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Application of Minimum Support Constraints To Seismic Traveltime Tomography

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We explore adaptive regularization methods based on compactness useful for imaging localized features. We apply Portniaguine & Zhdanov's (1999,2002) minimum support approach to generate compact solutions to the seismic traveltime tomography problem. We also explore a second type of compactness constraint based on a spatially varying exponential damping matrix. Both methods are applied to a simple synthetic and a more realistic test based on results from a 2-phase flow simulation of DNAPL infiltration.

1. Introduction : Why Regularize?

Geophysical inverse problems are often ill-posed in that an infinite number of models will fit a given dataset. When confronted with a multitude of answers, we can add secondary constraints to select for models which fulfill some independent notion of what a "good" solution should look like. *Regularization* accomplishes this by minimizing a weighted norm of the solution, typically the product of a low order differential operator and the model vector. By minimizing the 1st or 2nd order spatial derivatives of the model, flat or smooth solutions can be selected. Despite their wide (and often effective) use, 1st and 2nd order Tikhonov regularization are not fundamentally tied to the imaging problem; afterall, why should earth properties be flat or smooth?

We advocate choosing a regularization operator which incorporates the *physics* responsible for property variations such as subsurface flow, thermal diffusion, fracture propagation, or sediment deposition. Within this framework, we select models which both fit the data and reflect our understanding of the cause behind geophysical anomalies. Due to the complexity of these processes, we are currently examining heuristic techniques for including this class of constraints. In particular, we are interested in monitoring problems where changes in earth properties are due to fluid movement. Since flow processes tend to localize in zones of high permeability, regularization operators favoring *compact* and/or connected anomalies seem reasonable. Minimum support inversion, as developed by Last & Kubik (1983) and more recently Portniaguine & Zhdanov (1999,2002) is one approach for selecting compact models while still satisfying the data.

2. Minimum Support Inversion

Assume a linear problem where an operator **G** maps a model vector, **m**, to a dataset, **d** generating a system of the form **Gm = d.** Traditional Tikhonov regularization minimizes a combination of data misfit and weighted model length,

$$\Phi(m) = \|G\,m - d\|_2^2 + \lambda^2 \|W\,m\|_2^2 \longrightarrow \begin{bmatrix} G \\ \lambda W \end{bmatrix} m = \begin{bmatrix} d \\ 0 \end{bmatrix}$$

where **W** is a weighting matrix and λ is a trade-off or regularization parameter. In minimum support inversion we select an objective function of the form

$$\Phi_{ms}(m) = \|G\,m - d\|_2^2 + \lambda^2 \sum_{k=1}^{N_m} \frac{m_k^2}{m_k^2 + \beta^2}$$

where β is parameter designed to prevent singularities for **m = 0** . The solution to this objective function can be cast in terms of a model dependent diagonal weighting operator $\mathbf{W_e}$ where

$$W_e(m_p) = diag\left[m_p^2 + \beta^2\right]^{-1/2}$$

Since $\mathbf{W_e}$ is dependent on **m**, the resulting problem is non-linear and we must resort to a model-domain iteratively reweighted least squares approach (IRLS) – in contrast, hybrid L1/L2 solvers reweight data residuals, not the model space regularization term. At each iteration, a new $\mathbf{W_e}$ is calculated using the previous estimate of **m** and the resulting linear problem is solved. Of course a prior model is needed to start the process ....

When **m** is not homogeneous, $\mathbf{W_e}$ is spatially varying with higher weights in locations where **m** is close to 0. Consider a similar type of weighting matrix based on an scaled exponential of the the model,

$$W_e(m_p) = \lambda_1 + (\lambda_2 - \lambda_1)\, diag\, e^{\frac{-|m_p|}{\sigma}}$$

Since the exponential evaluates between 0 and 1, the two λ values control the maximum and minimum weights in **W** while σ controls the width of the objective function near 0 values. The best way to understand **W** is to examine the resulting weighting and penalty functions,

