"A Dynamic Multiscale Viscosity Method for Conservation Laws"

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Summary

We consider the spectral approximation of a conservation law in the limit of small or vanishing viscosities. In this regime, the continuous solution of the problem is known to become chaotic (turbulence) or to develop sharp spatial and temporal gradients referred to as shocks. Also, the standard Fourier–Galerkin solution is known to break down if the mesh parameter is large. We have developed a new dynamic, multiscale viscosity method that enables the spectral solution of such systems with relatively coarse discretizations. The key features of this method are: (1) separate viscosities are applied to the coarse and the fine scale equations; (2) these viscosities are determined as a part of the calculation (dynamically) from a consistency condition which must be satisfied if the resulting numerical solution is optimal in a user-defined sense. We have applied the proposed method to Burgers equation and Navier Sotkes equations and found that it yields very accurate results.

We have developed a numerical method for the spectral approximation of non-linear conservation laws. These laws describe a broad range of physical phenomena that includes the dynamics of gasses, the flow of traffic and the propagation of shallow water and non-linear acoustic waves. In all these systems we are interested in cases when the physical viscosity (or diffusivity) is small or zero. In the small viscosity case the solution to such systems becomes chaotic or develops local regions of large spatial and temporal gradients called shocks. The width of a shock reduces with reducing viscosity, and in the limit of zero viscosity the solution becomes discontinuous. In fact, in this limit in order to ensure unique solutions, the conservation law must be supplemented with an entropy production inequality and conditions that relate jumps in conserved quantities across the shock.

For small viscosities, the standard Fourier Galerkin approximation to non-linear conservation laws becomes unstable if the shock width is smaller than the grid size. For a large class of problems the computational cost of employing a grid which is fine enough to resolve a shock is prohibitive and as a result this method finds limited application. Further, in the limit of zero viscosity, even with sufficient grid refinement, the Fourier–Galerkin solution does not converge to the unique ''physical'' solution that satisfies the entropy production inequality. To overcome these difficulties associated with the Fourier Galerkin method, several methods have been proposed. A large proportion of these

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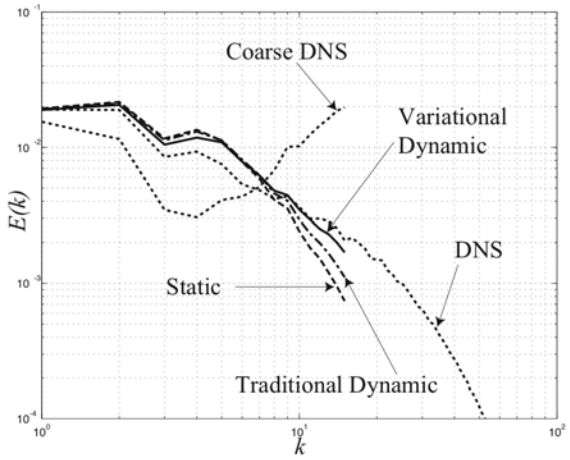
methods involve appending to the Fourier–Galerkin formulation a numerical viscosity term.

In several popular methods, such as the vanishing viscosity method, that guarantee the convergence of the numerical solution to the unique entropy solution, the numerical viscosity is applied to both the coarse and the fine scale equations. On the other hand, in the vanishing spectral viscosity method proposed by Tadmor, the viscosity is applied only to the fine scale equations. As a result, this method retains the spectral accuracy of the coarse or the large scale modes while guaranteeing convergence to the entropy solution. Motivated by the class of methods where the viscosity appears only in the fine scale equations, we propose a method where different numerical viscosities appear in the large and the small scale equations. In addition, in contrast to the methods described above, these viscosities are not determined *a-priori*, instead they are calculated as part of the solution (dynamically). The equations that are used to determine the viscosities are derived from the condition that the resulting numerical method be optimal in a certain user-defined sense. We call this method the dynamic multiscale viscosity method.

We remark that the equation used to dynamically determine the viscosities, is in effect the variational counterpart of the Germano identity. This identity has found widespread use in determining model parameters in the LES of turbulent flows. Recently, we have demonstrated how it may be used as a tool for determining unknown parameters in a numerical method aimed at solving an abstract partial differential equation. In Figure 1, we demonstrate the performance of this method (called Variational Dynamic) in predicting the turbulent energy spectrum of decaying of homogeneous isotropic turbulence. It is evident that is much more accurate than no model (called coarse DNS) the static and dynamic Smagorinsky methods (referred to as Static and Traditional Dynamic). In this figure DNS refers to the benchmark direct numerical simulation computed on a very fine grid.



Energy Spectrum for Decay of Homogeneous isotropic turbulence.

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