"Dimensional Reduction and Optimal Prediction for Nonlinear Systems"

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Summary

We have applied our methods for reducing the complexity of computational tasks to new problems, including the computation of the rate of decay of solutions of the Euler equations.

Many problems in science are too complex for direct numerical solutions, because there are too many variables, or the models are insufficient, or there are not enough data. In recent years the mathematics department at LBNL has developed tools for dealing with these situations through stochastic modeling and dimensional reduction, which go under the name "optimal prediction". We are now using these tools in a series of challenging problems.

In particular, we have succeeded this year in computing the rate of decay of solutions of the Euler equations, which describe incompressible flow in the limit of very high Reynolds number in two and three dimensions- i.e., we have computed the rate of decay of turbulence. This was done as follows. First, we reduced the solution of the equations to the solution of a very large system of ordinary differential equations for the coefficients in a series expansion, as is usually done. As is well known, turbulence is so complicated that the system one gets this way is far too large to be solved by any available method.

At this point we introduced our new tools. We picked out a privileged set of variables corresponding to the lowest frequencies in the system, and used optimal prediction methods to derive effective equations for these variables, in particular we applied the "t-model" version of optimal prediction which applies to systems with long memory, i.e. to systems where the effect of small perturbations persists for long times, as it does in the Euler system. Then we computed the rate of decay of the solutions from the resulting small system. In model problems and in the two-dimensional case, where the rate of decay was already known, the results were right on target; in the most interesting three-dimensional case, where there were no previous computational results, they were in line with the data.

These results are interesting for two reasons. First, the problem is a significant challenge to our methods. We computed explicitly the low frequency components of the solutions, which are the ones one normally observes and cares about, but the rate of decay of these components is actually determined by the behavior of the high frequency components, whose effect was represented solely through the machinery of optimal prediction. In other words, we used our tools to find correctly the effect of missing

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variables. That this was done without special adjustments is a powerful testimonial for optimal prediction.

Second, the computation of any property of turbulence from first principles is of great interest. The decay problem has some features which facilitate the application of optimal prediction, so what we did is not (yet?) a general solution of the turbulence problem, but still it is the only computation of a property of fully developed turbulence from first principles that we know of.

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