actual values, R and R_{avg}, respectively, are

$$E \sim Normal\left(O, \frac{Vn}{\left(\sum_{j=1}^{n} L_j\right)^2}\right) \qquad E_{avg} \sim Normal\left(O, V\sum_{j=1}^{n}\left(\frac{1}{L_j}\right)^2\right)$$

The variance of E is smaller than that of E_{avg}, so large observational errors are more likely if the average rate estimator rather than the aggregate ratio estimator is used.

RESULTS

Sea sampling data used for this study were collected from the large mesh otter trawl fishery operating in the Gulf of Maine for 1989. Observer data for this fishery consisted of 279 tow records from 39 trips (2,15,17, and 5 trips from quarters 1,2,3, and 4, respectively). For all bootstrapped estimates, the number of bootstrap replications (e.g. B) is 10000. Separate bootstrap samples are used to estimate the discard and catch rates of Cod, Haddock, and Yellowtail; this amounts to assuming that the distributions of discard and catch rates for these species are independent. Table 1 summarizes the sampling coverage for the Gulf of Maine large mesh otter trawl fishery in 1989 relative to total commercial effort.

Empirical comparisons of the estimators

While the aggregate ratio estimator is likely to be more robust than the average rate estimator, a direct comparison of the performance of the estimators may reveal potential bias in either. To perform an empirical comparison, the total observed catches in each quarter based on the observed sample of 279 tows are compared to the total estimated catches of Cod, Haddock, and Yellowtail flounder, respectively, in Tables 2.1, 2.2, and 2.3.

Table 2.1 shows that the R_{avg} estimator overestimates the catch of Cod in quarters 1 and 2, and overestimates the total Cod catch by 7408 pounds. The R_{avg} estimator also overestimates Cod discards in all quarters, and overestimates the total Cod discard by 784 pounds. In contrast, the R estimator performs well and its estimates are closer to the actual Cod catch and discard statistics. Although the R estimator overestimates Cod catch and discard by 76 and 14 pounds, respectively, the R overestimates are an order of magnitude smaller than the R_{avg} overestimates.

Table 2.2 shows that both estimators provide reasonable estimates of the total catch of Haddock, although the quarterly R_{avg} estimates are more variable. The quarterly estimates also show that the R estimator accurately estimates the Haddock discards, while the R_{avg} estimator overestimates discards in quarters 1 and 4.

Table 2.3 shows that the R_{avg} estimator overestimates the quarter 2 catch of Yellowtail flounder, and that both estimators produce reasonably accurate estimates of the observed Yellowtail discard. The R estimator underestimates the total Yellowtail catch by only 6 pounds. In comparison, the R_{avg} estimator overestimates the Yellowtail catch by 416 pounds, roughly 2 orders of magnitude larger than the R estimate. The Sea sampling data for quarter 1 do not adequately represent fishery impacts on Yellowtail (0 catch observed in 2 trips out of a total of 491 trips) since the total Yellowtail catch in the Gulf of Maine was 166,500 pounds for quarter 1 of 1989. Nonetheless, this comparison shows that the R estimator provides more reliable estimates of the observed catch and discard <u>within</u> the Sea sampling data.

Another comparison of the R and R_{avg} estimators was performed based on the estimated catch per day fished and total days fished by quarter (Table 1) to estimate the total catch for the three species. Actual catches in each quarter (based on all trips in the CFDBS) are compared to estimated catches of Cod, Haddock, and Yellowtail flounder, respectively, in Tables 2.4, 2.5, and 2.6.

Table 2.4 shows that the R and R_{avg} estimator perform poorly when estimating the Cod catch in quarters 3 and 4. In quarters 1 and 2, the R_{avg} estimator produces adequate estimates of Cod catch, while the R estimator underestimates the catch. For both estimators however, the approximate 95% confidence interval about the catch estimate does not contain the actual catch. Nonetheless, it is surprising that the R_{avg} estimator outperforms the R estimator in quarters 1 and 2 given the results of the other comparison for Cod (Table 2.1).

Table 2.5 shows that, while a few quarterly catch estimates for Haddock are reasonably accurate (R quarter 1; R_{avg} quarter 2), most estimates differ substantially from the actual statistics (R quarters 2,3,4; R_{avg} quarters 1,3,4). This is unexpected given the good performance of both estimators in the other comparison for Haddock (Table 2.2). Regardless, the catch estimates for Haddock are more accurate than those for Cod (Table 2.4) and Yellowtail (Table 2.6) and both estimators produce approximate 95% confidence intervals that contain the actual Haddock catch.

