Advanced Scientific Computing Research
Applied Mathematics
FY 2007 Accomplishment

"Multilevel Upscaling for Two-Phase Flow in Heterogeneous Porous Media"

E. T. Coon, Columbia University
J. D. Moulton[*], Los Alamos National Laboratory
S. P. MacLachlan, Delft Technical University

Summary

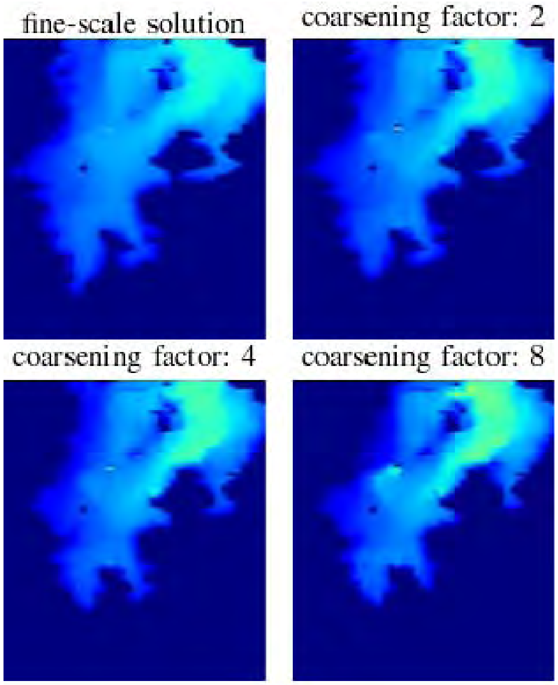
The study of two-phase flows in porous media is fundamental to our understanding of many important problems, including the maintenance of groundwater and the optimization of oil reservoir production. In these problems, fine-scale heterogeneity of the subsurface strongly influences large-scale features of the flow; however, fully resolved simulations are computationally intractable. Thus, the challenge is to capture the influence of the fine-scale structure, while performing the bulk of the computation at a coarse resolution. Recent work in Multilevel Upscaling (MLUPS) of single-phase flows has demonstrated the potential of leveraging variational coarsening techniques from robust multigrid solvers to develop an accurate hierarchy of coarse-scale models. Here, we investigate the use of this multilevel upscaling approach in a two-phase flow model of oil reservoir flooding.

The two-phase modeling of secondary oil recovery, or reservoir flooding, is a popular and challenging benchmark problem for upscaling methods. In this process, water is pumped into oil reservoirs, effectively pushing out the oil and, thereby, increasing the production rate of the reservoir. Flow in the reservoir is modeled by Darcy's Law coupled with conservation of mass of each phase. This results in a diffusion equation where flow is driven by gradients in pressure and scales with the relative permeability. The relative permeability is a function of the saturation, a measure of the water/oil content of the fluid, and the absolute permeability of the subsurface medium. In this way, the saturation, which is advected by the flow, also couples with the determination of the flow.

To simulate this flow we use a standard technique called IMPES. First, the hyperbolic saturation equation is solved, calculating the saturation at the next time step. Then, the pressure equation is updated implicitly using the multilevel upscaling (MLUPS) method [1], to obtain pressures and velocities. We note that the MLUPS generated velocity field, like many finite element based solvers, is not locally conservative. To recover this important property of the velocity we extended the post-processing algorithm of Cordes and Kinzelbach [2] to accurately treat source terms, such as wells, and non-trivial boundary conditions. Since MLUPS generates an approximation to the fine-scale pressure, smoothing of the reconstructed fine-scale pressure is required before post-processing. In this study, we used two sweeps of line relaxation.

---

[*] 505-665-4712, moulton@lanl.gov



The saturation front in a two-phase simulation of water flooding an oil reservoir is shown. Water is injected at a well in the upper-right corner while the oil-water mixture is pumped out at the lower-left corner. The top left shows the saturation for the fine-scale simulation (lighter colors indicate more water), while the other plots show the saturation obtained using an upscaled velocity field computed at the indicated resolution. The essential features of the flow are captured at all resolutions.

This simulation approach is applied to a single two-dimensional slice of the popular SPE 10 three-dimensional benchmark problem. The two-phase flow equations are solved on a 2160 ft by 1120 ft subdomain of layer 70. In this flooding problem, a water injection well is positioned in the upper right corner, and a production well that removes the bulk fluid is positioned at the lower left (see figure). The top plot zooms in on a region around the saturation front of the fine-grid simulation, which used a 57x217 grid. Below this plot, we show the saturation obtained with the MLUPS velocity field. In these cases the pressure is solved at the coarser resolutions, coarser by a factor of 8 in the last case. Here, the dominant features of the saturation front are well preserved in all cases.

In this preliminary work, we have demonstrated the potential for MLUPS to be used in two-phase flow modeling. MLUPS, like other multiscale methods, has the advantage that it upscales the fine-scale model and not just its parameters. However, many other multiscale methods use only two levels; the hierarchy of models given by MLUPS offers more versatility in resolution and accuracy. In the future we will investigate leveraging this versatility to develop multilevel simulation techniques that combine error estimation and adaptivity to enhance overall efficiency.

References:

[1] MacLachlan, S. P. and J. D. Moulton, "Multilevel upscaling through variational coarsening," *Water Resour. Res.*, **42**, W02418, doi:10.1029/2005WR003940, 2006.
[2] Cordes, C. and Kinzelbach, *Water Resour. Res.*, **28**:2903-2911, 1992.

Acknowledgements:

LANL Report LA-UR-07-4906. Funded by the Department of Energy at Los Alamos National Laboratory under contract DE-AC52-06NA25396, the DOE Office of Science's Advanced Scientific Computing Research (ASCR) program in Applied Mathematical Sciences, and the DOE Computational Science Graduate Fellowship.

For further information on this subject contact:

Dr. J. D. Moulton
Los Alamos National Laboratory
Phone: 505-665-4712
moulton@lanl.gov

Or

Dr. Anil Deane
Applied Mathematics Research Program
Office of Advanced Scientific Computing
Phone: 301-903-1465
deane@ascr.doe.gov

---

[*] 505-665-4712, moulton@lanl.gov

---

[*] 505-665-4712, moulton@lanl.gov