"Robust Algorithms for Large Scale Non-convex Nonlinear Optimization"

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Summary

Numerous engineering problems can be formulated as nonlinear optimization problems. Recent applications of these instances, which are usually of very large scale or involve highly structured degeneracies, impose challenges to existing solution approaches. In this project we developed an interior-point piecewise linear penalty algorithm for general nonlinear programming (NLP) problems and an active set algorithm for mathematical programs with equilibrium constraints (MPECs). These algorithms are robust in the sense that they possess strong convergence properties. Our numerical tests indicate that these algorithms are competitive with start-of-the-art solvers.

NLP problems have numerous applications ranging from traditional problems, such as production and distribution planning in the oil and gas industries, power dispatch in electric networks, plant design in process engineering and optimal control for dynamic systems, to emerging areas, such as integrated circuit design and partial differential equations for inverse problems. These applications, especially recent ones, often require the solution of large scale non-convex NLP problems. The non-convexities involved may include non-convex objective functions, nonlinear equality constraints and equilibrium constraints. Along with these non-convexities, various degeneracies may arise that are difficult for traditional NLP algorithms to handle.

To overcome these difficulties and enable large scale applications, we developed an interior-point penalty function method for non-convex NLP, where the penalty function is defined by the constraint violation measured in the Euclidean norm. We showed that this penalty function provides a perfect regularization to the Newton system that is solved at every interior-point method iteration. This enabled us to establish strong global convergence results for our method without assuming that the constraint gradients are non-degenerate. In particular, the linear independence constraint qualification, which is widely used in previous work, is relaxed to the more practical Mangasarian-Fromovitz constraint qualification. To the best of our knowledge, our method is one of the first that enjoys this feature. In addition, we also proved that our method enjoys fast local convergence at a superlinear rate.

On the practical side, in order to force convergence more efficiently, we recently adapted a piecewise linear penalty function (PLPF) approach to our interior point

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framework. The PLPF approach generalizes the traditional penalty approach while eliminating the need to predict a good penalty parameter in advance, and is closely related to the filter method proposed by Fletcher and Leyffer. Working with Andreas Wächter (IBM), we have implemented our method within the open source software package IPOPT for NLP. Extensive numerical tests show that our method is efficient and robust in solving large scale and ill-posed difficult problems. In particular, our algorithm is very competitive with state-of-the-art interior-point codes.

MPECs have received increasing attention in recent years for their numerous engineering and economics applications. For instance, MPECs have been used to model Stackelberg (leader-follower) games for analyzing electric power markets and traffic equilibrium problems.

MPECs are highly degenerate problems since their feasible region does not contain any interior point. This prevents the direct use of NLP algorithms, especially interior-point algorithms. The main bottleneck of current algorithms for MEPCs is that they are not able to guarantee convergence to first-order solutions of MPECs. To overcome this difficulty, we developed an active-set method for MPECs for which the complementarity constraints are linear and the objective function is allowed to be nonlinear and non-convex. Our method is a primal-dual active set projected Newton method. The projection space is defined by the active set. The algorithm is easily implementable: the major cost of each iteration involves only one matrix factorization and is comparable to that of an interior-point iteration. Our method has guaranteed convergence to first-order solutions of MPECs. To our knowledge, this is the first method that enjoys this property. Moreover, under additional second-order sufficient conditions and strict complementarity, the asymptotic rate of convergence of our method is quadratic.

References:

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