Energy Levels of One-Electron Atoms

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The table of precise one-electron atomic energy levels given by Garcia and Mack in 1965 is expanded to include all atomic numbers and more energy levels, updated by using more recent values of fundamental constants and radiative corrections, and extended to the maximum precision allowed by quantum electrodynamics (QED) calculations. All levels with $n \leqq 11$ are given for $Z \leqq 15$, with $n \leqq 5$ for $Z \leqq 39$, and with $n \leqq 3$ for $Z \leqq 105$. In addition, the $S_{1/2}$ and $P_{1/2}$ and $j=n-1/2$ levels with $n \leqq 20$ are given for $Z \leqq 15$, and with $n \leqq 13$ for $Z \leqq 39$. The uncertainty in the QED calculations is given for each level, and the level is given to that precision. Conversions to different units and corrections for changing the Rydberg or nuclear mass values are pointed out. The paper includes a comprehensive listing and brief discussions of all effects considered and of the uncertainties for the calculated and neglected terms. The Fine Structure Interval (difference between the $j=l\pm 1/2$ levels for given n and l) and its reduced mass and QED contributions are discussed in detail. All known measurements of Lamb shifts and other fine structure differences are compared with calculated values.

Key words: Atomic structure; electron structure; energy levels; fine structure; hydrogenic atoms; Lamb shift; level shifts; quantum electrodynamics; radiative corrections; relativistic corrections.

Contents

1. Introduction

Garcia and Mack published a table [1][1] of one-electron atomic energy levels in 1965, using the best values of masses and other constants [2] and quantum electrodynamics (QED) level shifts [3] known at the time. Since then, the constants have been better determined [4, 5], a discrepancy in the Lamb shift has been eliminated [6], and the precision of the QED level shifts has been improved [7] by an order of magnitude. The present table is intended to update the calculated energy levels, to extend them to more decimal places and larger values of n and Z, and to include the uncertainties in each level. The paper and other tables are intended to provide a complete summary of all of the terms included or considered in the calculations, including comparisons with all known Lamb shift and other fine structure measurements. Use of the table is explained in section 4 without going into details given elsewhere in the paper.

The calculations are discussed in section 2, starting in section 2.1 with the important nonrelativistic nuclear motion effects and relativistic corrections, adding the small but fundamentally important QED effects in section 2.2 and the small nuclear size and structure effects in section 2.3. Section 3 continues with the discussion of the estimated uncertainties in the calculations and fundamental constants and their effect on the uncertainties in the energy levels and their differences. Section 4 explains the entries in the table and their uncertainties and gives instructions on how they may be modified to change units or change the Rydberg constant or the electron-nucleus mass ratio.

The isotopes used (all stable isotopes plus tritium for $Z \leqq 5$, the single most abundant or longest-lived for $Z \geqq 6$) and their masses and sizes are given in table A and briefly discussed in Appendix A. Bethe logarithms (logarithmic average excitation energies occurring in the lowest order Lamb shift and related QED terms) are given in table B and the extrapolations used for uncalculated states explained in Appendix B. Electron

[1] Figures in brackets indicate literature references at the end of this paper.

structure corrections (Lamb shift remainders after the Bethe logarithm) are discussed in Appendix C and their small $Z\alpha$ expansion coefficients are tabulated for $n \leqq 4$ states in table C. Table D compares theory and experiment for all known Lamb shift and other fine structure measurements, including both microwave and optical measurements.

The Fine Structure Interval (difference ΔE_{nl} between states having the same n and l, but different $j=l\pm 1/2$) is distinguished from general fine structure separations throughout the paper, and its calculation is discussed in detail in section 2.4. Because the two states are nonrelativistically identical, most QED and nuclear corrections (and their uncertainties) cancel in ΔE, leaving a result which is more precisely known than either energy level alone or, in most cases, than their other fine structure separations. The calculated ΔE has been used in the past as the basis for the determination of the fine structure constant [2] and is still often (many times incorrectly) treated as "exact" compared to other fine structure separations. (The most common misuse is subtracting a measured $P_{3/2}-S_{1/2}$ separation from a calculated ΔE_{nP} in an attempt to obtain a Lamb shift $\mathcal{S}=S_{1/2}-P_{1/2}$ to compare with theory. What is usually overlooked here is that the $P_{1/2}$ uncertainty, which is larger by eq (2.45) than the $P_{3/2}$ uncertainty, enters both the calculated and "measured" $\mathcal{S}$, but would not enter either the calculated or measured $P_{3/2}-S_{1/2}$ separation. Of course, except in principle, this distinction is unimportant for measurements having much larger uncertainties than the P state uncertainties.) Because the energy level table is not precise enough to yield the most accurate possible ΔE's, we have given complete details for their calculation in section 2.4 and have referred to these in sections 3 and 4 and Appendix C.

The nuclear recoil effects ("reduced mass factors") can be written in many nearly-equivalent ways (differing only in unimportant higher order terms) and easily lend themselves to accidental double-counting or seductive assumptions about their exact form. Therefore, we have discussed them at length (to specify what is done in the present calculation) in section 2.1 and again (with comments on general rules) in section 2.4, using the Fine Structure Interval as an example since it appears in the literature with many different forms of recoil corrections.

Because of the inherent interest and fundamental importance of QED and its importance in determining precise energy levels, we have given a complete set of comparisons with experiment in table D and have given more than minimal discussion of the calculations in section 2.2 and Appendix C. These are intended to be descriptive, explanatory, and complete rather than critical evaluations since the primary purpose of this paper is the presentation of the energy level table, not a review of the status of QED. The QED electron structure calculations are especially important in the table because their uncertainties are dominant both for large n (the uncalculated Bethe logarithms in Appendix B) and for large Z (the approximate higher order remainders discussed in Appendix C).

2. Contributing Terms

In this section, we will discuss the various effects contributing to one-electron atomic energy levels, neglecting the hyperfine structure (which does not affect their centroids). Not all of the effects discussed are included in the calculated energy levels, but we will point out which terms are used (and what form is used), which are not, and which are included in the uncertainties treated in further detail in section 3. The discussion is roughly arranged according to the effects considered so that it may be used as a framework for further work.

The recoil ("reduced mass") corrections to different terms are the most confusing, since they overlap when mixed with relativistic effects, so we consider them at some length in section 2.1, refer to them briefly in section 2.2, and return to a discussion of (the lack of) general rules governing reduced mass factors in section 2.4, where the Fine Structure Interval is discussed in detail. Our results are not intended for atoms like positronium or muonium so, if forced to choose, we will assume that the electron-nuclear mass ratio m/M is a very small number rather than assuming the electron-nuclear coupling $Z\alpha = Ze^2/\hbar c$ is small.

Quantum Electrodynamics effects are considered in section 2.2, concentrating on two or three types of effects (self-energy and magnetic moment are electron structure, vacuum polarization is photon structure) and their dependence on m/M, on the basic QED expansion parameter α, and on the strength of the electron-nucleus electromagnetic coupling $Z\alpha$. The m/M corrections are the smallest and are discussed first. Higher order corrections in α/π are also small and are listed with few comments. The $Z\alpha$ dependence, although it relates as much to relativistic quantum mechanics as to QED, is presented in more length because it is more important for the results, its calculation presents most of the uncertainties in the QED contributions, and the reasons for the nature of the $Z\alpha$ dependence are not summarized elsewhere. The calculated $Z\alpha$ dependence used cannot be given exactly in closed form, but some closed-form terms are given as examples, and the coefficients in the small $Z\alpha$ expansion of the results used are given in Appendix C, with discussion of the calculations and comparison with other calculations.

