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"Ambiguous Chance-Constrained Problems"

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Summary

Blackouts are multistage stochastic events, where each event corresponds to the loss of a line or generator in a power grid. As lines go down, controllers have to decide how to regulate the generator outputs and loads so as to return the power grid to stability. This problem can be formulated as a multistage chance-constrained problem. In this project we show how to solve a one-stage problem efficiently and that the tractability of a two-stage problem is a function of the degree to which it is robustly feasible.

Multistage stochastic optimization problems are usually tackled by either stochastic programming or robust optimization. Stochastic programming is criticized for the enormous cost that is required by it to solve even small practical problems. On the other hand, the robust optimization framework is criticized as being overly pessimistic. This framework is closely related to Knightian uncertainty, which has been similarly criticized in the Economics literature. This pessimistic behavior is a consequence of the fact that in robust optimization one assumes that there is an adversary that is allowed to choose actual parameter values, or equivalently, choose distributions that are Dirac delta functions.

This suggests the following compromise: restrict the adversary to an appropriate class of probability measures, but instead of fixing one measure **P**, let **P** be uncertain within a specified uncertainty set. With a doctoral student Emre Erdogan, we investigate such a formulation in the context of *convex chance-constrained* problems. Such problems involve constraints on a random variable **X,** distributed according to the measure **P,** of the form that the probability that some function of **X** and some other variables **y** that is a convex function of **y** for each fixed value of **X**, is nonnegative is less than some given positive fraction. We consider a formulation where the probability measure **P** is *ambiguous* in the sense that it is only known to lie in an uncertainty set of measures that are within a certain distance of a given probability measure **Q** in an appropriate (Prohorov) metric.

We show that the corresponding *ambiguous* chance-constrained problem can be solved by combining techniques from statistical learning theory, coupling of random variables, and robust optimization. We also consider two-stage ambiguous chance constrained problems and show that the tractability of a two-stage convex chance constrained problem is a function of the

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degree to which it is *robustly* feasible. This is very similar to results relating the complexity of solving an optimization problem to its condition number. In addition to its applicability to controlling power blackouts, this work has important implications for solving other energy involving model uncertainty.

References:

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