Fig. A

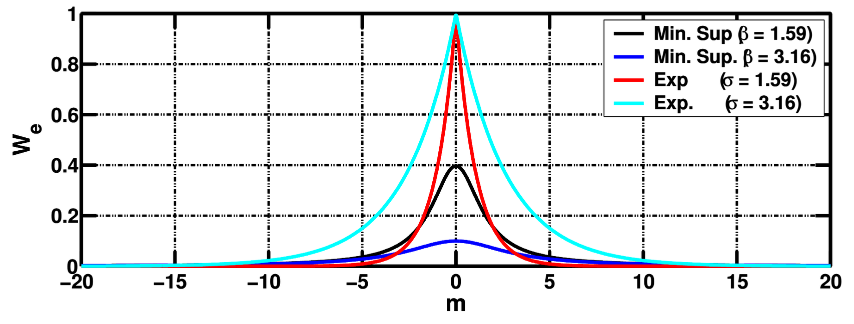


Fig. B

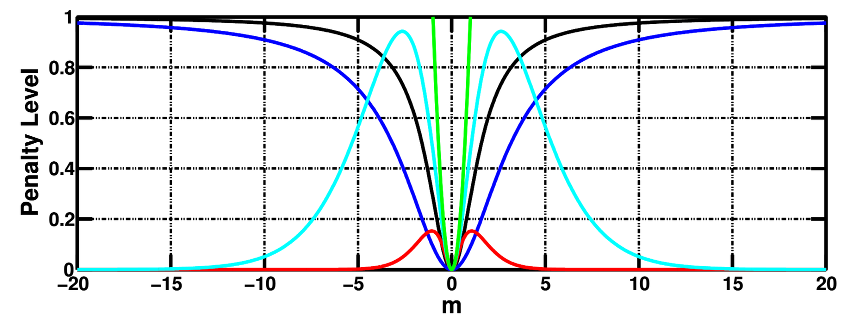


Figure A shows the value of an element of **W** as a function of **m** near 0 for the minimum support and exponential weighting regularization terms. Note that the maximum weights vary as a function of beta for the minimum support case. Figure B shows the corresponding penalty functions in comparison to standard damping (in green).

What does this mean? As we proceed, in each non-linear iteration **W** adaptively suppresses values of **m** near zero. In the case of the minimum support operator, no selective preference is given to large model values. In the exponential weighting case, model values near a selected band are suppressed. In both cases, zones of support are minimized as a function of iteration.

3. Traveltime Tomography : A Minimum Support Example

Traveltime tomography is ...

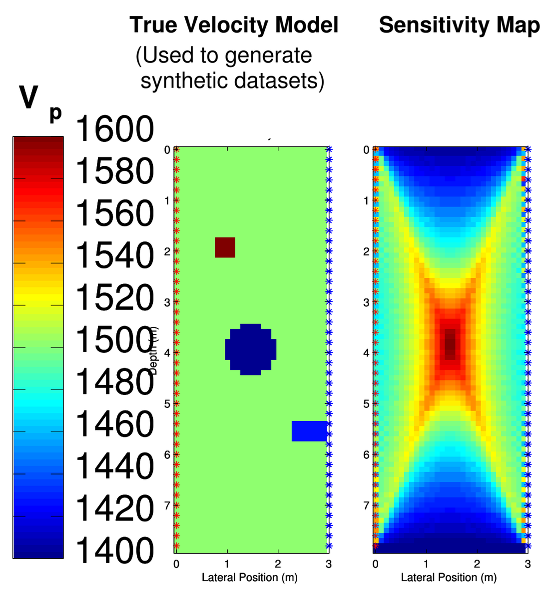
- Useful in a variety of near–surface applications
- Particularly powerful in a monitoring context
- Multiple modalities (seismic,, radar, etc)
- Often ill–posed (depends on descritization)
- Typically low–resolution (T1, T2 reg.)
- Suffers from aperture–related artifacts

Can we use minimum support regularization to improve image quality?