Section 2.3 discusses effects of the finite nuclear sizes (given in Appendix A) and the much smaller effects of nuclear structure. Their contribution to the tabulated energy levels is almost negligible compared to other uncertainties, so we treat them very simply. The secondary effects at very large Z are only briefly discussed, mostly in relation to the $Z\alpha$ dependence of the QED contributions, and their importance in muonic atoms is not even mentioned.

2.1. Mass Dependence and Relativistic Effects

The energy levels of an electron in the electrostatic potential of a fixed point charge Ze are given by

$$E_{\mathrm{NR}}=-\frac{(Z\alpha)^2mc^2}{2n^2}=-\frac{Z^2}{n^2}hcR_\infty \tag{2.1}$$

according to the nonrelativistic Schrödinger equation. There are two correction factors (for recoil and relativity) which are well-understood only if treated separately. Nuclear motion can be exactly accounted for nonrelativistically by replacing the electron mass m by the reduced mass $\mu\equiv mM/(M+m)$; which multiplies the energy levels by an overall reduced mass factor

$$\frac{\mu}{m}=\frac{M}{M+m}=\frac{R_M}{R_\infty}, \tag{2.2}$$

where R_M is the Rydberg for a nucleus of finite mass M. Spin and other relativistic effects given by the Dirac equation yield an exact correction factor

$$\frac{E_{\mathrm{D}}}{E_{\mathrm{NR}}}=\left(1-\left[1+\left(\frac{Z\alpha}{n-\epsilon}\right)^2\right]^{-1/2}\right)\frac{2n}{(Z\alpha)^2}$$

(with $\epsilon\equiv j+1/2-[(j+1/2)^2-(Z\alpha)^2]^{1/2}$)

$$=1+\frac{(Z\alpha)^2}{n^2}\left(\frac{n}{J+1/2}-\frac{3}{4}\right)+0(Z\alpha)^4 \tag{2.3}$$

which gives (fine structure) separation of levels with different values of total angular momentum $j=l\pm1/2$, but does not produce any (Lamb shift) splitting of levels with the same value of j but different values of orbital angular momentum l. The combination of these recoil and relativistic correction factors yields

$$E_{\mathrm{D}\mu}=-\frac{Z^2}{n^2}hcR_\infty\frac{\mu}{m}\left(1-\left[1+\left(\frac{Z\alpha}{n-\epsilon}\right)^2\right]^{-1/2}\right)\frac{2n^2}{(Z\alpha)^2}$$

$$=\mu c^2\left[1+\left(\frac{Z\alpha}{n-\epsilon}\right)^2\right]^{-1/2}-\mu c^2 \tag{2.4}$$

corresponding to a fictitious Dirac particle of reduced mass μ moving in the field of a fixed point nucleus. This is *not* the result of any correct treatment of the relativistic two-body problem, but is often (as here) arbitrarily taken as a convenient starting point for any such considerations. Subsequent corrections for the relativistic effects of nuclear motion usually utilize expansions, either in powers of m/M or in powers of $Z\alpha$.

To illustrate that the relativistic two-body problem does not have an exact "reduced-mass" reduction to an equivalent one-body equation, consider the simple formula proposed by Brezin, Itzykson, and Zinn-Justin [8],

$$\left(M+m+\frac{E}{c^2}\right)^2=M^2+m^2+2mM\left[1+\left(\frac{Z\alpha}{n-\epsilon}\right)^2\right]^{-1/2}$$

$$=(M+m)^2+2(M+m)\frac{E_{\mathrm{D}\mu}}{c^2}, \tag{2.5}$$

which seems to be correct (except for uncalculated spin effects only partially approximated by the term ϵ). This clearly cannot be written in terms of a single effective mass, but can be written as an energy-dependent correction to the reduced-mass Dirac energy levels $E_{D\mu}$,

$$\frac{E}{E_{D\mu}}=\frac{2}{1+[1+2E_{D\mu}/(M+m)c^2]^{1/2}}$$

$$=1-\frac{1}{2}\frac{E_{D\mu}}{(M+m)c^2}+0\left(\frac{E/c^2}{M+m}\right)^2, \tag{2.6}$$

and reduces to that result in the limit of the binding energy E being negligibly smaller than the total rest energy Mc^2+mc^2; this limit is given either by small m/M or by small $Z\alpha$. The leading correction (to lowest order in $Z\alpha$) is an energy shift

$$-\frac{1}{2}\frac{E^2_{NR\mu}}{(M+m)c^2}=-\frac{(Z\alpha)^4\mu^2c^2}{8n^4\,(M+m)}$$

$$=-\frac{\mu}{M+m}\frac{\alpha^2}{4}\frac{Z^4}{n^4}hcR_M, \tag{2.7}$$

previously obtained by others [9, 10], which (in this nonrelativistic or small $Z\alpha$ limit) is independent of j or l and thus does not contribute to fine structure or Lamb shift splitting.

A relativistic two-body Breit equation has been reduced to approximate forms corresponding to an expansion in powers of $Z\alpha$ by Barker and Glover [10], who obtained results to order $(Z\alpha)^4mc^2$ which are exact in their mass and angular momentum dependences. Besides the terms discussed in the previous paragraph (the Dirac energies (2.4) and the energy shift (2.7)), they obtain an additional correction factor of $1-(\mu/M)^2$ for the spin-orbit term; this shifts energy levels for non-S-states by

$$-\frac{\mu^2}{M^2}\frac{(Z\alpha)^4\mu c^2}{2n^3}\frac{C_{lj}}{2l+1}\text{ where }C_{lj}\equiv\frac{\langle\sigma\cdot\mathbf{L}\rangle}{l(l+1)}$$

$$=\begin{pmatrix}1/(l+1) & j=l+1/2\\ -1/l & j=l-1/2\end{pmatrix} \tag{2.8}$$

and thus not only contributes to fine structure but also to the Lamb shift splitting, although very slightly. This splitting shows that the simple formula (2.5) proposed by Brezin, Itzykson, and Zinn-Justin [8] is not correct to order $(m/M)^2\ (Z\alpha)^4\ mc^2$ even though it is exact in $Z\alpha$ to lowest order in m/M and exact in m/M to order $(Z\alpha)^2$. Barker and Glover also obtain reduced mass factors of $(\mu/m)^3$ for S-states and $(\mu/m)^2$ for non-S-states for intrinsic (anomalous) magnetic moment terms, but these will be discussed with other radiative corrections.

