Multiscale Modeling of Multiphysics Subsurface flows

M. F. Wheeler, The University of Texas at Austin
Ivan Yotov, University of Pittsburgh

Summary

The permeability of a porous media can vary on multiple scales, making brute force flow simulations computationally intractable. To alleviate this burden we have developed a multiscale mortar mixed finite element method for modeling complex multiphase subsurface phenomena. The method is based on domain decomposition and mortar finite elements. The solution is resolved globally on a coarse scale and locally on a fine scale. Error estimators have been derived for obtaining optimal coarse and fine grids. The method scales on massively parallel machines and is extendable to nonlinear multiphysics flows.

Water is of paramount social and economical value and its availability and use will considerably influence the development of our societies. A sustainable management and protection of water in the environment is one of the key problems of the 21st century, and numerical modeling will contribute considerably to its solution. For example, natural attenuation is used in tens of thousands of contaminated sites in the United States in place of or in conjunction with engineering remediation systems. Other important examples of complex hydrosystems include migration and storage of carbon-dioxide in the subsurface and atomic waste disposal sites. Hydrosystems consist of different regions or subdomains, e.g. the saturated and unsaturated zones. Different processes may occur within the different regions. These relevant processes may need to be considered on different spatial and temporal scales, and may require different models and data. Furthermore taking into account mutual interaction processes between the subdomains, coupling mechanisms must be employed in a mathematically and physically meaningful fashion in order to obtain a global solution.

To model these complex subsurface phenomena we have developed a multiblock methodology. The simulation domain is decomposed into subdomains. The governing equations are imposed locally on each subdomain and physically meaningful boundary conditions are imposed on interfaces via mortar finite element spaces. Some computational advantages of this approach include (1) **Multiphysics**, different physical processes/mathematical models in different parts of the domain may be coupled in a single simulation; (2) **multinumerics**, different numerical techniques may be employed on different subdomains; (3) **multiscale resolution and adaptivity**, highly refined regions or fine scale models may be coupled with more coarsely discretized regions, and dynamic grid adaptivity may be performed locally on each block; and (4) **multidomains**, highly irregular domains may be described as unions of more regular and locally discretized subdomains with the possibility of having interfaces with non-matching grids. The multiblock approach leads to a naturally parallelizable algorithm of the domain decomposition type. Figure 1 illustrates a parallel multiblock decomposition for modeling groundwater remediation or enhanced oil recovery. Here grid flexibility allows for application of advanced well models.

*512-475-8626, mfw@ices.utexas.edu; 412-624-8338, yotov@math.pitt.edu

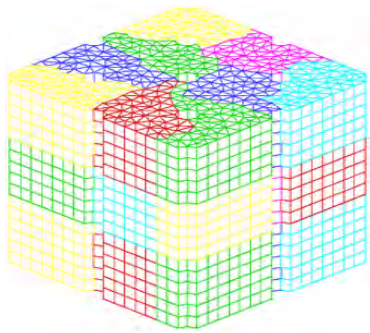

Figure 1. Multiblock decomposition on unstructured grids of a reservoir with multiple wells.

Since permeability of the porous media can vary on multiple scales, brute force simulations are computationally intractable. To alleviate this burden we have developed a multiscale mortar mixed finite element method for modeling subsurface flow [1,2]. This algorithm is built upon a multiblock formulation, in which the domain decomposition represents a coarse grid. The solution is resolved locally on each subdomain on a fine scale, while fluxes are matched on the interfaces using a coarse scale mortar space. By using a higher order mortar approximation, we are able to compensate for the coarseness of the grid scale and maintain good (fine scale) overall accuracy.

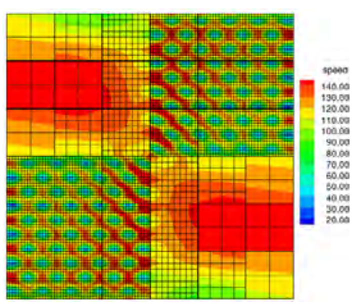

Figure 2. Magnitude of velocity in adaptive mesh refinement computation for a problem with a highly oscilatory coefficient. The solution oscilates on a finer scale in the lower left and upper right regions.

This approach is related to, but more flexible than existing variational multiscale methods, since it is easy to improve global accuracy by simply refining the local mortar grid where needed. That is, we can easily exploit adaptive meshing strategies based on a posteriori error estimates to improve where necessary the strength of the global coupling. This is illustrated in Figure 2 for a problem with highly oscilatory coefficient.

An additional advantage of the mortar multiscale method is that it can be implemented as a domain decomposition algorithm, which scales on massively parallel machines and allows for coupling of nonlinear multiphysics problems. A parallel application to compressible flow is demonstrated in Figure 3.

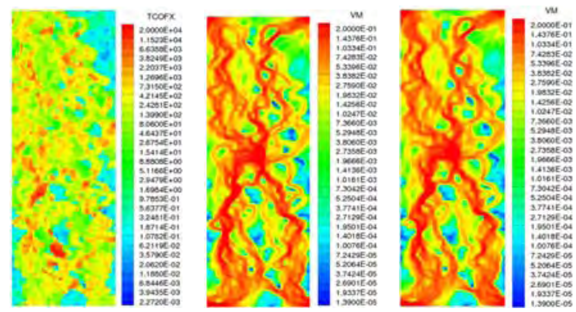

Figure 3. Log of velocity magnitude in a compressible flow simulation for SPE10 fluvial reservoir. Left: permeability field; middle: fine scale solution; right: multiscale mortar solution.

References

[1] T. Arbogast, G. Pencheva, M. F. Wheeler, and I. Yotov, *A multiscale mortar mixed finite element method*, SIAM J. Multiscale Modeling and Simulation, 6:1 (2007) 319-346.

[2] M. F. Wheeler, G. Pencheva, S. G. Thomas and I. Yotov, *Multiscale mortar mixed finite element method for multiphase flow in porous media,* Proceedings of ECMOR X, Amsterdam, the Netherlands, Sept. 2006.

For further information on this subject contact:

Dr. Anil Deane
Applied Mathematics Research Program
Office of Advanced Scientific Computing
Phone: 301-903-1465
deane@ascr.doe.gov