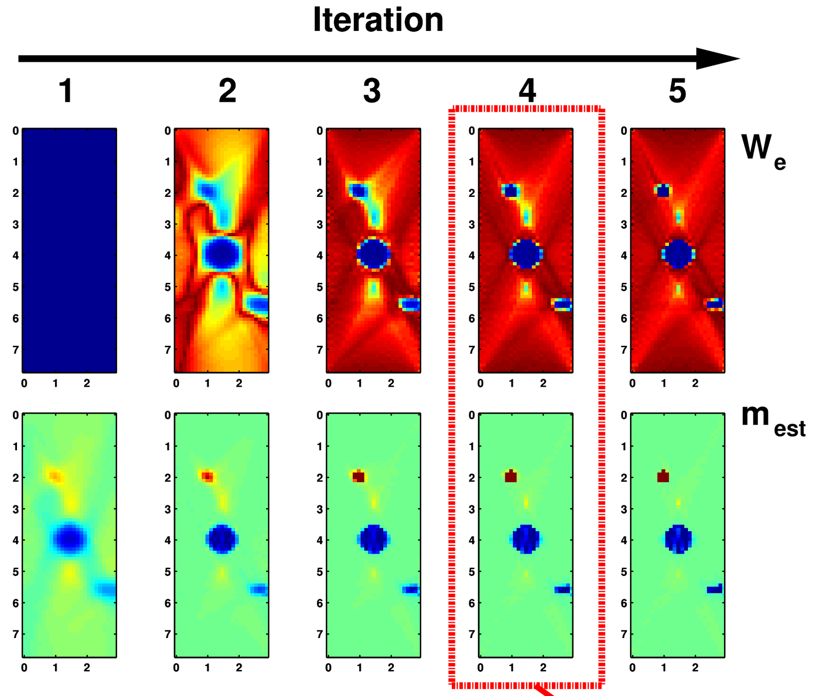
A Test Example

- Straight ray calculation of**G**
- Crosswell geometry (40 S x 40 R)
- Compact velocity anomalies
- Variable ray coverage (see map)

System solved using LSQR



Solutions within the IRLS process



How does this work in practice?

- Noise–free test case
- W is Exponential Weighting matrix
- Starting m is a 2nd ord. Tikh. solution
- m is a 25 x 75 mesh of rectilinear cells

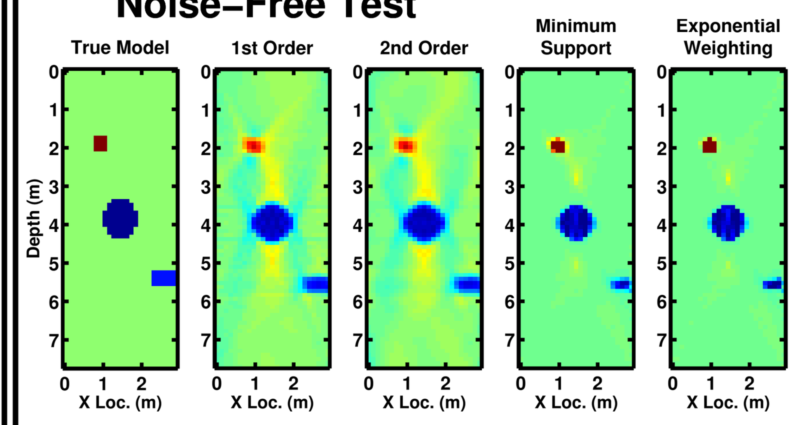
Top row, diag(W) as a function of # its (hot colors – high damping levels)

