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"Solving Large-scale Packing and Covering Problems Efficiently"

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Summary

The power flow recovery problem in a very large AC grid can be formulated as a path-packing problem with side constraints. Practical instances of these problems (e.g., with 100,000 nodes) are extremely difficult to solve. In this project we developed a new approximation algorithm for solving general packing and covering problems in a number of iterations that is proportional to the reciprocal of the accuracy of the approximation achieved.

A packing problem is an optimization problem in which the maximum of a finite number of linear functions is minimized over a set **P**, where **P** is a set over which linear optimization is easy. When **P** is a polytope, the packing problem is a linear program, and therefore, it can be solved in polynomial time. However, in practice, these linear programs are extremely difficult and are often unsolvable by state-of-the-art solvers. Consequently, many researchers have focused on developing algorithms that can very efficiently compute provably good, if not optimal, solutions.

Shahrokhi and Matula developed the first approximation algorithm for the maximum concurrent flow problem, an important special case of the packing problem. This problem is one in which given a network with capacities on its edges and a set of multi-commodity demands that must simultaneously be routed, one wishes to find a routing that maximizes the fraction of every demand that can be routed. Shahrokhi and Matula approximated the non-smooth objective of this packing problem by a smooth potential function and showed that, for a given accuracy $\boldsymbol{\varepsilon}$ ($\boldsymbol{\varepsilon} < 1$), one can choose the potential function so that a first-order descent method applied to it yields an $\boldsymbol{\varepsilon}$-optimal solution in a number of iterations that is proportional to $(1/\boldsymbol{\varepsilon})$-cubed. Since then, the work of a number of researchers, notably Plotkin, Shmoys, Tardos, Klein, Stein, Garg, Konneman, Grigoriadis, Khachiyan, and Fleischer, among others, generalized this technique to the broader class of packing and covering problems, and gradually reduced the dependence of the iteration count on $\boldsymbol{\varepsilon}$. Klein and Young proved a matching lower bound for the methods used. With Dan Bienstock at Columbia University, we were able to adapt a new technique proposed by Nesterov for minimizing non-smooth functions to show that an $\boldsymbol{\varepsilon}$-optimal solution to packing problems can be computed in a number of iterations that is proportional to $1/\boldsymbol{\varepsilon}$.

This work is a breakthrough in this field. There was almost a folk theorem that an

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iteration bound of better than (1/**ε**)-squared could not be achieved. In addition to power recovery problems, many other problems can be formulated as packing problems. Another important class of such problems is online learning problems. Our results imply that one can design learning algorithms that are an order of magnitude faster than those that currently exist.

References:

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