Office of Science
U.S. DEPARTMENT OF ENERGY

Advanced Scientific Computing Research
Applied Mathematics
FY 2007 Accomplishment

“Analysis of Stochastic Partial Differential Equations Quantifies Uncertainty in Transport Problems”

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Summary

Flow and transport in tubes with rough surfaces play an important role in a variety of applications. Often the topology of such surfaces cannot be accurately described in all of its relevant details due to either insufficient data, or measurement errors, or both. In such cases, this topological uncertainty can be efficiently handled by treating rough boundaries as random fields, so that an underlying physical phenomenon is described by deterministic or stochastic differential equations in random domains. To deal with this class of problems, we used a computational framework, which is based on stochastic mappings to transform the original deterministic or stochastic problem in a random domain into a stochastic problem in a deterministic domain. The latter problem has been studied more extensively and existing analytical/numerical techniques can be readily applied. We used our approach to describe transport of a passive scalar in Stokes' flow, and to quantify the corresponding predictive uncertainty.

Flow and transport in the vicinity of rough surfaces play an important role in a variety of applications that range risk analysis of glacier sliding to micro-electronic-mechanical system (MEMS) technology. Indeed, given a proper spatial resolution, virtually any natural or manufactured surface becomes rough. Often the topology of such surfaces cannot be accurately described in all of its relevant details due to either insufficient data, or measurement errors, or both.

The emphasis on uncertainty quantification suggests the use of probabilistic descriptions of rough surfaces. Such descriptions allow for more realistic, data-driven representations of surface roughness. This is in contrast to deterministic conceptualizations that treat rough boundaries as sinusoidal surfaces, a surface with a large number of periodically distributed humps, or self-similar and fractal surfaces.

Of course, not every stochastic representation of rough surfaces is conducive to uncertainty quantifications. For example, random fractals or fractional Brownian motion, which were used to represent rough surfaces, are not readily amenable to parameterization by data. Likewise, the representation of a rough surface as a Gaussian random field with a linear autocorrelation function might conflict with data. One of the goals of this

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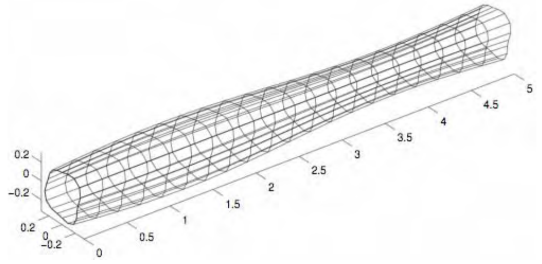
study was to propose a methodology for uncertainty quantification that is flexible enough to allow for non-trivial probabilistic descriptions of rough surfaces that are determined by data.

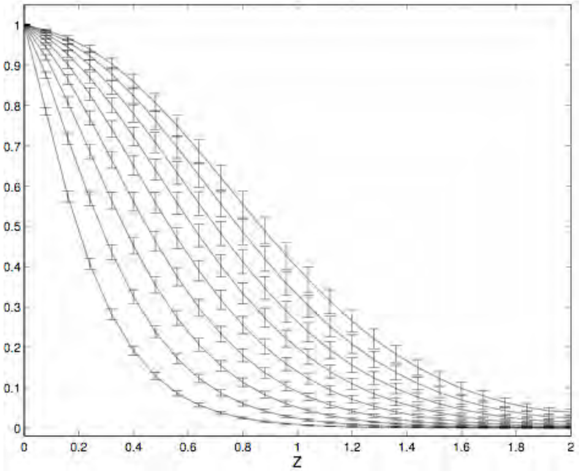
Topological uncertainty can be efficiently handled by treating rough boundaries as random fields, so that an underlying physical phenomenon is described by deterministic or stochastic differential equations in random domains. To deal with this class of problems, we are developing a computational framework, which is based on stochastic mappings to transform the original deterministic or stochastic problem in a random domain into a stochastic problem in a deterministic domain. The latter problem has been studied more extensively and existing analytical/numerical techniques can be readily applied.

To be specific, we employed both a generalized polynomial chaos and Monte Carlo simulations to solve the transformed stochastic problem. We used our approach to describe transport of a passive scalar in Stokes' flow in a pipe with rough walls (figure below), and to quantify the corresponding predictive uncertainty.



A tube whose surface exhibits roughness in longitudinal and angular directions.

The following figure shows the predictions of the concentration of a passive scalar, accompanied by error bars. The mean concentration profile remains practically unchanged for the degree of roughness varying between 0 and 5% of the tube's radius.



Prediction of the time evolution of concentration with the error bars.

This leads to a conclusion that, for this degree of roughness, the roughness-induced dispersion of a passive scalar can neglected if one is interested in the mean behavior alone. However, roughness is important for uncertainty quantification.

This analysis has been published in D. M. Tartakovsky and D. Xiu, Stochastic analysis of transport in tubes with rough walls, *J. Comput. Phys.*, 217(1), 248-259, 2006.

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