Besides the Breit equation contributions already considered, the fully covariant relativistic two-body Bethe-Salpeter equation contains additional recoil contributions of order $(m/M)(Z\alpha)^5mc^2$ originally derived for small m/M by Salpeter [11], slightly corrected and extended to arbitrary m/M by Fulton and Martin [12], and reconfirmed by an effective potential approach by Grotch and Yennie [13]. By combining the general m/M results of Fulton and Martin with the Z dependence discussed by Salpeter and the state-dependence given by Erickson [3, 14], we can write these in the form

$$\frac{\mu^3}{mM}\frac{4(Z\alpha)^5c^2}{3\pi n^3}\left(2\left[\left(\ln\frac{1}{(Z\alpha)^2}+\frac{11}{24}\right)\delta_{l0}+L_n\right]\right.$$
$$-\frac{7}{2}\left[\left(\ln\frac{n}{2Z\alpha}-\sum_{1}^{n}\frac{1}{i}+\frac{1}{2n}-1\right)\delta_{l0}+\frac{1-\delta_{l0}}{2l(l+1)(2l+1)}\right]$$
$$\left.-\delta_{l0}+\frac{2-7/2}{M^2-m^2}\left[M^2\ln\frac{m}{\mu}-m^2\ln\frac{M}{\mu}\right]\delta_{l0}\right) \qquad (2.9)$$

where L_n in the first bracket [2] here is defined with eq (2.10), and we see that this is a recoil correction for the QED self-energy contribution to be discussed early in the next section.

2.2. Quantum Electrodynamics Effects

Turning our attention from the minor mass dependences of the energy levels to the important radiative shifts and splittings of these levels, we find three different lowest order effects of quantum electrodynamics. The largest is the electron *self-energy* or vacuum fluctuation contribution, mostly for S-states,

$$\frac{4\alpha(Z\alpha)^4mc^2}{3\pi n^3}\left(\frac{\mu}{m}\right)^3\left[\left(\ln\frac{1}{(Z\alpha)^2}\right.\right.$$
$$\left.\left.+\ln\frac{m}{\mu}+\frac{11}{24}\right)\delta_{l0}+L_n\right], \qquad (2.10)$$

where L_n is the ($Z\alpha$- and m/M-independent part of the) Bethe logarithm discussed in Appendix B. The electron's anomalous *magnetic moment* interaction increases the 11/24 in (2.10) by 3/8 for S-states and contributes the largest non-S-state shift, a spin-orbit term

$$\frac{\alpha}{2\pi}\frac{(Z\alpha)^4mc^2}{n^3}\left(\frac{\mu}{m}\right)^2\frac{C_{lj}}{2l+1}; \qquad (2.11)$$

the magnetic moment reduced mass factors, $(\mu/m)^3$ for $l=0$ and $(\mu/m)^2$ for $l\neq 0$, are given by Barker and Glover [10]. The remaining lowest order radiative level shift arises from *vacuum polarization* (due to virtual electron-positron pairs) modifying the Coulomb potential and shifts S-states downwards by an amount given by adding $-1/5$ to the 11/24 in (2.10).

These radiative level shifts have three different types of higher order corrections, corresponding to expansions in powers of m/M, α, and $Z\alpha$. The smallest of these corrections, two-body effects obtained by Fulton and Martin [12] by interchanging the roles of the electron and the nucleus, are important and simple for positronium, but are only of relative order $(m/M)^2$ and will not be included here for the following reasons. The vacuum polarization effect of virtual nucleon-antinucleon pairs will not be included since it is smaller than the effect of less massive pairs; we will include only the muon pair vacuum polarization effect. The magnetic moment of the nucleus contributes to hyperfine splitting (via both a spin-orbit term and a spin-spin term) but each of these cancels in the weighted average over the hyperfine levels. The self-energy of the nucleus is actually a nuclear structure correction since it would require consideration of whether vacuum fluctuations in the electromagnetic field affect the nuclear constituents separately or as a whole. In obtaining the Z dependence of equation (2.9), Salpeter [11] treated the nucleus approximately as a structureless particle of charge Ze and mass M when it exchanges photons with the electron. Nuclear structure corrections to that approximation are probably of the same order of magnitude, $(m/M)^2(Z\alpha)^5mc^2$, as the nuclear self-energy effect and we will not include either, except as uncalculated terms contributing to the uncertainties.

Simple reduced mass correction factors are already included in the lowest order radiative level shifts (2.10) and (2.11) but will not be included in the higher order terms to be discussed, even though they may be known to be $(\mu/m)^3$. These reduced mass corrections are not included because they are the same order of magnitude, $(m/M)\ \alpha(Z\alpha)^5mc^2$, as the unknown α or $Z\alpha$ corrections to eq (2.9) and are expected to cancel to some extent, just as (2.9) partially cancels the reduced mass correction in (2.10).

Higher order radiative corrections yield terms of relative order α/π which only need to be evaluated to the lowest order in $Z\alpha$ (corresponding to the static

[2] The small $\ln(m/\mu)=m/M$ term from (2.10) was inadvertently included in the computer calculation of (2.9), but this is so much smaller than the uncertainties that none of the results given in the energy level table were affected.

limit of long wavelengths or zero momentum transfer). The vacuum polarization contribution [15] is simply

$$-\frac{82}{81}\frac{\alpha^2}{\pi^2}\frac{(Z\alpha)^4mc^2}{n^3}\delta_{l0}. \quad (2.12)$$

The self-energy or vacuum fluctuation contribution

$$\sigma_4\frac{4(Z\alpha)^4mc^2}{n^3}\delta_{l0} \quad (2.13)$$

has had a more difficult history [16], but is now reliably given by

$$\sigma_4 \equiv m^2\frac{d}{dq^2}F_1^{(4)}(q^2)\bigg|_{q^2=0}$$

$$=\left(\frac{\alpha}{\pi}\right)^2\left[-\frac{4819}{5184}-\frac{49}{432}\pi^2+\frac{\pi^2}{2}\ln 2-\frac{3}{4}\zeta(3)\right]$$

$$=\left(\frac{\alpha}{\pi}\right)^2[0.470^-]. \quad (2.14)$$

The higher order magnetic moment contribution

$$(a_4+a_6)\frac{(Z\alpha)^4mc^2}{n^3}\frac{C_{lj}}{2l+1}, \quad (2.15)$$

like (2.11), shifts all states and contributes to fine-structure splitting, so we include both the fourth-order coefficient

$$a_4=\left(\frac{\alpha}{\pi}\right)^2\left[\frac{197}{144}+\frac{\pi^2}{12}-\frac{\pi^2}{2}\ln 2+\frac{3}{4}\zeta(3)\right]$$

$$=\left(\frac{\alpha}{\pi}\right)^2[-0.328^+] \quad (2.16)$$

and the sixth-order coefficient [17, 5]

$$a_6=\left(\frac{\alpha}{\pi}\right)^3[1.285\pm0.057]. \quad (2.17)$$

Somewhat better values are available [18], but the effect on the energy levels is negligible, so we will use the value (2.17) for uniformity with reference [5]. Although a_6 gives a nearly negligible energy shift, it is the highest order term calculated in quantum electrodynamics and is directly tested by the measurement [19] of the electron's magnetic moment anomaly

$$\frac{g-2}{2}\equiv a=(1{,}159{,}656.7\pm3.5)\times10^{-9} \quad (3.0\text{ ppm})$$

$$=\frac{\alpha}{2\pi}+a_4+\left(\frac{\alpha}{\pi}\right)^3[1.62\pm0.30], \quad (2.18)$$

where

$$\alpha^{-1}=137.03612\pm0.00015 \quad (1.1\text{ ppm}) \quad (2.19)$$

is the "WQED" value [5] given by experiments that do not require quantum electrodynamics for their analyses. For a more recent comparison, see reference [18].

