"Suppression of magnetorotational instability (MRI)"

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Summary

Two important problems in the suppression of magnetorotational instability (MRI) in liquid metals have been addressed successfully by MHD stability analysis: (1) The small Prandtl number approximation with a constant applied axial magnetic field, (2) A constant applied axial current. Researchers at the Princeton Plasma Physics Laboratory (PPPL) are working to perform a series of liquid metal Couette flow experiments. Their purpose is to produce MRI, which, though they have predicted it theoretically, has not been observed in the laboratory so far. The theoretical results we have obtained indicate experimental regimes in which not to expect MRI.

Magnetorotational instability (MRI) is important to theoretical astrophysics because it is the only linear instability known to grow robustly under the conditions prevailing in most accretion disks: an electrically conducting rotating fluid, with local angular velocity $\Omega(r)$; a positive gradient of specific angular momentum, $\partial(r^2\Omega)^2/\partial r>0$; and a negative gradient of angular velocity $\partial\Omega^2/\partial r<0$. MRI was originally conceived as an ideal MHD instability but liquid metals are far from ideal on laboratory scales, especially in their magnetic diffusivity (η), which is typically $\sim 10^6$ times larger than their kinematic viscosity (ν). This makes MRI experimentally challenging. Until recently, the literature on liquid-metal Couette flow has treated magnetic effects as modifications to the Taylor instability, in which viscous and inertial forces are comparable and $\partial(r^2\Omega)^2/\partial r<0$. In this regime, as first shown by Chandrasekhar in 1953, the equations of motion can be scaled so that terms proportional to the magnetic Prandtl number $P_{m} \equiv \nu/\eta \sim 10^{-6}$ are manifestly negligible, permitting a reduction of the axisymmetric stability analysis from tenth to eighth order in radial derivatives. Chandrasekhar's "small-P_{m}" approximation governed essentially all analyses of magnetized Couette flow for the following forty years; none of these works predicted MRI, nor did contemporary experiments observe it. Since then, significant theoretical work by Goodman& Ji (2002) of the Princeton Plasma Physics Laboratory (PPPL), has provided this theory and buttressed the contention that MRI is physically realizable in the laboratory. The figure shows an early version of the apparatus in which water was used. Currently, they are at the final stage transitioning to liquid gallium in the experiment.

Two significant pieces of work have already resulted from the collaboration between the

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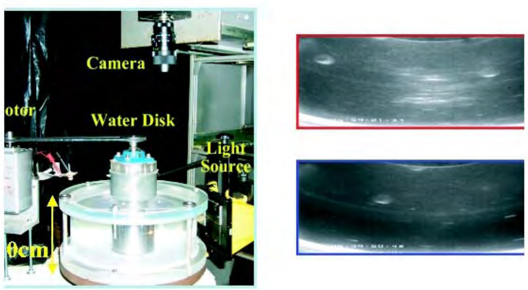
PI and the PPPL group. [1] The work of Goodman and Ji was extended. We were able to prove mathematically that the small magnetic Prandtl number approximation suppresses MRI. The result may be described as follows. Axisymmetric stability of viscous resistive magnetized Couette flow was re-examined, with emphasis on flows that would be hydrodynamically stable according to



Rayleigh's criterion: opposing gradients of angular velocity and specific angular momentum. In this regime, MRI may occur. The governing system in cylindrical coordinates is of tenth order. It was proved, by methods based on those of Synge and Chandrasekhar that by dropping one term from the system, MRI is suppressed; in fact no instability at all occurs, with insulating boundary conditions. This term is often neglected because it has the magnetic Prandtl number, which is very small, as a factor; nevertheless it is crucially important.
The outcome of the present paper was to extend results to wide-gap Couette flows. That is, we proved that Chandrasekhar's reduced system of equations predict stability when $\partial(r^2\Omega)^2/\partial r>0$, at least for insulating magnetic boundary conditions, which are particularly relevant to experiments. The proof made use of insights and techniques developed by Herron (2004) to treat the Rayleigh unstable regime.
[2] In collaboration with a graduate student, F. Soliman, we were able to establish the linear stability of Couette flow in a toroidal magnetic field. This result is described as follows. The stability of the hydromagnetic Couette flow was investigated when a constant current is applied along the axis of the cylinders. It was proved mathematically that if the resulting toroidal magnetic field depends only on this current, no linear instability to axisymmetric disturbances is possible. This was demonstrated using a method of quadratic functionals popularized by Chandrasekhar, originally used by Synge. Along with this, an operator notation was introduced, which aided in keeping track of the functionals involved.

References
[1] "The small Prandtl number approximation suppresses magnetorotational instability" *Journal of Applied Mathematics and Physics* (*Z. Angew. Math. Phys.*) 57, no. 4, 615-622 (2006) (with Jeremy Goodman).
[2] "The stability of Couette flow in a toroidal magnetic field", *Applied Mathematics Letters*, 19, 1113-1117 (2006) (with Fritzner Soliman).
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