

## **Outages as Rare Events in Information Transmission Systems**

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### **Summary**

*The most important parameter characterizing the performance of a communication system is bit-error rate. Low values of bit-error rate prohibit very expensive computer evaluation of statistical properties of this parameter using a standard Monte Carlo method. We showed that the probability distribution function of this parameter has long extended tails and verified it experimentally. These tails are characteristic of the outages in optical fiber transmission systems.*

In many systems, exceedingly rare events can have disproportionate impact. As a result, understanding and modeling such very rare events can be essential. For example, it is of critical importance to study the statistical characteristics of market crashes, even if their probability is very low. High magnitude earthquakes are quite rare; however, their impact and consequences are so serious that understanding their likelihood is of great practical value. Investigation of rare events is a challenging problem. Direct computer modeling is prohibitively expensive in resources required for accumulation of statistics of low-probability events. Theoretical modeling of such rare events is usually quite difficult due to the complexity of the underlying systems.

The performance of information processing, storage, and transmission systems is defined in terms of errors, which are very rare events. Current standards in information transmission systems require bit-error rates as low as  $10^{-12}$ . Such errors are consequences of noise in the system. We proposed an analytic method of evaluating

and controlling the statistical properties of rare events in the case of high-speed data transmission in optical fibers.

All of the different noise realizations contribute to the probability of an event. When the central portion of the distribution is studied, contributions from large numbers of different realizations must be taken into account. In contrast, the statistics of the rare events (far tails of probability distribution functions) is dominated by contributions of small numbers of (rare) noise configurations. These fluctuations are known as optimal fluctuations [Lifshitz, Brad Halperin]. The concept of optimal fluctuations was first worked out in the context of condensed matter physics; here investigation of rare events represents one of many problems studied. This concept is also very natural in information theory, where understanding rare events plays a central role. Recently it has been successfully applied to evaluating the efficiency of forward-error correction codes.

We studied the statistics of errors in optical fiber systems in the presence of temporal

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noise and spatial disorder. This optical system is an example of a problem with well-separated noise time scales. The separation of scales is a key ingredient that allows us to characterize the statistics of the errors in the system.

It was shown, first analytically and then experimentally, that, unlike what was previously thought, the bit-error rate fluctuates. Figure 1 shows experimental measurements of bit-error rate as a function of time, taken over the span of one hour. Close-up inspection of these data reveals that the nature of these fluctuations is not Poissonian, which is what one would expect if the probability of errors did not change in time.

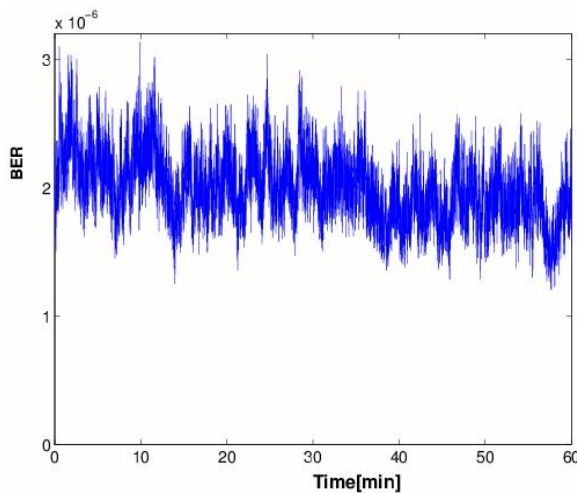


Figure 1: Bit-error rate changing in time.

Analytic consideration of this problem showed that, assuming Gaussian distribution for amplified spontaneous emission noise and structural disorder of the optical fiber, the errors are log-normally distributed. Such a distribution has “fat tails”, which means that large deviations from the mean are possible. Figure 2 shows the probability distribution functions for four different systems with different strengths of amplifier noise. Plotted on the log-log scale, these graphs demonstrate that the distribution

function of bit-error rate exhibits log-normal behavior, as predicted. The shift of probability the distribution function to the left with increase of temporal noise level is also in agreement with theoretical prediction. What is remarkable about Figure 2 is that decreasing the amplifier noise broadens the distribution function. This leads us to conclude that to improve the performance of the system it is not enough to decrease the amount of amplifier noise; the tails of distribution must be dealt with as well.

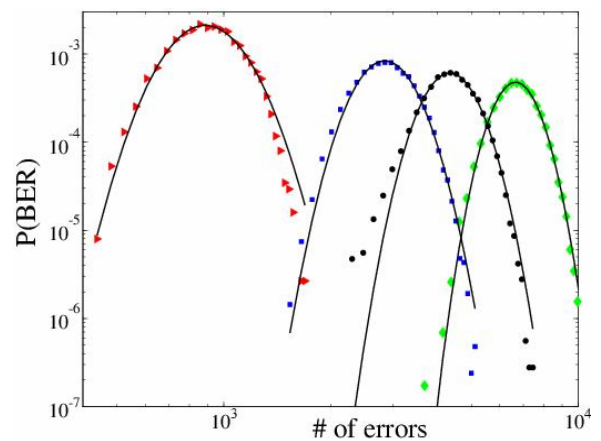


Figure 2: Probability distribution functions for four different systems. In red is the probability distribution function (PDF) for the system with smallest amplitude of spontaneous emission noise. In green is the PDF for the system with largest noise amplitude.

As a result of this work we can now understand better the statistical properties of fiber-optical communication system. These models have practical applications in improving of capacity of optical fiber links.

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