Bottom row, estimated model

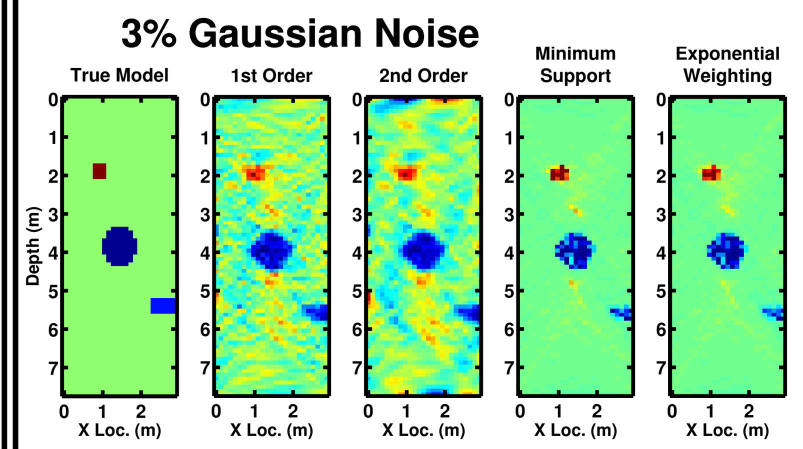
Note

- W remains low in anomalous regions
- model sharpens with each iteration
- artifacts suppressed (not needed for fit)
- How do we determine convergence?
- Data fit is almost exactly equivalent ...

Noise–Free Test



3% Gaussian Noise



Two Tests

- Comparison of 1st, 2nd order Tikh. vs. minimum support exponential weighting
- Compactness operators reduce smearing due to limited angular aperture
- Both types of compactness constraints generate similar results
- Both types of compactness improve quantitative property match (in comparison to T1, T2)

For The Noisy Case

- Compactness operators suppress back–projected noise in the tomogram
- However : high amplitude artifacts can be "focused" as well

Imaging results are similar
Main difference in practice : choice of reg. parameters

Heuristic Exponential Weighting vs. Traditional Minimum Support

Minimum Support : 2 params (beta + lambda)
beta is hard to select, changes effect of lambda

Exponential : 3 params (sigma + 2 x lambda)
lambdas (H,L) can be robustly chosen from Tikh. tests
Sigma controls zone of influence near m = 0

4. Imaging DNAPL Infiltration : A Realistic Synthetic

- Advantage of compactness : match anomalies induced by flow
- Example #2, 2–Phase flow modeling from Kueper & Gerhardt 95
- Permeability model from gaussian simulation
- TCE infiltrating from a point source on the surface (5 m^3)
- Subsurface initially water saturated

Key : A realistic fluid distribution, –> a good test

Building A Radar Synthetic

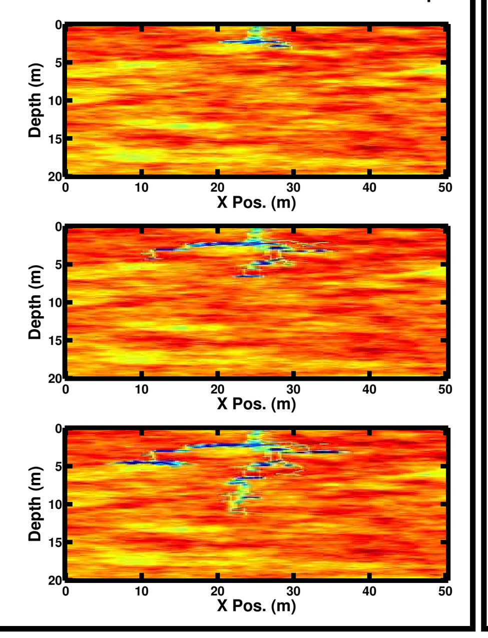
Transform from TCE saturation to dielectric using sequential BHS

Porosity linked to permeability

Fluids, TCE (k=3.35), Water (k= 80.1)

Quartz used used for grain dielectric (k=4.5)

