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Shape Optimization of Swimming Sheets

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Summary

Using a blend of asymptotic analysis and numerical modeling, we study the optimization problem of finding the shape of the deformation wave a gastropod or mechanical crawler should use to propel itself over a thin layer of viscous fluid. This is an important first step in the design of efficient mechanical crawlers for underground oil exploration in complex, unknown terrains.

Problems in fluid dynamics in which a fluid interacts with a flexible body are not well understood. One interesting regime that arises frequently in biological and engineering systems has the fluid confined to a thin region between the flexible body and a rigid substrate. In collaboration with A. E. Hosoi at MIT (who has built prototypes of mechanical crawlers that may someday be used in underground oil exploration), we have studied the problem of finding the optimal deformation wave that a periodically deforming sheet should use to propel itself over a viscous Newtonian fluid. The swimming speed is determined by the condition that the net horizontal force exerted by the fluid on the sheet is zero. We find that the sheet moves forward by passing close to the substrate in order to form a pressure drop across the fluid constriction to push the sheet forward against the viscous drag forces. Our goal is to find wave profiles that optimize this effect.

To make the optimization problem computationally tractable, we use Reynolds' lubrication approximation to model the fluid flow. This leads to explicit formulas for swimming speed and power consumption in terms of the wave profile $h(x)$. We then derive Euler-Lagrange equations to find the smoothest wave profile (minimizing arclength l) to achieve a given speed V, efficiency V/P and fluid loss Z. (A smooth wave profile is desirable to avoid singularities in the solution of the Stokes equations, and for the lubrication model to remain valid). Scale invariance of the lubrication model allows us to reduce the dimension of the parameter space by absorbing the fluid loss constraint into the vertical length scale H of the physical problem. This normalizes the remaining two-parameter family of optimal solutions (with speed and efficiency as parameters) to have the same cost in terms of energy associated with fluid production, making mechanical energy comparisons meaningful. The Euler-Lagrange equations turn out to be integro-differential equations, which we solve using a quadratically convergent multi-shooting algorithm we developed.

In studying the two-parameter family of optimal solutions, we encountered an interesting dilemma: optimization within lubrication theory drives us out of its realm of applicability. As the speed and efficiency constraints approach their maximal values, the optimal solutions develop self-similar, cusp-like singularities that cause the error in the lubrication approximation to grow without bound for any fixed choice of $\varepsilon=H/W$. This problem disappears as soon as we

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incorporate higher order corrections into the lubrication model. Doing this causes the region of attainable speeds and efficiencies to fold over itself, penalizing cusp-like singularities and causing non-singular wave profiles to emerge as those that strike the optimal balance between speed, efficiency and fluid loss.

References

J. Wilkening and A. E. Hosoi. *Shape optimization of a sheet swimming over a thin liquid layer*. J. Fluid Mech. (submitted). Also published as technical report LBNL-59395, 2006.

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