Table 2.6 shows that both estimators perform poorly for Yellowtail flounder in quarters 1,3, and 4, while reasonably accurate catch estimates are obtained in quarter 2. Both estimators produce approximate 95% confidence intervals that do not contain the actual Yellowtail catch, and the total catch is grossly underestimated using either estimator. Again, this is unexpected given the performance of the estimators, especially the R estimator, in the other comparison with Yellowtail (Table 2.3).

Discard estimates

Estimates of the total discard of Cod, Haddock, and Yellowtail flounder in the Gulf of Maine large mesh otter trawl fishery are calculated using the R and R_{avg} estimators. The mean squared error (variance + bias^2) of the estimated discard rates provides another comparison of estimator performance; the estimator with the lower mean squared error (MSE) should provide a better estimate (with respect to the squared difference between the estimate and the true value) of the discard rate. Tables 3.1, 3.2, and 3.3 show the discard estimates for Cod, Haddock, and Yellowtail flounder, respectively. Tables 4.1, 4.2, and 4.3 show the bias and standard deviation estimates for the discard rates of Cod, Haddock, and Yellowtail, respectively.

DISCUSSION

Discarding of Cod (Table 3.1) in the Gulf of Maine large mesh otter trawl fishery occurs throughout the year and peaks in quarter 4. The standard deviation of Cod discards is also largest in quarter 4. The estimated total discard of Cod is 307,075 pounds using the R estimator, and 392,324 pounds using the R_{avg} estimator. The approximate 95% confidence interval for the total amount discarded is (184160,426990) using the R estimator, while it is (247404,537244) using the R_{avg} estimator. The MSE of the R estimator is lower than that of the R_{avg} estimator in all quarters. Overall, the higher MSE for the R_{avg} estimator indicates that R_{avg} estimates of discard rate are more likely to vary around the true value. Estimates of the total discard as a percentage of total estimated catch are 12% for both estimators (12.3% and 12.4% for R and R_{avg}, respectively). The total catch of Cod (5,214,079 pounds) is underestimated using both estimators (R estimate is 2,500,858 pounds, R_{avg} estimate is 3,158,770 pounds). Neither estimator produces an approximate 95% confidence interval that contains the actual catch. The underestimates of total Cod catch may be due to a lack of sampling trips directed at Cod in quarters 3 and 4 (Table 2.4).

Discarding of Haddock (Table 3.2) peaks in quarters 1 and 4, and is negligible in quarters 2 and 3. Standard deviation and bias estimates for Haddock discards are also largest in quarters 1 and 4. The estimated total discard of Haddock is 11,134 pounds using the R estimator and 14,721 pounds using the R_{avg} estimator. The approximate 95% confidence intervals for the total amount of Haddock discarded are (0,25689) and (0,33012) for the R and R_{avg} estimators, respectively. Since the MSE for the R estimator is lower than that of the R_{avg} estimator in quarters 1 and 4, more reliable estimates can be expected using the R estimator. The estimates of total discard as a percentage of total estimated catch are 6.2% using the R estimator and 7.8% using the R_{avg} estimator. The total Haddock catch (162,424 pounds) is overestimated by both estimators (R estimate is 178,320 pounds, R_{avg} estimate is 189,325 pounds), yet both produce approximate 95% confidence intervals that contain the actual catch.

Discarding of Yellowtail flounder (Table 3.3) peaks in quarter 2, and is negligible in quarters 1, 3, and 4. The standard deviation and bias estimates for Yellowtail discards are also largest in quarter 2. The estimated total discard of Yellowtail is 9,410 pounds using the R estimator and 8,416 pounds using the R_{avg} estimator. The approximate 95% confidence intervals for the total amount of Yellowtail discarded are (3065,15754) and (3400,13432) for the R and R_{avg} estimators, respectively. The estimates of total discard as a percentage of total estimated catch are 10.4% using the R estimator and 6.7% using the R_{avg} estimator. Since the MSE for the R_{avg} estimator is lower than that of the R estimator in quarters 2 and 3, more reliable estimates might be expected from the R_{avg} estimator under the assumption of representative sampling. Since the total Yellowtail catch (464,053 pounds) is grossly underestimated by both estimators (R estimate is 85,228 pounds, R_{avg} estimate is 116,930 pounds) however, and since neither estimator produces an approximate 95% confidence interval that contains the actual catch, the quarterly estimates of Yellowtail discard for this fleet are suspect, despite the relatively small estimates of bias and standard deviation (Table 4.3).