Dielectric Model for 3 Time Steps

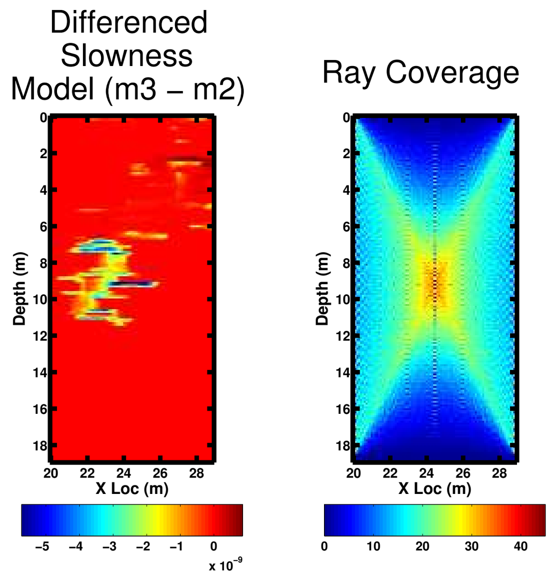


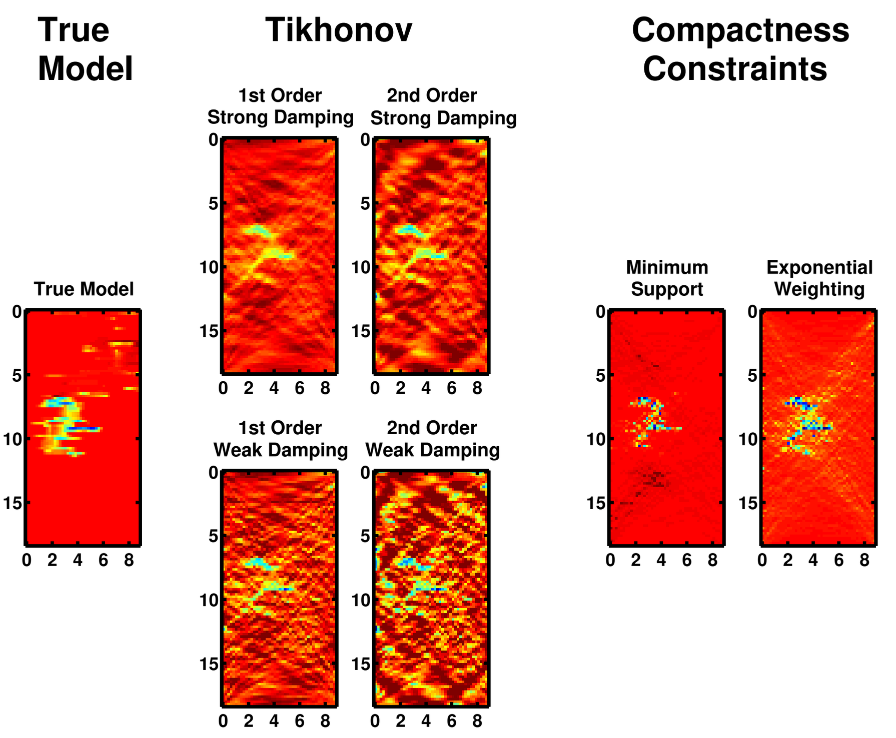
Synthetic Survey : Details

- Crosswell Geometry, 40 sources x 40 receivers
- 3% Gaussian noise added
- Straight ray synthetics (traveltimes)
- Note : main features lenticular pools

Tomographic Inversions : Details

- Reconstruction on 45 x 95 sample rectilinear mesh
- Inversion performed on differenced data (d3 – d2)
- Lambda values for compactness from T1, T2 solutions





Observations

Both compactness ops out–perform standard Tikhonov solutions, regardless of damping

Better suppression of artifacts
Closer match of pool properties

>5 non–linear iterations required for convg.
Search over multiple sigmas/betas
Factor of 5 to 10 more expensive

Convergence –

We always fit within tolerance
What's the right model norm?
Should we look at change in m?
Sometimes "too much" focusing

5. Conclusions

a) Compactness constraints seem to have advantages over traditional regularization assuming the target is, in fact, compact. Advantages include more quantitative estimates of anomaly shape and properties as well as suppression of artifacts due to survey geometry.

b) The traditional minimum support constraint and our heuristic exponential constraint have similar behavior for synthetic tests. The exponential constraint has one advantage, a more straight-forward approach to determining the regularization parameters.

c) Future work must include a better way of determining convergence for the non-linear IRLS iterations. Maybe through histogram analysis of model parameters (Hyndman and Harris 96)? A Bayesian interpretation of $\mathbf{W_e}$ (Clippard et. al., 95) might provide an interesting approach. The model-space IRLS might be more effective with a new class of constraints maximizing anomaly connectivity instead of compactness, a better match to flow effects.

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