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# *"Interior Point Methods for Very Large Nonlinear Optimization Problems"*

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## Summary

***In many large-scale optimization problems, it is not possible to form and factor Jacobian or Hessian matrices. We have developed novel interior point methods for such applications. One of our methods, designed for the case when the Hessian of the Lagrangian is not available, follows a step-decomposition approach and uses iterative linear algebra techniques accelerated by constraint preconditioners. The second method is suitable for PDE-constrained optimization; it follows a full-step approach and applies an iterative linear method directly to the primal-dual system. Our techniques have been tested successfully on a variety of practical applications.***

Many important optimization problems involve thousands of decision variables and a similar number of constraints. Often, these problems are nonlinear and finding their optimal solution quickly and reliably can be a formidable task. To give one example, consider the simulation of power flow in an electrical network --- a problem that is receiving renewed attention because its importance in understanding how to strengthen the electrical grid. The power flow simulation is typically cast as an optimization problem with nonlinear constraints formed by *sin* and *cos* functions; the number of variables can range in the tens of thousands or hundreds of thousands. This problem must be solved dozens of times in study of the topological strength of a regional power grid.

We wish to solve problems of this kind efficiently, but in practice the Hessian of the Lagrangian is often not available, which appears to preclude the application a Newton-like methods. Further, quasi-Newton techniques are typically very slow in these applications. To overcome these difficulties, we have developed an interior point algorithm in which the primal-dual equations are solved by the projected conjugate gradient method. The Hessian of the Lagrangian need not be formed because the iteration only requires products of this Hessian times vectors--- and these products can be approximated by finite differences of gradients of the Lagrangian. Preconditioning is essential since the primal-dual system is highly ill-conditioned due to the properties of barrier functions. We have pioneered the use of "constraint preconditioners" in interior point methods and have shown that they greatly accelerate the convergence of the conjugate gradient iteration. The overall algorithm is a Newton-type method and as such possesses desirable scale-invariance properties.

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To illustrate its efficiency, we report below the solution of an optimal power flow problem whose objective is reactive power injection in all network busses as a diagnosis of lack of reactive support in the system. The network represents a high voltage generation-transmission system with about 3500 busses and 5000 circuits. The optimization problem involves nearly 15000 decision variables and 6800 nonlinear constraints. The results are as follows:

1. A state of the art solver that uses traditional active-set techniques requires *35 minutes* to find the optimal solution
2. Our interior point algorithm described above solves the problem in *32 seconds.*
3. An interior point algorithm that estimates the Hessian using quasi-Newton approximations requires almost *16 minutes*.

These dramatic gains in performance are common in applications of this type and further improvements can be expected as the field of constrained preconditioning matures.

In a different class of applications, the number of variables can be much larger and the constraint Jacobian cannot be formed or factored. Such is the case in *PDE constrained optimization* in which the constraints are given by the discretization of three-dimensional partial differential equations and where the number of variables can be in the millions.

In some cases, we can solve these problems using interior point methods in which the step computation follows a full-space approach and an iterative method is a applied to the full primal-dual system. Up until recently, however, there were no theoretical guidelines on how inexact the step computation can be and still guarantee global convergence.

In collaboration with Ph.D student Frank Curtis and Prof. Richard Byrd (University of Colorado), we have established termination criteria for the inner iteration in these interior point methods. These results open the door for the development of practical algorithms, as they apply even in the nonconvex case. Numerical tests on computational tomography problems performed by Frank Curtis and Eldad Haber (Emory) have validated the usefulness of our approach.

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