The most important corrections to the radiative level shifts (2.10) and (2.11), those of relative order $\pi Z\alpha$, are due to electron structure of the order of its Compton wavelength, corresponding to quantum electrodynamics interactions in which the electron's momentum is of the order of mc rather than $Z\alpha mc$. For the magnetic moment form factor

$$\frac{\alpha}{2\pi}F_2(q^2)=\frac{\alpha}{2\pi}\int_0^1\frac{m^2du}{m^2+q^2(1-u^2)/4}, \quad (2.20)$$

we find the ground state contribution for a nonrelativistic wave function is

$$\frac{\alpha}{2\pi}\frac{(Z\alpha)^4mc^2}{n^3}\int_0^1\frac{du}{[1+Z\alpha\sqrt{1-u^2}]^2}$$

$$=\frac{\alpha}{2\pi}\frac{(Z\alpha)^4mc^2}{n^3}\left(1-\frac{Z\alpha\cos^{-1}Z\alpha}{\sqrt{1-(Z\alpha)^2}}\right)\frac{1}{1-(Z\alpha)^2}, \quad (2.21)$$

whose static ($q^2=0$) or small $Z\alpha$ limit is the lowest order contribution previously considered (the 3/8 already included with the 11/24 in eq (2.10)). The higher order remainder,

$$\frac{\alpha}{2\pi}\frac{(Z\alpha)^4mc^2}{n^3}\left(\frac{-\cos^{-1}Z\alpha}{\sqrt{1-(Z\alpha)^2}}+Z\alpha\right)\frac{Z\alpha}{1-(Z\alpha)^2}\delta_{l0}, \quad (2.22).$$

is found to be a well-behaved function of $Z\alpha$ (even for $Z\alpha\geqq 1$) whose leading term for small $Z\alpha$,

$$\frac{\alpha}{2\pi}\frac{(Z\alpha)^4mc^2}{n^3}\left(\frac{-\pi}{2}\right)Z\alpha\,\delta_{l0}, \quad (2.23)$$

is exactly the same as obtained for an arbitrary state by a complete relativistic calculation [20] up to terms of order $\alpha(Z\alpha)^6mc^2$. Although (2.22) is not exactly correct, we will use it since it is a better approximation for all $Z\alpha$ than is the small $Z\alpha$ leading term (2.23) alone. The error is of order $\alpha(Z\alpha)^6mc^2$ for small $Z\alpha$ and probably smaller than (2.22) itself for large $Z\alpha$. Similarly, for the vacuum polarization (photon structure) modification

$$\Pi(q^2)-\Pi(0)=\frac{\alpha}{\pi}\int_0^1\frac{q^2u^2(1-u^2/3)}{4m^2+q^2(1-u^2)}du \quad (2.24)$$

of the Coulomb interaction, we use its contribution for the nonrelativistic ground state wave function,

$$-\frac{\alpha}{\pi}\frac{(Z\alpha)^4mc^2}{n^3}\delta_{l0}\int_0^1\frac{u^2(1-u^2/3)\,du}{[1+Z\alpha\sqrt{1-u^2}\,]^2}, \qquad (2.25)$$

whose small $Z\alpha$ behavior,

$$\frac{\alpha}{\pi}\frac{4(Z\alpha)^4mc^2}{n^3}\left(\frac{-1}{15}+\frac{5\pi}{192}Z\alpha-\frac{3}{35}(Z\alpha)^2+\frac{7\pi}{256}(Z\alpha)^3+\ldots\right) \qquad (2.26)$$

(where $m^2\Pi'(0)=-\alpha/15\pi$ gives the $-1/5$ already included with the 11/24 in eq (2.10)), agrees with the correct calculation through order $\alpha(Z\alpha)^5mc^2$. Here, however, the relativistic wavefunction yields a small additional contribution of lower order than $\alpha(Z\alpha)^6mc^2$,

$$-\frac{2}{15}\frac{\alpha}{\pi}\frac{(Z\alpha)^6mc^2}{n^3}\ln(Z\alpha)^{-2}\delta_{l0}, \qquad (2.27)$$

which we will arbitrarily represent by

$$-\frac{2}{15}\frac{\alpha}{\pi}\frac{(Z\alpha)^6mc^2}{n^3}\delta_{l0}\int_0^1\frac{dz(1+2z)(1-z)}{z+(Z\alpha)^2(1-z)}$$

$$=-\frac{2}{15}\frac{\alpha}{\pi}\frac{(Z\alpha)^6mc^2}{n^3}\delta_{l0}\left(\frac{\ln(Z\alpha)^{-2}}{1-(Z\alpha)^2}[1-3(Z\alpha)^2]+2(Z\alpha)^2\right)\frac{1}{[1-(Z\alpha)^2]^2} \qquad (2.28)$$

in order to have the same small $Z\alpha$ behavior without the unrealistic large $Z\alpha$ behavior of (2.27).

The magnetic moment and vacuum polarization calculation details in the previous paragraph were given not because of their numerical importance, but because they are relatively simple examples of the electron and photon structure effects involved and of the nature of the $Z\alpha$ approximations used. The self-energy contribution is by far the largest part of the radiative level shift, due to the high probability of emission and re-absorption of long-wavelength virtual radiation ("infra-red divergence"). Its calculation is especially complicated by the corresponding importance of the binding of the electron in its intermediate state, which yields the large "infra-red Bethe logarithm", $\ln(Z\alpha)^{-2}+L_n$ in eq (2.10). In the total electron structure contribution,

$$\frac{4\alpha(Z\alpha)^4mc^2}{3\pi n^3}\left(\left[\ln\frac{1}{(Z\alpha)^2}+\frac{11}{24}\right]\delta_{l0}+L_n+\frac{3}{8}\frac{C_{lj}}{2l+1}+H(Z\alpha)\right), \qquad (2.29)$$

$H(Z\alpha)$ is defined to be the exact remainder after the lowest order self-energy (2.10) and magnetic moment contributions. The approximation we use for the higher order electron structure correction $H(Z\alpha)$ is similar to its magnetic moment part (2.22) and the vacuum polarization terms (2.25) and (2.28) in that it results mostly from exact calculations with nonrelativistic wave functions, is well-behaved for large $Z\alpha$, and agrees with known small $Z\alpha$ behavior up to order $\alpha(Z\alpha)^6mc^2$. The uncertainty in this approximation, $\delta H(Z\alpha)$, is estimated to be of the order of $\pm0.5(Z\alpha)^2$ for small $Z\alpha$ and ±0.25 for large $Z\alpha$ for S-states, and various fractions of this for non-S-states. The approximation and uncertainty used are discussed further in Appendix C, where it is pointed out that the expansion coefficients given in table C and cited in section 2.4 are correct only for C_5, C_{62}, and C_{61}, while those for C_{60} and C_7 are correct coefficients only for our approximation for $H(Z\alpha)$.

