Office of Science
U.S. DEPARTMENT OF ENERGY

Advanced Scientific Computing Research
Applied Mathematics
FY 2007 Accomplishment

"Algebraic Multigrid Methods for Large-Scale Electromagnetic Problems"

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Summary

The goal of this research is to develop scalable multilevel methods for solving large linear systems of equations that arise from the discretization of partial differential equations with emphasis on electromagnetism. We are looking at multilevel solution strategies, as they are capable of scaling up to the very large problem sizes needed. Our algorithms research is application-driven and is presently focused on the area of electromagnetism, and may also prove relevant in other problems such as flow problems (Navier-Stokes). The problems of interest are defined on a variety of grids, but we are mostly focusing on unstructured grids. This research effort involves both programmatic partners and academic collaborators.

The Scalable Linear Solvers team in CASC has proven record of developing parallel multigrid methods for solving the large linear systems of equations that arise from the discretization of partial differential equations. As we are interested in solving very large systems on massively parallel computers, it is important that our solvers be scalable, or very nearly so. For the most part, we are concerned with how the solver performs as both the size of the problem and the number of processors is increased. In general, we would like the computational resources required to solve increasingly larger problems to grow only linearly with problem size (which is the optimal rate). In such a case, one could double both the size of the problem and the number of processors, while keeping the solution time constant. This gives one possible definition of scalability.

In iterative methods for solving linear systems, solver scalability can be divided into two aspects. The first is *algorithmic scalability*, which requires that the computational work per iteration is a linear function of problem size and that the convergence factor per iteration is bounded independent of problem size. The second aspect is *implementation scalability*, which requires that a single iteration is scalable on the parallel computer. Both algorithmic and implementation scalability are required for the iterative solver to be scalable. Many of the linear solvers used in today's simulation codes are based on standard simple iterative methods like Jacobi preconditioned conjugate gradient methods. These methods are often unscalable in the algorithmic sense defined above (e.g., for elliptic model problems the convergence factor approaches one as the mesh is refined). Our research has its focus on multigrid solvers because they can provide algorithmic scalability. AMG (algebraic multigrid) is a viable alternative to the more classical geometric multigrid for problems discretized on unstructured meshes. Previously, we have worked on more standard elliptic type of PDEs for which AMG (algebraic multigrid) still had challenges (in the non M-matrix case) but

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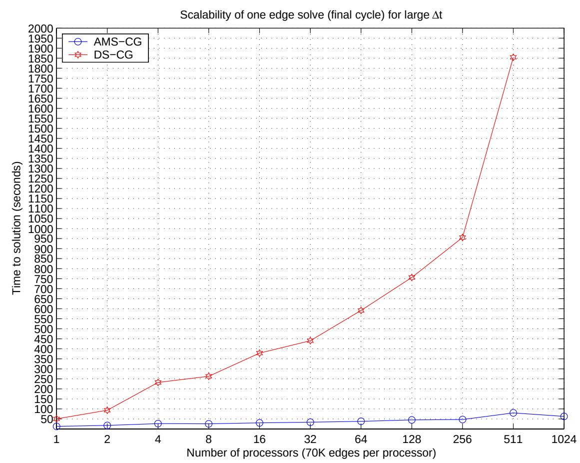
nevertheless the success of the research was for the most part guaranteed motivated by the fact that geometric multigrid is ideally suited (and originally designed) for elliptic PDEs.

The focus of this effort is on long-term fundamental mathematics research aimed at the development of theory (mostly two-level) for AMG methods for unstructured-grid problems for definite time domain Maxwell problems discretized by the commonly used Nédélec finite elements, as well as for (moderately) indefinite Maxwell (time domain) problems. The functional space where the problem is posed is denoted by H(curl). Many of the multilevel algorithms developed here are and will be made available to computational scientists through *hypre* (http://www.llnl.gov/CASC/hypre/), a library of parallel high-performance preconditioners funded in part by the DOE's SciDAC and ASC programs.

We have begun exploring a number of new approaches for the development of optimal order multilevel methods for the time domain Maxwell equations. One approach is based on the so-called auxiliary mesh method that we have analyzed in collaboration with J. Pasciak from Texas A & M University. This method is not purely algebraic; it requires additional geometric information about the fine-grid problem. An auxiliary mesh is used that needs only approximate the original domain and does not have to completely match it. Because of the need for additional geometric information, however, the method may be not very appealing in practice since it requires the redisretization of the original H(curl) problem on an auxiliary mesh. While not necessarily practical, our result is important since it can be viewed as an "existence" result; namely, it shows that there is a hierarchy of spaces that can be used to construct multilevel methods for definite H(curl) problems on general unstructured meshes.

The second approach that we have also explored is based on the so-called auxiliary space technique. It relies on certain stable space decompositions of the lowest order Nédélec space into components from spaces that can successfully be handled by existing multilevel methods. The decomposition by Hiptmair and Xu (2006) offered such possibility. Using this method, we have developed and provided perhaps the first provably scalable definite Maxwell solver in our library *hypre*. Its parallel performance on a real application problem is illustrated in the Figure.

A third approach that we have begun working on, is to construct a hierarchy of commuting de Rham complexes using element agglomeration. The assumption is that we are given an exact de Rham sequence of finite element spaces on the original fine unstructured mesh. This is a general approach for constructing element based AMG (or AMGe) methods, not only for H(curl) but also for H(div) (a space of functions with weakly defined divergence) as well as for elliptic problems, all on general unstructured meshes. The preliminary results are very encouraging since they seem to offer potential for constructing AMGe methods as well as providing powerful tools for upscaling a wide range of linear and nonlinear PDEs.



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