High Order Schemes for Complex Shocked Flow

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Summary

Richtmyer-Meshkov instability (RMI) describes the evolution of a perturbed interface separating two gases of different densities accelerated impulsively by an impinging strong shock. The interface evolves into complex bubble and spike structures as well as vortices along the interface mainly due to the deposition of strong vorticities by the misalignment of the pressure gradient and density gradient. The physics of the RMI is modeled by a system of hyperbolic partial differential equations (PDE). For a long term simulation, high order schemes are the method of the choices for this class of problems as the lower order schemes are not capable for capturing the evolution of the small scales accurately over time. The applications of the RMI include but not limited to the supernova explosion and fusion implosion. The complex flow field containing large and small scales can be further handled adaptively by sub-dividing the physical domain into a number of sub-domains. The multi-domain nature of the algorithm allows the PDE be solved efficiently and adaptively with appropriate high order schemes (spectral and WENO schemes) temporally and spatially based on a high order measurement of the local smoothness of the solution in each sub-domain.

The Richtmyer-Meshkov instability is a fundamental fluid instability that develops when perturbations on an interface separating gases with different properties grow following the passage of a shock. This instability is typically studied in shock tube experiments, in which an incident shock passes through an initially perturbed interface separating the gases. Following the passage of the shock, the interface is set in motion along the direction of shock propagation and a transmitted shock enters the second gas. The misalignment of the density and pressure gradients causes a deposition of vorticity on the interface through baroclinic vorticity production. The vorticity deposited by the shock on the interface drives the instability, which results in inter-penetrating bubbles and spikes. At later times, complex roll-up structures form on the spike and the vorticity forms strong cores. The transmitted shock reflects from the end wall of the shock tube and interacts with the evolving interface during reshock phase, further contributing to the development of complex interacting fluid and wave structures.

The detail understanding of the physics of the RMI and RT are critical for an accurate performance prediction and design improvement of the National Ignition Facilities (NIF) that is expected to start operation in the near future. We have performed a long time simulation of the Mach 1.5 RMI flow field with a perturbed membrane with random noise using the 9th order WENO finite difference scheme on the

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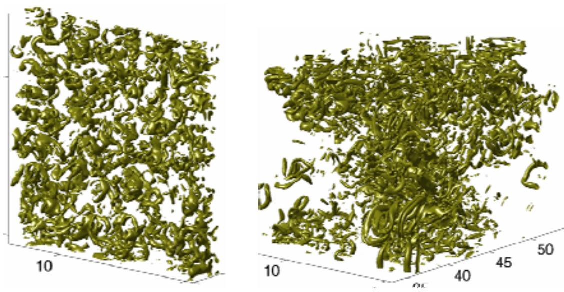
IBM-Blue machine at the Lawrence Livermore National Laboratory and the evolution of the enstrophy of the flow field is shown at the early time and late time in the figure below. The 9th order scheme has been shown to be more efficient and accurate in capturing the fine scale structures than the 5th order scheme.

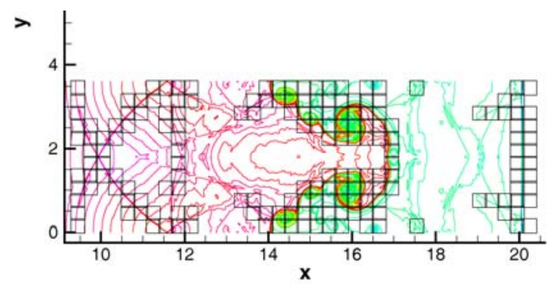


The enstrophy, magnitude of the vorticity, of the three dimensional RMI simulation with a perturbed membrane coupled with random noise shows the spreading and fragmentation of small scales structures in the early time (left) and the late time (right) after the reflection of the shock from the end wall of the shock tube.

We have also developed a high order multi-domain hybrid Spectral-WENO finite difference scheme (Hybrid) for hyperbolic conservation laws. The idea is to apply distinct methodologies to smooth and discontinuous parts of the solution in a temporal and spatial adaptive multi-domain framework. The computational domain is subdivided into a number of sub-domains. In each sub-domain

- High order WENO scheme for local treatment of high gradients such as shocks and contact discontinuities;
- Global spectral representation of the smooth and complex fine scale structures;
- High order multi-resolution analysis is used to measure the smoothness of the solution.

The Hybrid algorithm was developed and tested on the shock tube problems with Riemann IVP such as the Lax, Sod and 123 and shock-density wave interaction. The Hybrid algorithm was also used to simulate the two dimensional Mach 4.46 and Mach 8 RMI with a sinusoidal perturbation of the interface separating the Xenon and Argon gases as shown in the figure below. The WENO sub-domains are enclosed by the boxes and the rest of the sub-domains are the spectral one. A typical speedup factor is in the range of 1.5-2 when compared with the pure WENO runs.



The density of Mach 8 RMI as computed by the Hybrid scheme is shown here. The enclosing boxes are WENO sub-domains and the rest are spectral.

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