**Advanced Scientific Computing Research**
**Applied Mathematics**
**FY 2007 Accomplishment**

# *"Analysis of Multidimensional Conservation Laws"*

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**Summary**

***The PI, working with Suncica Canic, Gary Lieberman, Eun Heui Kim, Katarina Jegdic and Allen Tesdall (the last three partially supported by the DOE grant), has pioneered an approach to multidimensional conservation laws that is yielding the first analytic results for an important class of problems, including the equations of compressible gas dynamics. Related to this, careful numerical simulations are revealing new structures that shed light on the famous "von Neumann paradox". The results so far on the interplay between analytical techniques and physical phenomena in high-speed flow, and the relation of both to computational fluid dynamics, are encouraging.***

The landscape of two-dimensional self-similar solutions has changed dramatically during the period of this grant, owing in part to the research of the PI and her team of co-workers. Our program has inspired other researchers, working in several groups (Gui-Qiang Chen and Mikhail Feldman at Northwestern and Wisconsin; Yu-Xi Zheng and colleagues at Penn State; Marshall Slemrod and Seung-Yeal Ha at Wisconsin; Tai-Ping Liu and Volker Elling at Stanford; and others) to take up the self-similar approach and extend it in many directions.

The specific features of our method that have lent themselves to extension are the centrality of a second-order quasi-linear partial differential equation that changes type (hyperbolic/elliptic) at a sonic line; the appearance of a reflected shock as a free boundary connecting a known constant flow to an undetermined elliptic (subsonic) flow, with the possible appearance of a sonic degeneracy in the elliptic equation; and the use of compactness methods, mainly the Schauder fixed-point theorem, to produce a convergent subsequence. Collectively, these results have created, for the first time, an understanding of the analytical properties of the regular reflection regime. Our method, which does not assume a potential, has required us to introduce a cut-off function, and thus the validity of our solution is confined to a finite neighborhood of the reflection point. However, since the actual gas-dynamic flows are not of potential type (the curved shock necessarily introduces some vorticity), it will prove useful in the long run to work with some non-potential form of the equations.

We have recently extended our approach to a case involving the equations of gas dynamics, where there is, again, a second-order equation that changes type, in this case coupled to a set of evolution equations for the velocity variables. Obtaining the right framework for compactness and convergence has been much more difficult than we anticipated, but we have found a method (involving a contraction mapping

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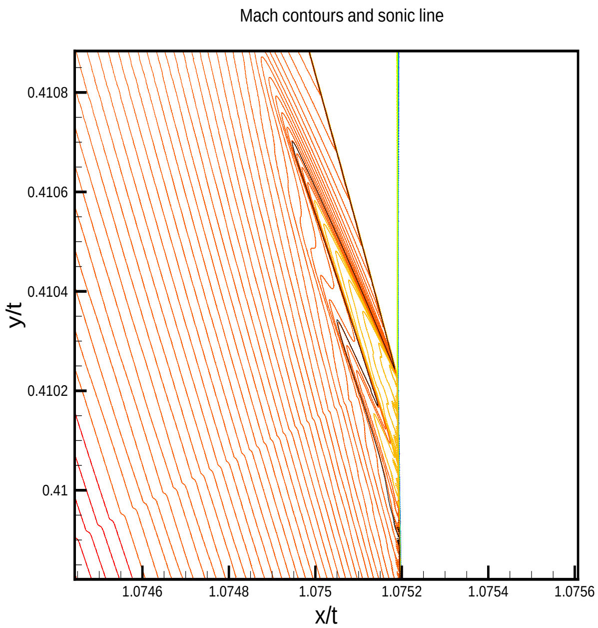
argument for the velocity variables) that appears, again, to give a local solution.




**Guderley Reflection in Gas Dynamics**

In a second breakthrough, Allen Tesdall has extended his earlier work with John Hunter. This work exhibited numerically a new phenomenon, named by the discoverers "Guderley Mach reflection", consisting of a sequence of as many as six separate supersonic patches in the wake of a Mach-type shock reflection, in a regime where neither regular nor standard Mach reflection is possible. Despite the reference to earlier work of Guderley that had predicted a single supersonic bubble, formed by a rarefaction reflecting off the sonic line as a shock, no one had expected to see the complex phenomenon of a series, possibly infinite, of such patches. Following the numerical solutions, the phenomenon has now been confirmed experimentally by Skews and Ashworth.

The original numerical simulation found this solution in the model unsteady transonic small disturbance equations; Tesdall's new work has shown it in the so-called nonlinear wave system, and, most recently, in the adiabatic gas dynamics equations themselves.

This work has been recognized in a number of ways: I received the 2005 Krieger-Nelson Prize of the Canadian Mathematical Society. I was also honoured to receive the 2006 Farfel Award of the University of Houston. This is the highest award given to faculty at the University of Houston, and is based on research, teaching and service. Besides the Krieger-Nelson lecture, I have also received a number of prestigious invitations to lecture on the research funded by this grant. These include

- Invited speaker, Conference on Advances in PDE in honor of the eightieth birthdays of Peter Lax and Louis Nirenberg, Toledo, Spain, 2006;
- Joint Plenary speaker, Analysis of PDE Conference and Topical speaker, SIAM Annual Meeting, Boston, 2006;
- Invited speaker, International Congress on Industrial and Applied Mathematics, Zurich, 2007;
- Invited speaker, conference in honour of Olavi Nevanlinna and Juhani Pitkaranta, Helsinki University of Tech, 2008.

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