

## Multiscale Mathematics For Plasma Kinetics Spanning Multiple Collisionality Regimes

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### Summary

*This project is developing more efficient kinetic algorithms for the simulation of plasma systems that span a wide range of collisionality. Kinetic modeling of such plasmas is highly challenging, is in need of improved algorithms, and is key to understanding plasma systems that underlie controlled fusion (magnetic and inertial), semiconductor processing, and space plasmas. We have demonstrated that such hybrid algorithms have excellent accuracy and a significant speedup relative to existing algorithms for the collisional relaxation of a velocity distribution that represents a beam injected into an ambient plasma. We are continuing to refine, extend, and optimize such algorithms on a succession of more realistic paradigm plasma physics problems.*

Kinetic modeling is needed to understand moderately collisional or “near-continuum” (NC) plasmas. In a NC plasma, collisions are frequent for most of the particles of one or more charged particle species, but subpopulations of that species important to key phenomena are present for which collisions are sufficiently weak that a fluid description fails. NC plasmas are a key component in various laboratory and technological situations such as controlled fusion (both magnetic and inertial) and semiconductor processing, and in natural settings, e.g., space plasmas.

Many phenomena in NC plasmas are inaccessible to kinetic modeling for two reasons. The main dependent variable in kinetic modeling is a density defined on a position-velocity phase space. This phase space has roughly twice the number of

dimensions than the configuration space on which fluid variables would be defined, thus requiring many more (by a factor ranging from 10 to more than  $10^6$ ) degrees of freedom (mesh nodes or simulation particles) when discretized than in a fluid model. Present kinetic models also require time steps significantly shorter than a typical collision time. This poses a difficulty since a significant fraction of the NC species particles have collision rates much more rapid than the phenomena of interest.

We are developing accelerated hybrid (fluid+particle) kinetic algorithms that show promise to greatly reduce the computational resources needed for, and significantly expand the range of problems accessible to, kinetic modeling of NC plasmas.

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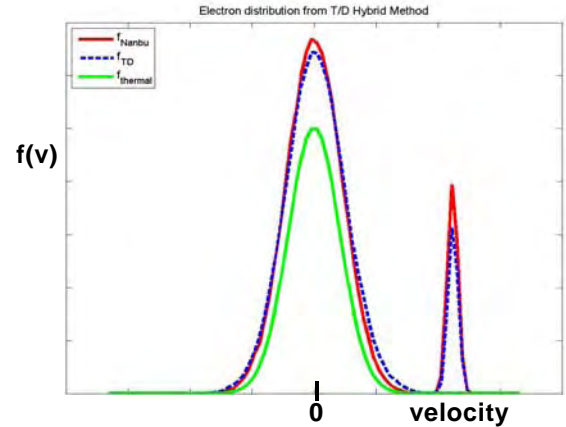
Two new classes of hybrid-kinetic schemes have been formulated. The first is similar to the interpolated-fluid Monte-Carlo (IFMC) scheme successfully used in rarefied gas dynamics (RGD). The second scheme is a hybrid of a  $\mathcal{F}$  particle-in-cell (PIC) Monte-Carlo (MC) method with a fluid method.

We have derived the key (kinetic + fluid) equations for both schemes. The fluid equations are amenable to standard compressible fluid schemes, as they resemble the standard compressible Euler equations, albeit with additional sources that represent their closure by solutions of the associated kinetic equations.

The kinetic equations are tailored to specific PIC-Monte-Carlo (MC) methods. The IFMC-like algorithm uses a full PIC-MC scheme, with fixed-weight particles. These may be created or destroyed both due to physical sources and sinks and through “thermalization/dethermalization” (TD), which passes their content (mass, momentum, energy) to/from the fluid part of the system. The TD step limits the simulation particle population to those parts of phase space where it is needed, and gives correct behavior to those particles for which the change in velocity due to collisions is their most rapid change.

We have implemented a TD-hybrid algorithm (see *Fig. 1*), and have demonstrated excellent accuracy and a 3-to-4-fold speedup relative to existing kinetic algorithms on a key test problem. We expect that further refinements will result in significant additional speedups at given solution accuracy.

We are implementing and testing these hybrid algorithms on, and have initiated simulations of, a succession of more realistic paradigm plasma physics problems: (a)



*Fig. 1. Velocity distribution function from hybrid (blue) and standard Monte-Carlo (red) simulations of relaxation by Coulomb collisions of a “bump-on-tail” distribution. The hybrid solution agrees well with the standard solution. The particle component of the hybrid solution (difference between the blue and green curves) is a factor of  $\sim 4$  (representing computational savings) smaller than the total solution.*

collisional temperature equilibration between parallel and perpendicular temperatures, (b) collisional relaxation of a velocity distribution that represents a beam injected into an ambient plasma “bump-on-tail” velocity distribution, (c) a collisional electrostatic sheath, (d) nonlocal collisional electron thermal transport along magnetic field lines, and (e) a collisional ion-acoustic wave.

We have also completed a careful computational convergence study (with a manuscript submitted to the *Journal of Computational Physics*) of the (Takizuka-Abe and Nanbu) Coulomb collision algorithms that form a key component of our hybrid schemes.

**For further information on this subject contact:**

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