Fast Marching Methods for the Continuous Traveling Salesman Problem

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Summary: We consider a problem in which we are given a domain, a cost function that depends on position at each point in the domain, and a subset of points (``cities'') in the domain. The goal is to determine the cheapest closed path that visits each city in the domain once. This can be thought of as a version of the Traveling Salesman Problem, in which an underlying known metric determines the cost of moving through each point of the domain, but in which the actual shortest path between cities is unknown at the outset.
We describe algorithms for both a heuristic and an optimal solution to this problem..

Consider a collection of cities on a map, in which the cost of traveling from one city to the other is known from the outset. The goal of finding the cheapest path that visits each city once is the well-known Traveling Salesman Problem (TSP). We have developed a collection of numerical algorithms to solve the associated Continuous Traveling Salesman Problem. In this version, we are given a domain, a cost function which depends on position at each point in the domain, and a subset of points (``cities") in the domain. The goal is to determine the cheapest closed path that visits each city in the domain once. This can be thought of as a version of the Traveling Salesman Problem, in which an underlying known metric determines the cost of moving through each point of the domain, but in which the actual shortest path between cities is unknown at the outset. A large collection of physical problems fall into this category, including finding optimal paths to navigate terrain with various speeds and obstacles while reaching a given number of fixed stations.

One way to solve this problem is to use the Fast Marching Method to find the shortest path between all possible pairs of points. This then discretizes the continuous problem into a graph, which can then be handled by any of a large number of discrete algorithms to find both optimal and near-optimal solutions to the Traveling Salesman Problem.

However, we have been able to find a far more efficient approach by intertwining the Fast Marching Method inside the solution to the Traveling Salesman Problem, and only computing paths "on-the-fly" as needed. The essential idea is the following. For optimal solutions, we use branching algorithms in which we solve the Eikonal equation from each city using the Fast Marching Method, and only proceeding as far as an associated time clock allows. By gluing together paths from each city onto existing paths, we can be assured that the algorithm terminates as soon as the shortest path is found. This avoids calculating a host of unnecessary shortest paths.

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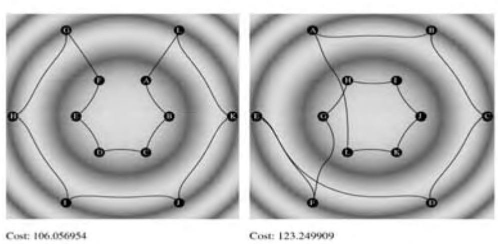
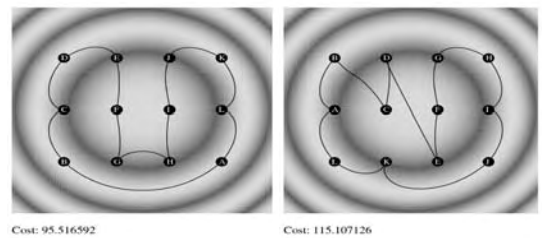
If a heuristic shortest path is desired, we were to develop a continuous version of Christofides' algorithm, which can find a path which is no more than 1.5 times the actual shortest optimal path. To do so, we again embed the Fast Marching Method in each step of the algorithm. First, we embed our fast Eikonal solver inside Kruskal's algorithm to build a minimum spanning tree. We then can construct a perfect matching, and finish by finding the Eulerian path and subsequent Hamiltonian path, which then gives us a heuristic Christofides-type solution to the continuous Traveling Salesman Problem. The average runtime of the heuristic algorithm is linear in the number of cities and $O(N \log N)$ in the size N of the mesh.

Two examples are shown below. Each shows the optimal solution and the heuristic solution. In the examples below, the darker the region, the higher the cost



Optimal Cost=210 Heuristic Cost=246



Optimal Cost=152   Heuristic Cost=182

References:

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J.A. Sethian, Level Set Methods and Fast Marching Methods, Cambridge University Press, 1999.

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