To summarize, we use the reduced-mass-corrected lowest order self-energy contribution (2.10), including an additional 3/8 and $-1/5$ for S-state magnetic moment and vacuum polarization contributions and (2.11) for non-S-state magnetic moment contributions. For the higher order corrections in $Z\alpha$, we use $H(Z\alpha)$ in (2.20), as discussed in Appendix C, for the electron structure correction and (2.28) plus the higher order remainder of (2.25) plus $(m/m_\mu)^2$ times (2.25) (with the internal $Z\alpha$ replaced by $(m/m_\mu)Z\alpha$) for the photon structure correction including vacuum polarization due to virtual muon pairs. The photon structure (vacuum polarization) corrections could be done better than this, but they are so small for our atoms that even the higher order corrections included are almost negligible. The photon structure uncertainty is included with the electron structure uncertainty in $\delta H(Z\alpha)$. For the higher order corrections in α, we use (2.12) and (2.13) and (2.15). The contribution of (2.9) is used, but whether it is considered a relativistic (sec. 2.1) or QED (sec. 2.2) correction is a matter of semantics or taste, as it has no powers of α (only $Z\alpha$ factors) but arises from the Bethe-Salpeter equation.

2.3. Nuclear Size Effects

Finally, let us consider the effects of the finite size of the nucleus. The only contribution we will explicitly include is the lowest order shift of the nonrelativistic energy levels,

$$\frac{\langle r^2\rangle}{(\hbar/mc)^2}\frac{2(Z\alpha)^4mc^2}{3n^3}[1+C_{\mathrm{str.}}] \quad \text{for } l=0, \qquad (2.30)$$

where the RMS radius of the nucleus,

$$<r^2> = 6G'_E(0), \tag{2.31}$$

is obtained primarily from low-momentum-transfer electron-scattering measurements of the nuclear electric form factor $G_E(q^2)$, as noted in Appendix A, and the nuclear structure correction $C_{str.}$ is -0.004 ± 0.002 for Hydrogen and -0.0147 ± 0.0055 for other nuclei [21]. For $l \neq 0$, the $1+C$ in (2.30) is replaced by

$$\frac{5}{5+2l}\left(\frac{<r^2>}{a^2}\right)^l, \tag{2.32}$$

where $a = n\hbar/Z\alpha mc$ is the Bohr radius; this assumes a uniform spherical charge distribution, but the contribution is not appreciable in any event. No other corrections of relative order $(Z\alpha)^2$ or $<r^2>/a^2$ have been included.

The finite nuclear size changes the electron's wave functions (and energy levels and potential energy operator) and thus affects the QED shifts, but only by a negligible amount

$$-\frac{8}{3}\frac{Z\alpha<r^2>}{(\hbar/mc)^2}\frac{4\alpha(Z\alpha)^4mc^2}{3\pi n^3}\delta_{l0} \tag{2.33}$$

for small $Z\alpha$; the order of magnitude of this effect has been double-checked, but not the numerical coefficient, so we will include it as a small uncertainty rather than as a contribution.

For large $Z\alpha$, the important effect of nuclear size is not its shift of nonrelativistic energies but its removal of spurious singularities at $Z\alpha = 1$ in relativistic energies and wave functions. These singularities are related to the singularity at small distances of the term $V^2 = (Z\alpha)^2/r^2$ arising from the use of a pure Coulomb potential in the Dirac wave equation, and take the form of cusp factors like (2.3),

$$\left.\frac{E_D}{E_{NR}}\right|_{n=1} = \frac{2}{1+[1-(Z\alpha)^2]^{1/2}} \tag{2.34}$$

(where $E_D + mc^2 = [1-(Z\alpha)^2]^{1/2}mc^2$), which yield nearly negligible effects except for large $Z\alpha$, where they are finite but have infinite derivatives at $Z\alpha = 1$. A finite nucleus prevents these singularities and allows the ground state energy, as a function of atomic number, to go smoothly past $Z\alpha = 1$ (where $E + mc^2 \simeq 0$) down to join the negative energy continuum ($E + mc^2 < -mc^2$) at [22, 23] about $Z \simeq 168$. We account for this finite nucleus effect only by freely using nonrelativistic Coulomb wave functions (which do not have any such singularities) for all $Z\alpha$ in QED terms such as (2.22) and (2.25), and by approximating effects of relativistic correction wave functions, such as (2.27), by functions, such as (2.28), which smoothly extrapolate the known behavior at small $Z\alpha$ to finite values at $Z\alpha = 1$ and $Z\alpha \to \infty$. An improved approximation would determine the $Z\alpha = 1$ values directly rather than merely let them follow arbitrarily, as they do now, from simplicity requirements on the approximating function. This improvement was not attempted for the present calculation because the only atoms affected would be those ($Z > 70$) for which better results [24–27] already exist.

2.4. Reduced Mass Factors and the Fine Structure Interval

Let us discuss the following observation on the nuclear mass dependence of higher order terms and the "correct" number of reduced mass factors: There seems to be no single proven rule, merely conjectures or preferences. The form of mass dependence used for a term of a given order depends both on the amount of reduced mass dependence contained in lower order terms (which determines how many powers of m/M *must* be present) *and* on the amount left to still higher order terms (which depends on how many powers of μ/m are *arbitrarily* used in this term). A *single* reduced mass factor like (2.2) is appropriate for the lowest order term (2.1) and might be arbitrarily chosen as an overall factor in any or all other terms, such as the Dirac energy (2.4). On the other hand, *three* reduced mass factors are appropriate for small distance perturbations of the nonrelativistic two-body problem, like (2.10), which are proportional to the square of the wave function at the origin (and thus has the dimensions of inverse volume) and contains 3 powers of the reduced mass μ, $(\mu c/\hbar)^3$; this assumes that the small distance perturbation (which has dimensions of energy times volume and provides 2 inverse powers of some mass, $mc^2(\hbar/mc)^3 = \hbar^3/m^2c$) has been arbitrarily chosen to scale like electron mass m rather than, say, reduced mass μ. There is also the possibility of using *two* reduced mass factors for non-S-state contributions of intrinsic magnetic moment terms, as given by Barker and Glover [10].

Consider now the Fine Structure Interval ΔE between the $2P_{1/2}$ and $2P_{3/2}$ states, which is given by

$$\Delta E = \mu c^2\left(\left\{1-\left(\frac{Z\alpha}{2}\right)^2\right\}^{1/2} - \left\{\frac{1+\sqrt{1-(Z\alpha)^2}}{2}\right\}^{1/2}\right),$$

$$= \frac{Z^4\alpha^2R_\infty}{16}\frac{\mu}{m}\left(1+\frac{5}{8}(Z\alpha)^2+\frac{53}{128}(Z\alpha)^4+\ldots\right) \tag{2.35}$$

when we use the common starting point of the Dirac equation for a particle of mass μ, as in (2.4). If we follow Barker and Glover [10], who start with such a Dirac equation and include the anomalous magnetic moment contribution (2.11)

$$\frac{Z^4\alpha^2 R_\infty}{16}\left(\frac{\mu}{m}\right)[g-2], \tag{2.36}$$

then we find an additional spin-orbit term (2.8),

$$\frac{Z^4\alpha^2 R_\infty}{16}\frac{\mu^3}{mM^2}[-1], \tag{2.37}$$

which they cite as "arising as a result of writing the Dirac equation in terms of the reduced mass". Let us note that, by using

$$\frac{\mu}{m}-2\left(\frac{\mu}{m}\right)^2-\frac{\mu^3}{mM^2}=-\left(\frac{\mu}{m}\right)^3, \tag{2.38}$$

we can rewrite the sum of (2.35), (2.36), and (2.37) as

$$\frac{Z^4\alpha^2 R_\infty}{16}\left[\left(\frac{\mu}{m}\right)^2 g-\left(\frac{\mu}{m}\right)^3+\frac{5}{8}(Z\alpha)^2 +0\left(\frac{m}{M}(Z\alpha^2)\right)+0\,(Z\alpha)^4\right]. \tag{2.39}$$

Because this contains no corrections of order m/M or $(m/M)^2$, it might be interpreted as a proof to lowest order in $Z\alpha$ that all magnetic moment terms (both anomalous and Dirac) have 2 reduced mass factors and all other terms have 3 reduced mass factors, and leads to the conjecture that this might be true to all orders in $Z\alpha$. Note that the mass dependence of $5/8(Z\alpha)^2$ must be considered as unknown or arbitrary since Barker and Glover did not calculate the contributions of order $(m/M)(Z\alpha)^2$.

