A novel least-squares method for mesh tying

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Summary

In the finite element method, a standard approach to mesh tying is to apply Lagrange multipliers. If the interface is curved, though, discretization generally gives rise to adjoining surfaces that do not coincide spatially. In this case, standard Lagrange multiplier methods have difficulty passing even a first-order patch test. We developed a new mesh-tying method that uses least-squares minimization ideas to joint the subdomains. A unique feature of the new method, that is not present in any other mesh-tying approaches, is that it passes a patch test of the order of the finite element space by construction.

Mesh tying, or domain bridging methods are the opposite of domain decomposition (DD). A DD method solves a boundary value problem using subdomains formed by clustering finite elements from a given discretization of a domain Ω. A mesh tying method solves the same problem by using a discretization of Ω, composed of subdomains that were meshed completely independently. This computational setting arises in modeling and simulation of complex engineering structures in which the bottleneck, as measured in calendar time, is mesh generation. One example is certification of aerospace structures where creating a monolithic mesh is hugely impractical and time consuming. In such cases, for practical and efficiency reasons, grid generation on Ω is replaced by independent meshing of its subdomains; see Fig.1. Other examples that lead to mesh tying settings include transmission, contact, and domain-bridging problems.

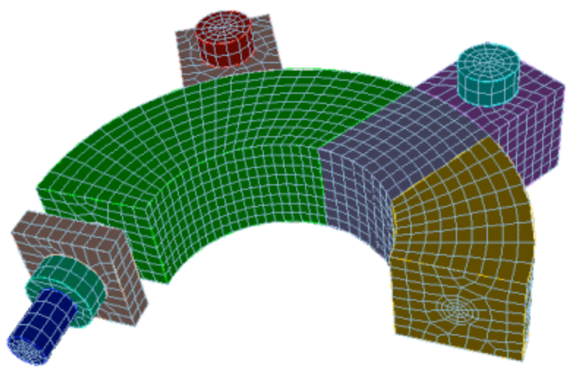

Fig. 1. A typical mesh-tying setting.

A major difficulty in mesh-tying problems arises when independent meshing of a single curved interface gives rise to two discrete non-matching surfaces.

Typically, mesh-tying methods rely on Lagrange multipliers to join together the subdomains. However, this approach is prone to difficulties when independent meshing of a single curved interface gives rise to two non-matching discrete surfaces. In this case, the multipliers can only be defined on one of the surfaces and the matching condition requires a projection operator, or additional meshing between the non-coincident interfaces. Besides being

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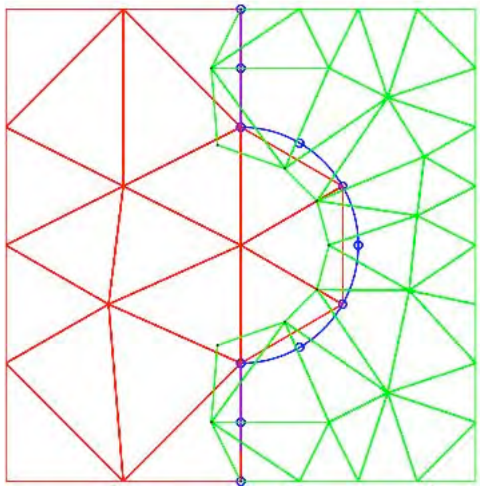

Fig.2. Interface perturbations give subdomains with no gaps between them. The solid line is the interface σ. The domain $\Omega = [-1,1]^2$ has two overlapping subdomains. Only the interface nodes on the right domain are perturbed. The original interface node locations are marked with circles.

computationally challenging, this approach is not guaranteed to pass even a first-order patch test, which is considered to be a minimal requirement for such methods.

Our approach for dealing with non-matching interfaces utilizes least-squares principles and extends a least squares finite element method to mesh-tying configurations with non-matching interfaces. A least squares functional is defined as the sum of the residuals of the differential equations measured in Sobolev space norms. As a result, such a functional always vanishes at the exact solution. By exploiting this property, we formulated a least squares method for mesh tying that automatically passes a patch test of the same order as the finite element space employed in its definition. We start by perturbing the discrete interfaces until there are no void regions between the subdomains; see Fig.2. Then, least-squares principles for each subdomain are joined together by generalized jump terms defined on the overlap region between the subdomains. This resembles the approach used in the Arelquin method, however, by measuring residual energy and not physical energy, a least squares functional may measure energy redundantly in subdomain intersections. This fact greatly simplifies our algorithm. In contrast, methods that minimize physical energy, subject to appropriate constraints on the interfaces, require special efforts to avoid counting energy twice in the overlap regions.

Figure 3 shows an example finite element solution computed with the new mesh-tying method, using the region from Fig.2. Besides being able to pass a patch test of an arbitrary order, our new method does not require interface projection operators or additional meshing between the non-coincident interfaces.

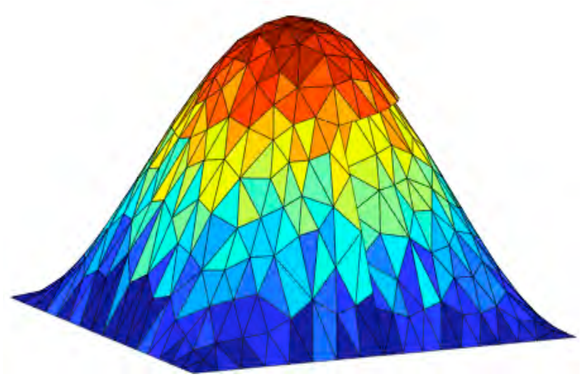

Fig.3 Finite element approximation of $\varphi(x,y)=\cos(\pi x/2)\cos(\pi y/2)$ by the least squares mesh-tying method using the overlapping subdomains from Figure 2.

For further information on this subject contact:

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