# Using Spatially Integrated Crosswell Geophysics For Environmental Site Assessment

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## Motivation

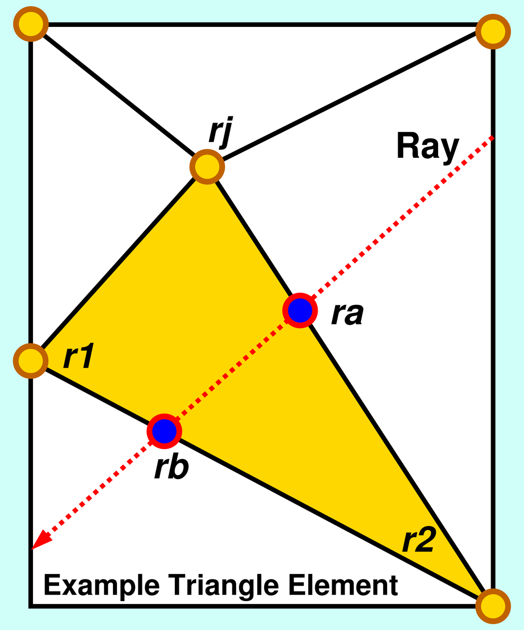
Crosswell seismic and radar techniques provide high resolution subsurface images with excellent depth control and quantitative estimates of acoustic and dielectric properties. We present an adaptive traveltime tomography flow which allows simultaneous inversion of multiple crosswell surveys with constraints on minimum model resolution for all inverted parameters. The proposed algorithm is tested on a set of crosswell surveys acquired at the Pinellas DOE facility, a site with known NAPL contamination. Simultaneous inversion of the entire site profile allowed examination of several features overlooked in the initial independent inversions.

## Formulation of Forward and Inverse Problems

Our formulation of the forward problem has several advantages over traditional cell based tomography parametrizations,

1. Mesh can be adapted to coverage or resolution constraints
2. Smooth basis functions allows a smaller number of parameters for the inversion

Although our current implementation only accomodates straight rays, a more sophisticated version using a semi–analytic shooting method is being developed.



Example Triangle Element

### A Tomographic System For Unstructured Trigonal Meshes

The model is parameterized with an unstructured triangular mesh. Inside of each triangular cell the slowness varies linearly. The slowness at any point $r$ is given by

$$s(r) = \sum_{j=1}^{N} s_j \phi_j(r)$$

where

$$\phi_j(r) = \begin{cases} \frac{(r-r_1)\times(r_2-r_1)\cdot e_3}{(r_j-r_1)\times(r_2-r_1)\cdot e_3}, & \text{if } r \text{ is inside of a triangle with vertex } r_j \\ 0, & \text{otherwise.} \end{cases}$$

$r_j$ is the position vector of node $j$;

$r_1$, $r_2$ are the position vectors of the other two nodes of the triangle containing $r$ and $r_j$;

$e_3$ is the unit vector normal to the plane.

The tomographic matrix

$$G_{ij} = \int_{\text{ray } i} \phi_j dl,$$

has a simple analytical expression in this kind of model,

$$G_{ij} = \sum_{\substack{\text{Cells that share node } r_j \\ \text{and are intersected by the ray i}}} \|r_a - r_b\| \frac{(\frac{r_a+r_b}{2} - r_1) \times (r_2 - r_1) \cdot e_3}{(r_j - r_1) \times (r_2 - r_1) \cdot e_3}$$

where $r_a$ and $r_b$ are the intersections of the raypath with the cell edges.

Once G, the Frechet derivative matrix is calculated, the tomography problem is solved using a regularized least squares formulation.

Anisotropic 1st order Tikhonov regularization is used to provide lateral continuity constraints (flattest model)

Regularization parameters are chosen by inspection.

$$\begin{bmatrix} G \\ \lambda_x D_x \\ \lambda_z D_z \end{bmatrix} m = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

LSQ Formulation

For the iterative refinement of the mesh (see next panel) the model resolution matrix is calculated using the natural generalized inverse of G. For large problems R is computed using the implicit Arnoldi algorithm. For smaller problems a dense SVD code is used. Future work will incorporate an efficient iterative scheme for calculation of R (see Fomel et.al.).

$$R = G^{-g} G \qquad m^{est} = R\, m^{true}$$

## Adaptive Mesh Refinement Algorithm

**General Idea : Use model resolution threshold to generate an adaptive mesh with quality guarantee**

Our method guearantees that every unknown is resolved to within a given limit. The technique is similar to that presented by B. Cox (see references) . However we use a new heuristic updating algorithm that generates higher quality meshes in most situations.