Before returning to the Fine Structure Interval, let us consider the lowest order (in α or $Z\alpha$) contributions of the Bethe-Salpeter equation to the Lamb shift—(2.10), (2.9), and a nuclear self-energy term obtained from (2.10) by interchanging m and M (treating the nucleus as a Dirac particle of charge $+Ze$ and mass M). The terms containing the Bethe logarithm L_n have the sum

$$\frac{4\alpha(Z\alpha)^4c^2}{3\pi n^3}\left(\frac{\mu^3}{m^2}+2Z\frac{\mu^3}{mM}+Z^2\frac{\mu^3}{M^2}\right)L_n =\frac{4\alpha(Z\alpha)^4c^2}{3\pi n^3}\mu^3\left(\frac{1}{m}+\frac{Z}{M}\right)^2 L_n. \tag{2.40}$$

We may interpret this result as showing that the mass scaling for this perturbation is μ', where

$$\frac{1}{\mu'}=\frac{1}{m}+\frac{Z}{M}. \tag{2.41}$$

For $Z=1$, we have $\mu'=\mu$ and obtain only a *single* reduced mass factor

$$\frac{\mu^3}{m}\left(\frac{1}{m}+\frac{Z}{M}\right)^2\Bigg|_{Z=1}=\frac{\mu}{m}, \tag{2.42}$$

just as in the nonrelativistic two-body problem (2.2)! The point of considering the results in this and the preceding paragraph is to note that terms with "obvious" μ/m dependence, like (2.35), can (or should) be combined with related terms to yield a $(\mu/m)^3$ dependence like (2.38), and that terms with "obvious" $(\mu/m)^3$ dependences, like (2.10), combine with related terms to yield a simple μ/m dependence (2.42). This should constitute a strong warning that mass dependences can be rearranged into a number of useful forms and the success or utility of one form should not be taken as denying the possibility of another form also being successful or useful.

For consistency in this and future calculations, we have arbitrarily taken the Dirac energy levels and wave functions for a particle of reduced mass μ, (2.4), as our starting point, and then must add on the necessary recoil corrections (2.7), (2.8), etc. Then, in the lowest order QED terms (2.10), (2.11), and (2.9), the reduced mass is used for the wave functions, but not the operators, and we treat (2.9) as a separate correction distinct from (2.10) and drop the related nuclear self-energy as an unknown term of higher order $(m/M)^2$. For the higher order QED contributions, we sidestep the alternatives of using 1 or 3 factors of μ/m by taking the simpler option of using none.

Our resulting Fine Structure Interval is then

$$\Delta E=\frac{Z^4\alpha^2R_\infty}{16}\Bigg(\frac{\mu}{m}\left[1+\frac{5}{8}(Z\alpha)^2+\frac{53}{128}(Z\alpha)^4+\ldots\right. \left.-\frac{\mu^3}{mM^2}+2\left(\frac{\mu}{m}\right)^2\left[\frac{\alpha}{2\pi}\right]+2[a_4+a_6]\right. +\frac{16}{3}\frac{\alpha}{\pi}[\Delta H(Z\alpha)\pm\delta\Delta H(Z\alpha)]\Bigg), \tag{2.43}$$

where $\Delta H(Z\alpha)\equiv H_{2P_{3/2}}-H_{2P_{1/2}}$ is the contribution of the higher order QED approximation whose series expansion for small $Z\alpha$ is[3]

$$\Delta H(Z\alpha)\simeq-\frac{3}{16}(Z\alpha)^2\ln(Z\alpha)^{-2} +\Delta C_{60}(Z\alpha)^2+\Delta C_7(Z\alpha)^3 \tag{2.44a}$$

$$\simeq-\frac{1}{4}\left(1-\frac{1}{n^2}\right)\delta_{l1}(Z\alpha)^2\left[\ln\frac{1}{(Z\alpha)^2}+\underline{\frac{11}{24}}-\underline{7.476}Z\alpha\right]-\frac{9}{32}\frac{\pi(Z\alpha)^3}{l(l+1)}, \tag{2.44b}$$

[3]The underlined numbers are approximations, as noted in Appendix C.

and where $\delta\Delta H$ is the uncertainty in this Fine Structure contribution, estimated to be

$$\pm\delta\Delta H(Z\alpha)\simeq\pm\frac{3}{8}\left(1-\frac{1}{n^2}\right)(Z\alpha)^2 \quad (2.45)$$

for small $Z\alpha$. The terms

$$-\frac{1}{4}\left(1-\frac{1}{n^2}\right)(Z\alpha)^2\left[\ln\frac{1}{(Z\alpha)^2}+\frac{11}{24}\pm\frac{3}{2}\right] \quad (2.46)$$

account for the leading small $Z\alpha$ QED contributions (calculated and uncalculated) due to the $P_{1/2}$ Dirac wave functions having an S-state small component. For $l>1$, there are no S-state components in the Dirac wave functions, and we use a smaller uncertainty,

$$\pm\delta\Delta H(Z\alpha)\sim\pm\frac{3}{16}\frac{(Z\alpha)^2}{l(l+1)}, \quad (2.47)$$

corresponding to the QED contributions

$$\frac{\alpha}{2\pi}\frac{2Z^4\alpha^2R_\infty}{l(l+1)n^3}\left[1\pm\frac{(Z\alpha)^2}{2}-\frac{3\pi}{4}(Z\alpha)^3\right] \quad (2.48)$$

of the lowest order anomalous magnetic moment term (2.11). Since the uncertainty is smaller than the combination of the individual uncertainties in the energy level table (because of cancellation of many uncertainties common to both $j=l+1/2$ and $j=l-1/2$), the table may not provide sufficient accuracy, and (2.45) and (2.47) may be used to find the uncertainty, and a modification of (2.43) used for precise values of ΔE for $l>1$. The first term in (2.43) is the exact difference of the Dirac energy levels, but its small $Z\alpha$ expansion

$$\frac{Z^4\alpha^2R_\infty}{l(l+1)n^3}\frac{\mu}{m}\left(1+\frac{(Z\alpha)^2}{4}\left[\frac{l+1}{l^2}-\frac{l}{(l+1)^2}+\frac{3}{n}\frac{2l+1}{l(l+1)}-\frac{6}{n^2}\right]\right) \quad (2.49)$$

should be sufficient for most cases. The next three terms in (2.43) are spin-orbit terms and therefore are simply multiplied by $16/l(l+1)n^3$ to obtain general results. The higher order term

$$\frac{8\alpha}{3\pi}\frac{Z^4\alpha^2R_\infty}{n^3}[\Delta H(Z\alpha)\pm\delta\Delta H(Z\alpha)], \quad (2.50)$$

using (2.44b) for ΔH and (2.45) or (2.47) for $\delta\Delta H$, is a correct small $Z\alpha$ expansion of the approximation used.

