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Super-rotation of Venus atmosphere- exactly-solvable model"

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Summary

Venus' middle atmosphere – a 40km layer of greenhouse gas – rotates almost like a solid top once every 4 earth days while the planet itself rotates once every 260 earth days. The formation and robustness of super-rotation defies an easy explanation- naively speaking, friction in the planetary boundary layer should have dissipated the super-rotation long ago. A predictive model for this mystery should be based on the notion of a steady-state involving primarily energy and angular momentum balance. Highlighting our work on the statistical mechanics of planetary atmospheres, is a recently solved lattice model for a coupled atmosphere-planet system that provides qualitative predictions of transitions to super-rotation in terms of a few key planetary parameters such as its rate of spin, and the density, thickness and enstrophy – a measure of vorticity - of its atmosphere. A fundamental question answered here in the affirmative is whether a solvable model can be found that is realistic enough to include the principles of energy and angular momentum conservation in a coupled atmosphere-planet system, and powerful enough to characterize the asymmetry between very high-energy super-rotating and very low- energy counter-rotating states.

Energy balance alone cannot account for the formation and maintenance of super-rotation - the super-rotating atmosphere has to extract angular momentum from the massive rotating planet through an unknown combination of topographical torque and Hadley's circulation. The full complexity of this mechanism is beyond the reach of today's computational capabilities**.**

Instead, we seek a tractable model based on statistical mechanics that will provide qualitative predictions of most probable end-states in terms of a few key parameters in the initial states. To be relevant to the Venus problem, we incorporate the fundamental principles of the conservation of angular momentum and energy of the coupled system, and use the rest-frame kinetic energy H of barotropic flow coupled to an infinitely massive rotating sphere – this is the simplest energy functional recognized by geophysical fluid dynamicists that does not fix angular momentum of the flow.

However it is known that this energy H alone does not yield a well-defined Gibbs canonical formulation of statistical mechanics. It turns out from arguments based on vanishing of the forward enstrophy cascade in the interior flow in the zero viscosity limit, that a natural constraint is to microcanonically fix the entrophy – a measure of vorticity in the atmosphere.

Evidence that such a tractable and realistic model exists – see figure (where blue/red denotes negative/positive relative vorticity) of a near solid-body sub-rotational end-state at very low energy levels - came from the extensive Monte-Carlo simulations and approximate analyses performed by the PI's team at RPI and NUS – J. Nebus [2], X. Ding [3] and R. Singh Mavi [4].

Condensation of random vorticity at extreme energies into macroscopic angular momenta – shown by exact solution of a Kac-Berlin

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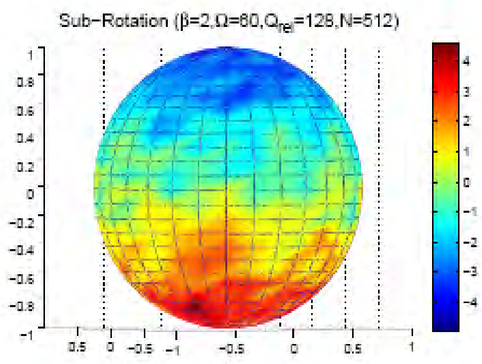
spherical model - occurs through phase transitions because the spin-lattice model of the fluid component is coupled by torque to a massive solid sphere. Such coupling allows the solid substrate to absorb / contribute the angular momentum required by conservation laws. Intuitively, as pointed out earlier, this coupled system can be viewed as classical 2D fluid motion restricted to the surface of a solid sphere with which it exchanges energy and angular momentum. Thus, spontaneous organization of random local vorticity of the fluid (with initial total angular momentum set to zero) into a macroscopic rotational flow with nonzero angular momentum occurs, compensated according to conservation of angular momentum, by an opposing rotation of the solid sphere, which changes the duration of the Venusian day. This was observed.



The key to an exact solution of this lattice model is that fixing the enstrophy - a large (high dimensional) sum of the squares of local vorticity or spins – is equivalent to a spherical constraint on state space, which makes the partition function – a huge sum over states – integrable in the thermodynamic limit by the saddle point method [2].
Further qualitative confirmations in the Venus, Titan and Jupiter problems of the predictive capabilities of this lattice model will consist of the asymmetry in the transitions predicted for very low-energy sub-rotating and very high-energy super-rotating states – the super-rotational transition occurs for all values of planetary spin and enstrophy, but the sub-rotational transition occurs only when the planetary spin is large compared to the atmosphere's enstrophy; Venus spins very slowly (once in 260 earth days) and its atmosphere super-rotates instead of sub-rotating.

In conclusion, we highlighted here the exact analytical solution of a spin-lattice model based on the atmospheric kinetic energy H [1] and discuss the potentially profound impact of its phase transitions on the study of planetary atmospheres, including future extensions to richer divergent models for Jupiter's spots and jets and the double vortex on Venus' south pole that was discovered in the European Space Agency's 2006 Venus Express mission. These collection of results have been announced in several invited talks including a Plenary Talk at the IUTAM Moscow 2006.

[1] C.C. Lim**, A Classical version of the Einstein – de Haas Effect,** submitted 2006**.**
[2] C.C. Lim and J. Nebus**, *Vorticity, Stat Mech and Simulations,* Springer-Verlag 2006.**
[3] X. Ding and C.C. Lim, **Physica A** 374, 152-164, 2006.
[4] C.C. Lim and R. Singh Mavi, **Physica A** 380(1), 43-60, 2007.

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