"Optimization Approaches to Image Restoration and Inverse Problems"

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Summary

Image restoration and multi-scale decomposition problems are so-called inverse problem. More complicated examples include determining the properties of large basins to predict ground motion under strong earthquakes. The traditional approach for solving inverse problems using total variation regularization is through the solution of partial differential equations. In this project we describe how to formulate and solve these problems by interior-point algorithms for second-order cone programs and parametric maximum flow algorithms. The latter approach is extremely fast and facilitates the solutions of large-scale three-dimensional problems. We also derive a new iterative regularization paradigm based on Bregman functions.

The aim of inverse problems is the reconstruction of the cause for an observed *identification* or a desired *design* effect (e.g., topology optimization). Inverse problems arise in military, imaging, biomedical, geological, and financial applications. An important example of an inverse problem is the determination of the locations of sources of radiation from measurements taken at an array of sensors.

Inverse problems tend to be ill-posed in the sense that noise in the data (measurements, model parameters) may result in large errors in the solution; hence, regularization techniques are usually applied to obtain stable solutions. In this project we are studying the properties and efficient methods for solving total variation-based regularized inverse problems. Our focus thus far has been on inverse problems in imaging.

Working with Stanley Osher (UCLA), Martin Berger (Univ. of Linz, Austria) and one of my doctoral students, Wotao Yin, we developed an iterative regularization procedure based on Bregman functions. For image denoising this procedure works as follows. A total-variation based denoising algorithm is applied to a noisy image **f**, producing a denoised image **u**. The difference **v** between **f** and **u** contains the remaining noise and some of the desired image since the denoising is not perfect. This "noise" **v** is then added to **f** and the denoising algorithm is then reapplied to **f + v**. Remarkably, this gives an improved image. Moreover, we have been able to prove that by repeating this process, the images produced get closer and closer to the true noiseless image (which we don't even know) in the sense that the Bregman distance between the generated images **u** and this true image keeps getting smaller, until the norm of **f-u** is less than the norm of

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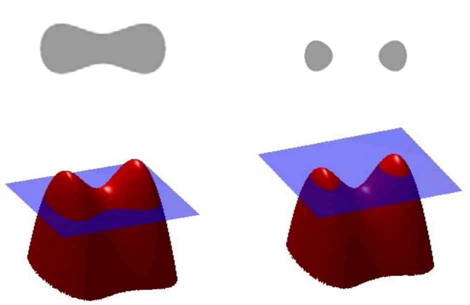
the noise in **f**. These results are applicable to more general inverse problems. This breakthrough has led to a completely new and fertile area of research.

We have shown how to apply interior-point algorithms combined with nested dissection ordering heuristics to solve image denoising problems efficiently both in theory (i.e., in polynomial time) and in practice.

Our most recent work on image restoration and multi-scale decomposition has focused on developing extremely fast algorithms to solve these problems, using both Gallo, Grigoriadis and Tarjan's parametric version of the Goldberg-Tarjan preflow-push max-flow algorithm and a divide-and-conquer approach. This revolutionary approach is based on a nesting property of the level sets of an image and it makes it possible for the first time to denoise high resolution 3-D and 4-D images in a reasonable amount of time.

We have also studied how to separate image and signal features according to their geometric scales and extract texture information.



Level set decomposition of an image

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