3. Uncertainties

All uncertainties discussed in this paper, as in [4, 5, 7], correspond to one standard deviation or roughly a 68 percent confidence level uncertainty, rather than a limit of error used in [3]. They are not simply added, but in quadrature, with the combined uncertainty given by the square root of the sum of the squares (RSS) of the independent uncertainties.

Each of the entries in the energy level table is uncertain by ± 0.075 ppm due to the value used for the Rydberg constant [5],

$$R_\infty=109\,737.3177\pm0.0083\ \text{cm}^{-1}; \quad (3.1)$$

this uncertainty could be eliminated by dividing out exactly this value of R_∞ to get energy levels expressed in units of Rydbergs. Additional uncertainties introduced by converting to units of MHz or eV are discussed in section 4 along with other details of obtaining uncertainties. The energy levels are also uncertain due to the various reduced mass factors

$$\frac{\mu}{m}=\frac{M}{M+m}=\left(1+\frac{m}{M}\right)^{-1} \quad (3.2)$$

using an electron mass m (given in Appendix A) that is uncertain by ± 0.38 ppm and nuclear masses (given in table A) that are uncertain by various amounts; the reduced mass factor may be known to an accuracy of better than a part per billion, but applications depending critically on mass values should consider the mass dependence of their application directly, whether it is a transition energy or an isotope shift.

When energy levels having the same n are subtracted to obtain differences of the order of magnitude of $Z^4\alpha^2R_\infty\mu/m$, the uncertainties in the Rydberg or overall reduced mass factor become negligible compared to the ± 1.6 ppm uncertainty in α^2 due to the fine structure constant used [5],

$$\alpha^{-1}=137.03604\pm0.00011. \quad (3.3)$$

(See (4.8) for a different value of α^{-1}.) This ± 1.6 ppm uncertainty is associated with the fine structure differences themselves (such as (2.49)) rather than the individual level shifts. On the other hand, when energy levels having the same n and j are subtracted to obtain differences (Lamb shifts) of the order of magnitude of $Z^4\alpha^3R_\infty$, the uncertainties due to α are usually small compared to the uncertainties due to various incompletely or approximately calculated terms.

The uncertainty given with each level is the combined uncertainty due to α and the various uncalculated terms to be discussed here (but ignores the uncertainties due to the Rydberg constant or the electron/nuclear mass ratio). It is the purpose of this section to note these uncertainties, giving the Z- and state-dependence expected for each, so their relative importance may be assessed. For ease in making these comparisons, the contribution of each uncertainty to the $2S_{1/2}-2P_{1/2}$ Lamb shift in Hydrogen ($\mathcal{S}_H$) will be given (in units of kHz), where the combined uncertainty is ± 10 kHz.

For all except the lowest values of Z (or larger values of n), the dominant uncertainty is in the approximation used for the electron structure correction $H(Z\alpha)$ in (2.29). This is discussed in Appendix C, and the uncertainty $\delta H(Z\alpha)$ is given in eq (C. 2). For small $Z\alpha$, this is of the order of

$$\pm\frac{2\alpha(Z\alpha)^6mc^2}{3\pi n^3} \qquad (\pm 4.4 \text{ kHz for } \mathcal{S}_H). \qquad (3.4)$$

For large n (roughly $n \geqq 4$), however, the dominant uncertainty is in the lowest order electron structure calculation, the Bethe logarithm discussed in Appendix B. The extrapolation used to estimate uncalculated values has an uncertainty of the order of

$$\pm 0.001\,\frac{4\alpha(Z\alpha)^4mc^2}{3\pi n^3}, \qquad (3.5)$$

but these are negligible compared to present experimental uncertainties. For small Z and small n, where the precision tests of quantum electrodynamics have been done, there are a number of sources of uncertainty comparable in size to (3.4) and we will consider them here.

The overall factors

$$\frac{4\alpha(Z\alpha)^4mc^2}{3\pi n^3}=\frac{8Z^4}{3\pi n^3}\,\alpha^3R_\infty hc \qquad (3.6)$$

in most of the QED contributions are uncertain by

$$3(\pm 0.8 \text{ ppm}) = \pm 2.4 \text{ ppm} \qquad (\pm 2.3 \text{ kHz in } \mathcal{S}_H) \qquad (3.7)$$

due to the α^3 factor, by ± 0.075 ppm due to R_∞ and ± 0.004 ppm due to c. The ± 5.4 ppm uncertainty in h does not occur unless we use units of Joules (or ± 2.6 ppm for units of eV). These may be completely neglected compared to (3.4) for values of Z larger than 2.

The nuclear size uncertainties listed in table A produce energy level uncertainties primarily through eq (2.30); this is ± 6 kHz for $\mathcal{S}_H$. The nuclear structure uncertainties given for C_{str} in eq (2.30) are small (± 0.25 kHz for $\mathcal{S}_H$). These effects for non-S-states are essentially negligible in our calculation, as noted, and their negligible uncertainties, taken as $\pm$ 10 percent for the size effect and $\pm$ 1 percent (of the S-state C_{str} contribution) for the structure, are included only for completeness. Similarly, the effect (2.33) of nuclear size on the QED shifts is negligible, but is included, as an uncertainty ($\pm$ 0.01 kHz for $\mathcal{S}_H$), for completeness. It should again be noted that our comments about the unimportance of these nuclear size effects should only be taken in the context of our calculations, which do not attempt to be accurate at large $Z\alpha$.

The uncalculated terms of order α and $Z\alpha$ relative to the contributions (2.12), (2.13), and (2.15) are estimated to be of the order of magnitude of

$$\pm\left(2\pi Z\alpha+\frac{\alpha}{\pi}\right)\left[\frac{1}{2}\left|(a_4+a_6)\frac{C_{lj}}{2l+1}\right| + \frac{1}{3}\frac{\alpha^2}{\pi^2}\delta_{l0}\right]\frac{(Z\alpha)^4mc^2}{n^3} \qquad (\pm 5.7 \text{ kHz for } \mathcal{S}_H). \quad (3.8)$$

The uncalculated terms of order $Z\alpha$ relative to (2.9) and order m/M relative to (2.22) and the higher order parts of (2.25) and (2.29) are estimated to be of the order of magnitude of

$$\pm\frac{2}{3}\frac{m}{M}\frac{4(Z\alpha)^6mc^2}{n^3} \text{ for } l=0 \quad \text{and}$$
$$\pi Z\alpha \text{ times (2.9)} \quad \text{for } l\neq 0 \quad (\pm 3.4 \text{ kHz for } \mathcal{S}_H). \quad (3.9)$$