*Start With Coarse Mesh Enclosing S/R Array*

*While valid refinements exist*

- Compute delaunay tri. for point set
- Compute model resolution matrix
- If Even Iteration (Edge Refinement)
  - *Collect edges where both endpoints satisfy resolution criterion*
  - *Choose n longests valid edges – add bisection points to pnt. set*
- If Odd Iteration (Centroid Refinement)
  - *Collect triangles where all nodes satisfy resolution criterion*
  - *Choose n largest valid triangles – add centroid points to pnt. set*

*While coarsening locations exist*

- Compute delaunay tri. for pnt. set
- Compute model resolution matrix
- Scan nodes which do not fulfill resolution crit.
- Remove worst m nodes

Triangulation computed using Shewchuck's Triangle code (quality delaunay)

Computing R

R = v' v

For small problems Dense SVD

For large problems : Implicit Arnoldi Alg.

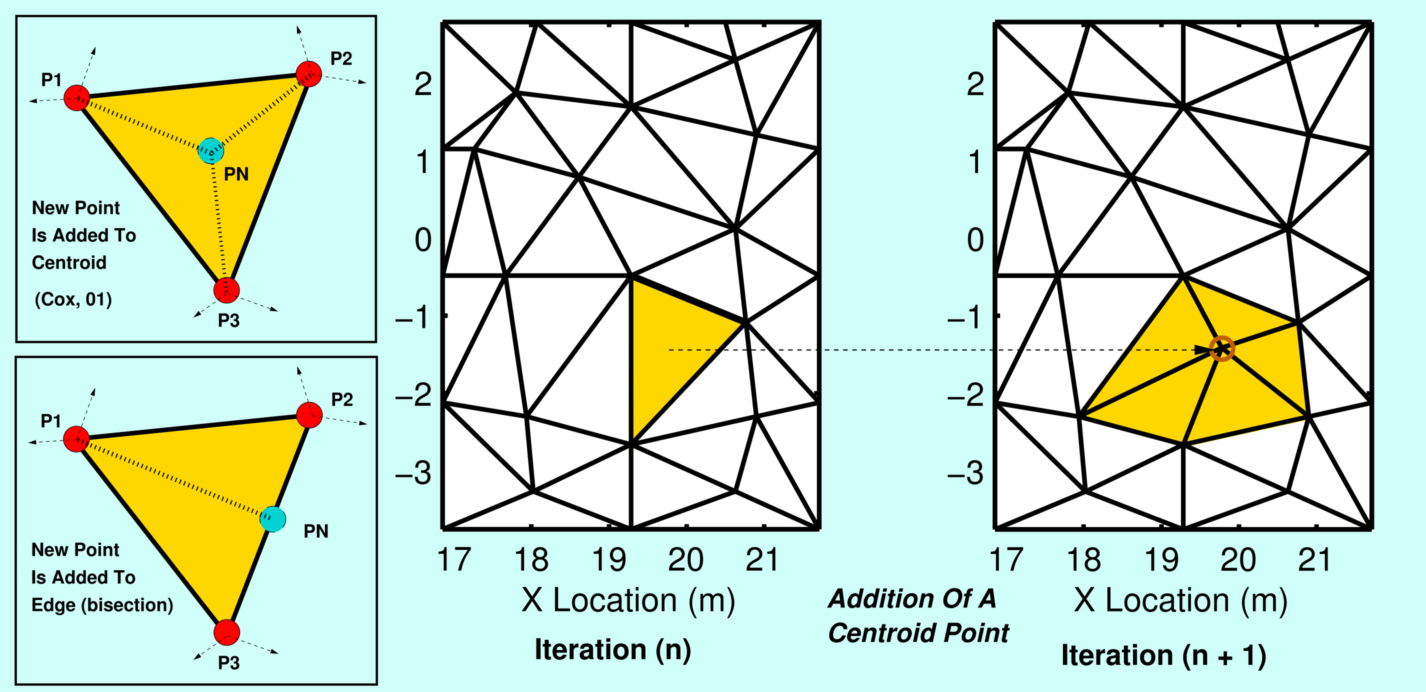
Future : Iterative Res. Estimate via Berryman's method

