Multilevel Upscaling of Heterogeneous Media

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Summary

A fundamental challenge in modeling flow through porous media is the need to resolve the multiscale structure of geological formations, which range in scale from millimeters to kilometers. Consequently, fully resolved flow simulations are likely to remain intractable for the foreseeable future; however, current upscaling techniques, which are used to generate computationally tractable coarse-scale models, are either extremely costly or fail to capture the true influence of fine-scale heterogeneous structure on the flow. To address this critical weakness, we explore a new multilevel upscaling (MLUPS) methodology that accurately and efficiently treats the multiscale properties of the underlying porous medium and flow model. This approach leverages key components of robust variational multigrid solvers to generate a complete hierarchy of coarse-scale models, which naturally defines multiscale basis functions that not only capture the influence of fine-scale structure on the coarse scale, but may also be used to recover additional fine-scale flow information as well.

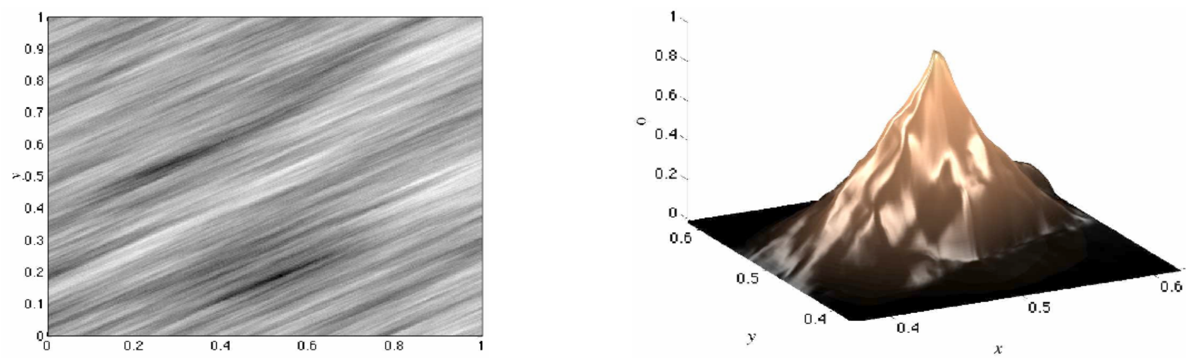

Figure 1: A randomly generated realization of a strongly heterogeneous permeability field with variation (from light to dark) of 6 orders of magnitude (top). Our multilevel upscaling algorithm constructs a self-consistent hierarchy of coarse-scale models for single-phase saturated flow, as well as the corresponding multiscale basis functions, without solving any local or global fine-scale problems. The multiscale basis function for the center of the domain, shown in the lower figure, was generated using this algorithm. The fine-scale structure is clearly visible in the surface, which accurately represents the influence of this structure on the flow.

In this research, we explore a new multilevel upscaling (MLUPS) methodology that accurately and efficiently treats the multiscale properties of the underlying porous medium and flow model [1]. MLUPS builds on the observation that the operator-dependent variational coarsening central to robust multigrid algorithms

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generates a complete and self-consistent hierarchy of coarse-scale models, along with their corresponding basis functions. In the MLUPS method, this hierarchy is created by coarsening the fine-scale discretization and with Dendy's Black Box Multigrid (BoxMG) to the desired computational scale. This operator-induced variational coarsening effectively reduces the dimension of the fine-scale operator by selecting an appropriate local, low-energy basis for the coarse scale. The coarse-scale model is solved, with this solution yielding a fine-scale representation via the multiscale basis functions. This approach provides a natural setting for adaptivity, error estimation, and extensions to more complex regimes such as unsaturated, multiphase, and reactive flows.

To demonstrate the accuracy and efficiency of this approach we consider a permeability field generated by the GSLIB software Package [2]. This field, shown in the first figure, has a range of permeabilities from approximately 10^{-3} (light) to 10^{3} (dark). A coarse-scale pressure gradient is imposed on a fine computational grid of 256 x 256 elements, with impermeable boundary conditions on the top and bottom edges, to induce flow from left to right. We compare the results of the MLUPS method with the current state of the art, the Multiscale Finite Element Method (MSFEM) [3], for a coarse computational scale of 8 x 8 elements.

Errors in both the average flux across the line $x=x_1$, $\mathbf{q}(x_1)$, and the pressure, $p(x,y)$, are shown in the table below. A 2048 x 2048 grid calculation, which predicts a constant flux in the x-direction of 1.13, is used to represent the true solution of the PDE, while an important benchmark is the bilinear finite element (BLFEM) solution on the 256 x 256 grid that indicates the "best" accuracy that we can, in general, expect at the fine computational scale. This computation takes 1.94 s on a 1.6 Ghz Athlon machine, only slightly more than the 1.88 s required by MSFEM, while the MLUPS computation requires only 0.18 s, less than one tenth of the MSFEM cost.

Errors in Flow Properties

Measure	BLFEM	MSFEM	MLUPS
$\|\|e(\mathbf{q})\|\|_\infty$	$2.96\text{x}10^{-2}$	$3.32\text{x}10^{-1}$	$2.08\text{x}10^{-1}$
$\|\|e(\mathbf{q})\|\|_2$	$2.96\text{x}10^{-2}$	$1.55\text{x}10^{-1}$	$1.06\text{x}10^{-1}$
$\|\|e(p)\|\|_\infty$	$1.20\text{x}10^{-2}$	$8.38\text{x}10^{-2}$	$9.52\text{x}10^{-2}$
$\|\|e(p)\|\|_2$	$8.92\text{x}10^{-4}$	$1.33\text{x}10^{-2}$	$9.58\text{x}10^{-3}$

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