Advanced Scientific Computing Research Applied Mathematics FY 2007 Accomplishment

Simulation based optimization and application to electromagnetic inverse problems

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Summary

This project develops new techniques for optimization problems constrained by PDE's and apply these techniques to the solution of inverse-Maxwell's equations. In the last year we have been developing adaptive mesh refinement techniques for forward and inverse Maxwell's equations. Both theory and practical implementation have been explored. As a result we have gained a factor in speed over existing codes. This allows us to solve inverse Maxwell's problems of sizes never attempted before.

Optimization problems with Partial differential equations as constraints appear in many applications including optimal control and inverse problems. In this project we develop new approaches for the solution of such problems and apply them to electromagnetic inverse problems. Electromagnetic inverse problems are solved on a daily basis for undestructive testing, shallow, subsurface imaging of hydrological targets, the exploration of carbohydrates and more. A typical survey used for the evaluation of contaminated water is presented in Figure 1.

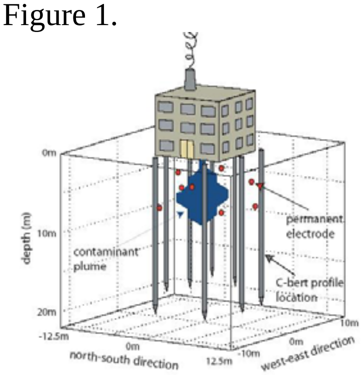

Figure 1: Geophysical experiments for the detection of contaminated plums

In this case, boreholes are drilled around the area of interest and electromagnetic currents are injected to the ground. Electric or magnetic fluxes are measured and the goal is to determine if and where the contaminated plum is traveling to. To achieve this goal one requires solving Maxwell's equations in areas where the resolution may change. Using regular mesh leads to a large number of unknowns which in return makes the simulation very expensive. There are two approaches to reduce the size of the problem. First, one can use an unstructured mesh. While this has done extensively in the past it has some major drawbacks. First, computing a mesh is a non-trivial issue. Since we deal with an inverse problem such computation needs to be repeated whenever we update the geometry. Second, unstructured meshes may lead to ill-conditioning of the system induced by low quality mesh. Finally, unstructured meshes require algebraic multigrid solvers and it is difficult to use the underlying geometry in order to develop scalable solvers.

Our approach was to use adaptive local mesh; in particular Octree meshes for the solution of the problem. Such a mesh is presented in Figure [2].

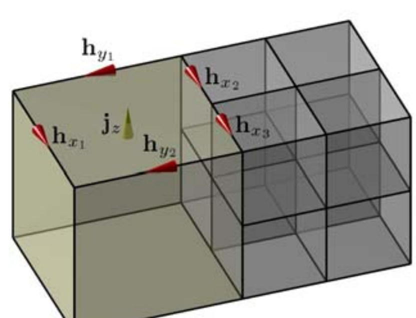

Figure 2: Discretization of the curl on an Octree

While meshing using Octree's is a relatively trivial task, the solution of Maxwell's equations on such meshes, were never attempted before. We therefore had to derive a mimetic discretization to Maxwell's equations on Octree grids. Our discretization is a simple extension of the well-known Yee's method. We have shown in [1] that this discretization is mimetic, conservative and that efficient multigrid methods can be developed for the solution of Maxwell's equations on such meshes.

Next, we used Octree discretization for the solution of the inverse problem. The idea and basic implementation details are presented in [2]. For the solution of inverse problems, further complications arise because we need to discretize the electric fields, the conductivity and the Lagrange multipliers. We have found that the conductivity and the fields are best discretized on different meshes. Based on our ideas we have coded a matlab package for the inversion of resistivity data. The package is described in [3] and it is publically available. We have used this package to invert field data collected by Stanford Environmental Group. Figure [3] presents the borehole data and Figure [4] presents the inversion obtained by our code. The inversion matches known geology of the region.

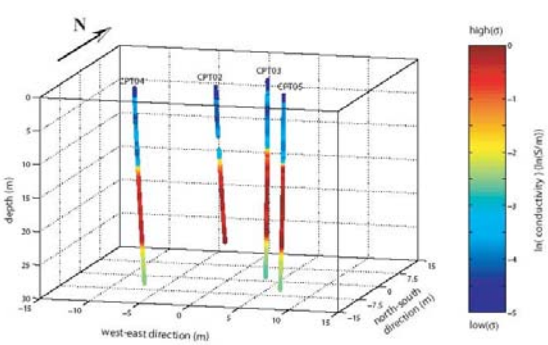

Figure 3: Resistivity data from boreholes

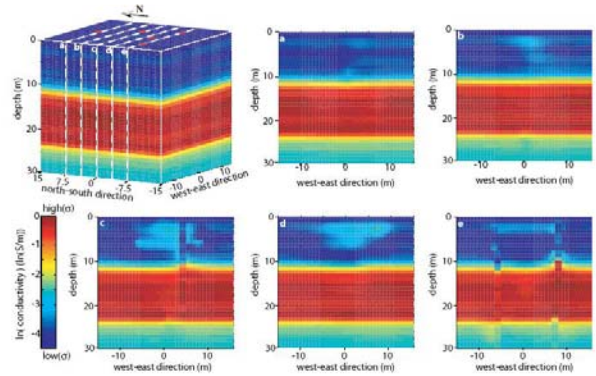

Figure 4: Inversion results.

References:

[1] E. Haber, S. Heldmann, To appear, J. of Comp. Phys. An OcTree Multigrid Method for Quasi-Static Maxwell's Equations with Highly Discontinuous Coefficients

[2] E. Haber, S. Heldmann, U. Ascher, To appear in Inverse Problems Adaptive finite volume methods for distributed non-smooth parameter identification

[3] A. Pidlisecky, E. Haber, R. Knight, To appear in Geophysics RESINVM3D: A matlab 3D resistivity inversion package

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