Resolution Criterion

diag(Ri) > threshold

***Final Mesh – Perform LSQR Tikh 1. Tomography***

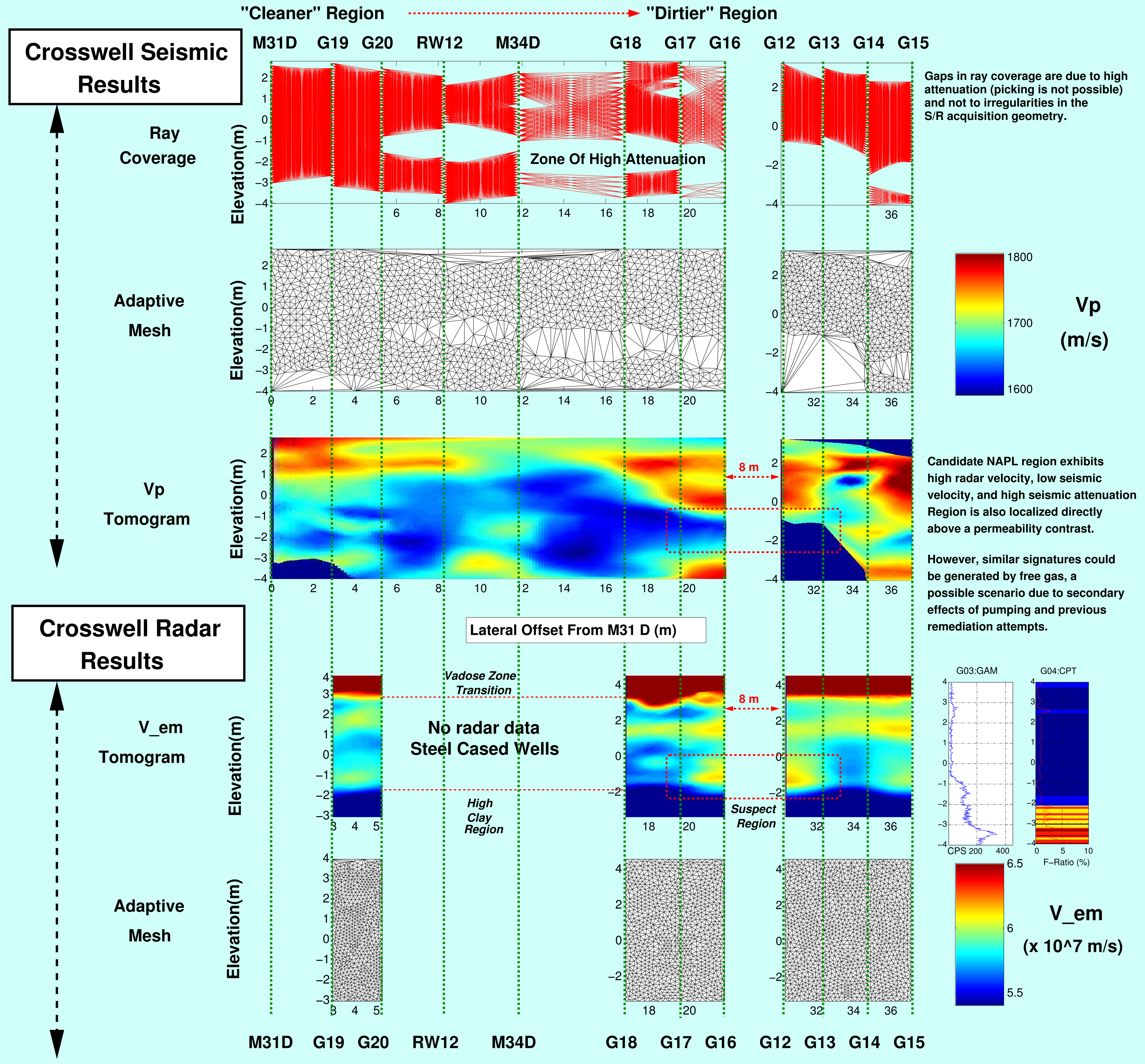
### Mesh Refinement Geometry

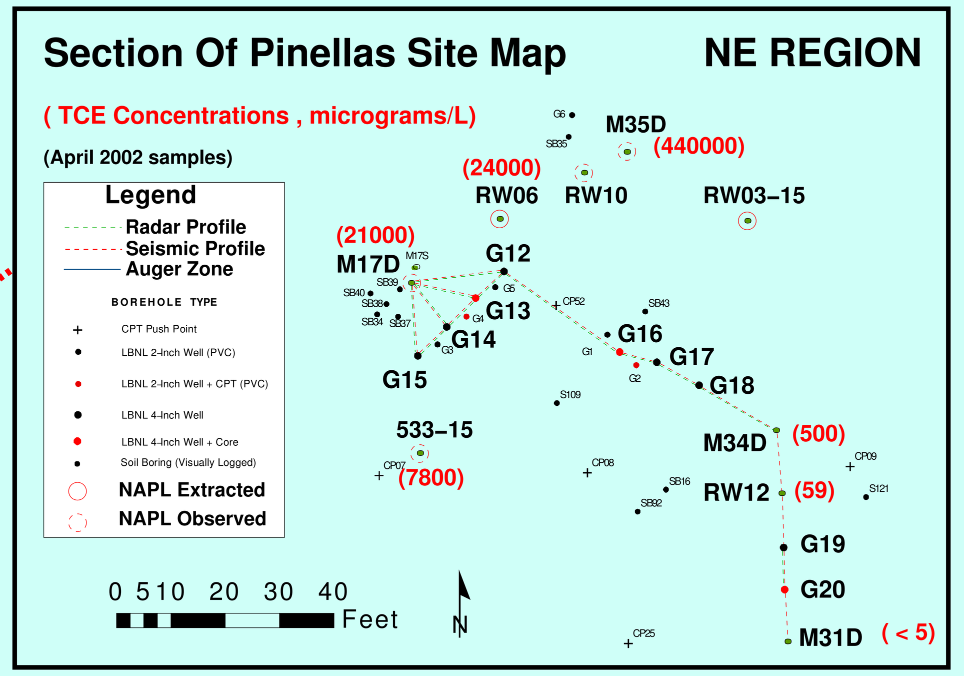



### Site Details

The DOE Pinellas site, located near Tampa, is a legacy manufacturing facility with extensive saturated zone NAPL contamination. A large set of crosswell seismic and crosswell radar surveys were acquired during 2002 across a profile spanning both a relatively clean zone and a region with suspected NAPL contamination. The imaging targets at Pinellas are hypothesized "pools" of dense contaminants localized at the bottom of the surficial aquifer. Candidate contaminant locations would exhibit high radar velocities, low seismic velocities, and possible seismic attenuation. Initial sequential processing of our traveltime data did not reveal any likeley features. After integrated reprocessing using our adaptive tomography flow, a possible region of DNAPL contamination has been identified.

## Spatially Integrated Crosswell Results : Pinellas DOE Site




## Results and Interpretation

The panel to the left depicts the application of our adaptive tomography flow to the Pinellas crosswell radar and seismic profiles. As is shown by the mesh generated for the seismic inversion, the adaptive gridding algorithm eliminates control nodes in regions with insufficient model resolution.

In zones with no ray coverage, the result is a conforming interpolation. For the primary seismic profile, 7 surveys with a total of 5905 picked rays were used for inversion on an unstructured mesh of 2400 unknowns. Since the SVD of G is computed to estimate the model resolution at each iteration of refinement, this problem is on the edge of what we can handle using dense SVD algorithms. Larger problems will require the development of more efficient iterative methods for calculating R

In the case of the radar tomogram the excellent ray coverage and absence of high conductivity regions allowed use of relatively uniform trigonal meshes.

The primary advantage we have observed in simultaneously inverting large multiwell profiles is the enhancement of laterally continuous features. Abrupt transitions between sequentially inverted crosswell images prevent easy interpretation of subtle variations in velocity. The suspect region of high radar velocity depicted to the left (between G16–G12) was obscured in previous independent inversions due to mismatch at the adjoining wells.We have found that tomographic algorithms based on adaptive meshing are stable and capable of handling large transitions in ray coverage, an inevitable component of large crosswell profiles. An alternative approach to handling the coverage problem might be a spatially varying regularization filter paired to a more traditional cartesian mesh.

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