While the comparison of the estimators using the Sea sampling alone (Tables 2.1, 2.2, and 2.3) suggests that the R estimator is superior, the consistency check based on the total catch estimates (Tables 2.4, 2.5 and 2.6) shows that both estimators underestimate total catch for Cod and Yellowtail. This suggests that the quarterly Sea sampling data may not be representative of large mesh otter trawl trips directed at Cod and Yellowtail in the Gulf of Maine. Nonetheless, the total discard estimates for Cod and Yellowtail (Tables 3.1 and .3) are likely to be conservative since both estimators understate the total catch.

Discard rates are consistently higher for Cod than for Haddock and Yellowtail (Tables 3.1, 3.2, and 3.3). Bias and standard deviation estimates for the Cod discard rate are also larger than those for Yellowtail and Haddock, although Haddock discard rates (Tables 3.2 and 4.2) have relatively large variances. Bias does not appear to be a serious problem for either estimator because standard deviation estimates are an order of magnitude larger than bias estimates (Tables 4.1, 4.2, and 4.3). Estimated discard rates for Haddock appear reasonable based on the concordance of the actual and estimated total catch (Table 3.2) as well as the their relatively small bias and MSE (Table 4.2). Nonetheless, it is recommended that the Sea sampling data be used cautiously to estimate discard rates on a quarterly basis.

REFERENCES

Efron, B. 1982. The jackknife, the bootstrap and other resampling plans. Monograph 38. S.I.A.M. Philadelphia, PA.

Table 1. Sampling coverage for Gulf of Maine large mesh otter trawl trips in 1989

Quarter	#Trips	Total Days Fished	Sampled D. F.	% D.F. Sampled	Sampled Trips (Tows)
1	491	1,037	9.42	0.9%	2 (61)
2	563	943	12.63	1.3%	15 (96)
3	421	806	16.73	2.1%	17 (89)
4	413	915	6.05	0.7%	5 (33)
Total	1,888	3,701	44.83	1.2%	39 (279)

Table 2.1. Comparison of actual Cod catch and discard to R and R_{avg} estimates for observed tows

	CATCH (pounds)			DISCARD (pounds)		
Quarter	Actual	R	R_{avg}	Actual	R	R_{avg}
1	7,578	7,616	10,307	290	292	387
2	13,433	13,463	17,522	850	852	1,074
3	4,000	4,004	4,289	1,276	1,278	1,378
4	3,060	3,064	3,361	981	989	1,342
Total	28,071	28,147	35,479	3,397	3,411	4,181

Table 2.2. Comparison of actual Haddock catch and discard to R and R_{avg} estimates for observed tows

	CATCH (pounds)			DISCARD (pounds)		
Quarter	Actual	R	R_{avg}	Actual	R	R_{avg}
1	289	292	497	67	67	78
2	250	249	186	2	2	1
3	2,169	2,170	2,059	4	4	4
4	152	152	143	22	22	39
Total	2,860	2,863	2,885	95	95	122

Table 2.3. Comparison of actual Yellowtail flounder catch and discard to R and R_{avg} estimates for observed tows

	CATCH (pounds)			DISCARD (pounds)		
Quarter	Actual	R	R_{avg}	Actual	R	R_{avg}
1	0	0	0	0	0	0
2	1,042	1,046	1,456	102	102	101
3	68	68	75	18	18	16
4	35	25	30	1	1	1
Total	1,145	1,139	1,561	121	121	118

Table 2.4. Comparison of actual and estimated Cod catch (pounds) in all trips using the R and R_{avg} estimators

Quarter	Actual Catch	R Catch/DF	R Catch	R_{avg} Catch/DF	R_{avg} Catch
1	1,119,936	808.4	838,311	1094.1	1,134,582
2	1,354,482	1066.4	1,005,615	1387.9	1,308,790
3	1,266,226	239.6	193,118	256.4	206,658
4	1,473,435	506.9	463,814	556.0	508,740
Total	5,214,079	-	2,500,858	-	3,158,770
S.E. of total			351,921		430,147

