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"Fast Discrete Fourier Transforms on Polar Grids and Their Applications"

G. Beylkin[1], C. Kurcz[1], L. Monzon[1] and G. Fann[2]
University of Colorado at Boulder
Oak Ridge National Laboratory

Summary

New fast discrete Fourier Transforms and their adjoints to map a square in space to a disk in the Fourier domain were developed. Unlike the Discrete Fourier Transform (DFT) the new transforms are not unitary and have a numerical null space; they also, in special situations, permit the use of adjoint in lieu of inverse transform. As a part of this work a general approach for constructing useful analogues of the FFT to and from a polar grid in the Fourier domain was derived. These transforms and their associated grids arise in many applications such as X-ray tomography, linearized inverse scattering, synthetic aperture radar, and signal processing. In all of these applications band-limited functions play an important role, and we anticipate these new transforms to play a role similar to FFTs as a standard tool for computing Fourier integrals. We are investigating the application of these methods.

Fast Fourier Transform (FFT), an efficient algorithm for computing the DFT and its inverses, is recognized as one of the most important algorithms of computational science. However the traditional DFTs are restricted to mapping a square region in space to a square region in the Fourier domain. Furthermore, the usual FFT methods are algebraically exact and, in some instances, this imposes significant limitations on their use.

To address such limitations, a number of authors have developed unequally spaced FFTs [1, 2, and 3] for treating non-uniformed discretization of the Fourier transform. Recently, a new method for constructing a pseudo-polar grid in a square was developed [4]. However, it is not possible to perform high order interpolation using the pseudo-polar grid and the inversion algorithm still requires data in a square region in the Fourier domain. In [5] we derive a new fast discrete Fourier transform (approximate but with controlled accuracy) from a square in the spatial domain to a disk in the Fourier domain. Such transform is needed in many applications and a critical contribution of this work is in developing algorithms with a full accuracy control at nearly minimal computational cost.

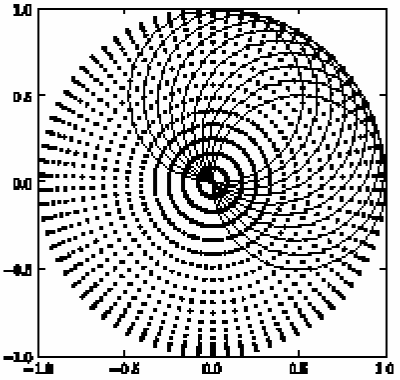

Fig. 1: A special grid on rotating circles in the Fourier domain. This grid is constructed by placing equally spaced nodes (37 in this

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[*] 865-576-2374, fanngi@ornl.gov

example) on the circles rotated full 360 degrees around the origin in the same number of steps. Remarkably, the resulting nodes line up on the diameters of the disk.

Constructing such transforms lead to several additional areas of applications. In particular, we developed algorithms that use

1. Grids which provide quadratures and interpolation with controlled accuracy for band-limited functions within a disk in the Fourier domain.

2. Special grids in the Fourier space (motivated by inverse scattering problems) that are excellent for trigonometric interpolation on rotating circles within a disk.

Furthermore, we developed grids adapted to particular radially symmetric kernels which we use as a part of an algorithm for fast application of the oscillatory Helmholtz kernel. This work is forthcoming.

We expect a number of applications in which these new algorithms should have a significant impact. In particular, using new grids in Magnetic Resonance Imaging (MRI) should result in reduced data collection time. Similarly, in other methods of non-destructive evaluation, e.g., cone-beam tomography, electron microscope tomography, synthetic aperture radar, X-ray tomography, etc., the near optimal design of grids and accuracy control should result in an improved performance of reconstruction algorithms.

We are currently investigating several applications of this new methodology.

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For more information, contact:

Dr. George Fann
Oak Ridge National Laboratory
Phone: (865) 576-2374
fanngi@ornl.gov

Or

Dr. Anil Deane
Applied Mathematics Research Program
Office of Advanced Scientific Computing
Phone: 301-903-1465
deane@ascr.doe.gov

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[*] 865-576-2374, fanngi@ornl.gov