Table 2.5. Comparison of actual and estimated Haddock catch (pounds) for all trips using the R and R_{avg} estimators

Quarter	Actual Catch	R Catch/DF	R Catch	R_{avg} Catch/DF	R_{avg} Catch
1	36,636	31.0	32,147	52.7	54,650
2	11,805	19.7	18,577	14.7	13,862
3	71,756	129.7	104,538	123.1	99,219
4	42,227	25.2	23,058	23.6	21,594
Total	162,424	-	178,320	-	189,325
S.E. of total			31,275		37,868

Table 2.6. Comparison of actual and estimated Yellowtail catch (pounds) for all trips using the R and R_{avg} estimators

Quarter	Actual Catch	R Catch/DF	R Catch	R_{avg} Catch/DF	R_{avg} Catch
1	166,500	0	0	0	0
2	93,098	82.8	78,080	115.3	108,728
3	8,966	4.1	3,305	4.5	3,627
4	195,489	4.2	3,843	5.0	4,575
Total	464,053	-	85,228	-	116,930
S.E. of total			17,015		23,161

Table 3.1. Cod discard estimates (in pounds)

Quarter	MSE R	MSE R_{avg}	Discard/Day Fished R	Discard/Day Fished R_{avg}	Total Discard R	Total Discard R_{avg}
1	292.4	342.3	31.0	41.1	32,109	42,575
2	185.0	256.0	67.5	85.1	63,671	80,244
3	338.6	364.8	76.4	82.4	61,563	66,390
4	3637.5	5535.4	163.6	222.0	149,732	203,115
Total	-	-	-	-	307,075	392,324
% of Actual Catch (5,214,079)					5.9%	7.5%
% of Estimate (R 2,500,858; R_{avg} 3,158,770)					12.3%	12.4%

Table 3.2. Haddock discard estimates (in pounds)

Quarter	MSE R	MSE R_{avg}	Discard/Day Fished R	Discard/Day Fished R_{avg}	Total Discard R	Total Discard R_{avg}
1	42.3	55.8	7.2	8.3	7,417	8,583
2	0.0	0.0	0.2	0.1	149	105
3	0.0	0.0	0.2	0.2	193	196
4	11.6	33.6	3.7	6.4	3,375	5,837
Total	-	-	-	-	11,134	14,721
% of Actual Catch (162,424)					6.9%	9.1%
% of Estimate (R 178,320; R_{avg} 189,325)					6.2%	7.8%

Table 3.3. Yellowtail flounder discard estimates (in pounds)

Quarter	MSE R	MSE R_{avg}	Discard/Day Fished R	Discard/Day Fished R_{avg}	Total Discard R	Total Discard R_{avg}
1	0.0	0.0	0.0	0.0	0	0
2	11.6	7.3	8.1	8.0	8,392	7,520
3	0.3	0.1	1.1	1.0	866	775
4	0.0	0.0	0.2	0.1	152	121
Total	-	-	-	-	9,410	8,416
% of Actual Catch (464,053)					2.0%	1.8%
% of Estimated Catch (R 85,228; R_{avg} 116,930)					11.0%	7.2%

Table 4.1. Bias and standard deviation estimates for Cod discard rates (in pounds per day fished)

	Bias		Standard Deviation	
Quarter	R	R_{avg}	R	R_{avg}
1	0.18	0.01	17.1	18.5
2	0.19	-0.03	13.6	16.0
3	0.11	-0.01	18.4	19.1
4	1.71	0.09	60.3	74.4
Standard error of total amount discarded			61,181	73,939

Table 4.2. Bias and standard deviation estimates for Haddock discard rates (in pounds per day fished)

	Bias		Standard Deviation	
Quarter	R	R_{avg}	R	R_{avg}
1	0.04	-0.01	6.5	7.4
2	-0.00	0.00	0.1	0.1
3	0.00	0.00	0.2	0.2
4	0.05	-0.02	3.4	5.8
Standard error of total amount discarded			7,426	9,332

Table 4.3. Bias and standard deviation estimates for Yellowtail flounder discard rates (in pounds per day fished)

	Bias		Standard Deviation	
Quarter	R	R_{avg}	R	R_{avg}
1	0.00	0.00	0.0	0.0
2	0.01	0.01	3.4	2.7
3	-0.00	-0.00	0.5	0.3
4	0.00	0.00	0.2	0.1
Standard error of total amount discarded			3,237	2,559