The square root of the sum of the squares of the uncertainties listed here from (3.3) through (3.9) is the QED uncertainty given in parentheses after each tabulated energy level. The QED uncertainty for the difference between energy levels is the similar quadrature (RSS) combination of the uncertainties of the two levels if they are independent. Let us briefly consider the few cases in which a "blind" RSS combination of tabulated uncertainties (and the Rydberg (3.1) and reduced mass (3.2) and fine structure (3.3) uncertainties) might yield overestimates, especially the two situations (Fine Structure Interval and isotope comparisons) in which differences may be known better than the tabulated values. The QED uncertainties are sufficiently independent (or sufficiently different in magnitude that their dependence is immaterial) except for states having the same n and l values. These are either the same levels for different isotopes or the Fine Structure Interval for one isotope. As noted in section 2.4, the Fine Structure Intervals ΔE are known to higher precision than given in the table, but can be calculated as discussed with eqs (2.43–50) if higher precision is required. In comparing different isotopes, the QED uncertainties are usually irrelevent if they are smaller than the ± 0.075 ppm Rydberg uncertainty (3.1) or the uncertainty in the difference in reduced mass factors (3.2), which is usually the case for energy level differences between states with different n. This leaves isotope comparisons of fine structure (same n, but not necessarily the same l), such as a Lamb shift difference

$\mathcal{S}_H - \mathcal{S}_D$ or a comparison $(P_{3/2}-S_{1/2})_{^3He^+}-(P_{3/2}-S_{1/2})_{^4He^+}$, as the only cases in which the table (or paper) might not yield the most precise calculated difference. So far, however, no measurements of such isotope comparisons even come close to the tabulated uncertainties. If more precise values than tabulated are needed, the author can provide results for researchers requiring them.

4. Use of the Table

For both theoretical and practical reasons, the zero of energy is taken to be the ionized atom (i.e., the nucleus with one electron infinitely far away, and no other electrons nearby), so all bound state energy levels are negative. The ground state energy (the negative of the ionization energy) is by far both the largest and most uncertain energy level calculated for each atom, so we rejected the alternative of taking it as zero and subtracting it from each entry, which would have yielded a table with positive, but much longer, entries. We keep the minus sign in each entry to maintain algebraic consistency with energies increasing from the ground state to excited and ionized states. For ease in locating levels, we have not arranged them in the order of their energies, but in the order of increasing n, l, and j. Hyperfine splitting is not included, so the energy levels given are the centroids of the hyperfine structure.

In order that the differences between most levels will be as accurate as their QED shifts can be calculated, each entry is extended to the decimal place left uncertain by the QED calculations (with the QED uncertainty in that place given in parentheses) by treating the mass values (Appendix A) and Rydberg value (3.1) as exact constants. Therefore, every entry has an absolute uncertainty of ±0.075 ppm (due to the uncertain Rydberg constant), its difference with the same level for another isotope of the same atomic number has an uncertainty of ±0.38 ppm (due to the uncertain electron mass), and only its differences with other levels of the same isotope having the same value of n and j have uncertainties as small as the QED uncertainties given in the table. Other fine structure differences have an additional ±1.6 ppm uncertainty due to the fine structure constant used (3.2). The QED uncertainty gives a *limiting* value of the usefulness of this table in the sense that improvements in our knowledge of the masses or of the Rydberg can be used to find correction factors, and other improvements might yield additive corrections, but changes by amounts much smaller than the QED uncertainties, even improvements substantially reducing the QED uncertainties, cannot be satisfactorily accounted for without a complete calculation including more decimal places.

Let us consider how changes in the units or in the value of the Rydberg constant can be carried out and how this would affect the ±0.075 ppm uncertainty presently associated with each entry. We pointed out in section 3 how we could remove this uncertainty by dividing each entry by 109 737.3177 cm^{-1} to get units of Rydbergs. We could then multiply by an improved value of the Rydberg constant and use the uncertainty associated with that value. For example, if we wish to use the Rydberg recently determined by Hänsch et al. [28]

$$R_\infty = 109\ 737.3143 \pm 0.0010\ \text{cm}^{-1}, \tag{4.1}$$

we may multiply every entry we need by the ratio

$$\frac{R_{\text{new}}}{R_{\text{table}}} = \frac{109\ 737.3143}{109\ 737.3177} \tag{4.2}$$

(i.e., decrease the entries by 0.030 983 079 ppm) and decrease the overall uncertainty from ±0.075 ppm to ±0.009 ppm. On the other hand, if we wish to convert units from cm^{-1} to MHz, we may multiply by the speed of light,

$$c = 29\ 979.2458 \pm 0.00012\ \text{MHz/cm}^{-1}, \tag{4.3}$$

and combine its ±0.004 ppm uncertainty with the overall uncertainty due to the Rydberg. To convert further from MHz to eV, we divide by

$$\frac{e}{h} = 241\ 796.96 \pm 0.63 \times 10^3\ \text{MHz/eV} \tag{4.4}$$

(or, equivalently, convert from cm^{-1} to eV by dividing by

$$8065.479 \pm 0.021\ \text{cm}^{-1}/\text{eV} \tag{4.5}$$

or from Ry to eV by multiplying by

$$13.605\ 804 \pm 0.000\ 036\ \text{eV/Ry}) \tag{4.6}$$

and include an overall uncertainty of ±2.6 ppm. For consistency, all fundamental constants given in this paper (except for eqs (4.1) and (4.8)) are those of the 1973 adjustment of Cohen and Taylor [5]. We might note that we have used their final recommended values, including

$$\alpha^{-1} = 137.03604 \pm 0.00011 \qquad (0.82\ \text{ppm}), \tag{4.7}$$

rather than their "WQED" values, (2.19), since we wish to present the best results rather than a QED test not using QED theory, and in any event the results are rather insensitive to changes in α or its uncertainty. Even the improved value [29]

$$\alpha^{-1} = 137.035\ 987 \pm 0.000\ 029 \qquad (0.21\ \text{ppm}) \tag{4.8}$$

would not appreciably change any results given in the tables; its primary effect would be to reduce the additional ± 1.6 ppm fine structure uncertainty (noted with eq (3.3) and earlier in this section) to ± 0.4 ppm.

If we wish to account for a change in the electron-nucleus mass ratio

$$\delta \equiv \frac{m}{M} - \frac{m_0}{M_0}, \tag{4.9}$$

we primarily need to correct for the change in the (nearly) overall reduced mass factor $(1 + m/M)^{-1}$ by adding the fraction

$$\frac{(1+m/M)^{-1}}{(1+m_0/M_0)^{-1}} - 1 = -\delta\left(1+\frac{m}{M}\right)^{-1} \tag{4.10}$$

of the table entry or difference being corrected. Since $m/M \lesssim 0.5 \times 10^{-3}$, and small changes in m or M will be of the order of 0.2×10^{-6} or less, then $\delta \lesssim 10^{-10}$ is a small correction and its effect on QED terms may be completely ignored. However, for precise work with the energy levels themselves (or transitions between levels of different n), we may need to account for the lowest order change in the term (2.7) by adding an amount

$$-\delta \frac{(Z\alpha)^2}{4n^2} |E_n| \tag{4.11}$$

to each energy level E_n. In that case, the total correction to be added to each table entry is a fraction

$$-\delta\left[\left(1+\frac{m}{M}\right)^{-1} - \frac{(Z\alpha)^2}{4n^2}\right], \tag{4.12}$$

of that entry.

Differences between the same level for different isotopes are roughly proportional to

$$\left(1+\frac{m}{M_2}\right)^{-1} - \left(1+\frac{m}{M_1}\right)^{-1} \simeq \frac{m}{M_1}\left(1-\frac{M_1}{M_2}\right) \tag{4.13}$$

and thus have an uncertainty of ± 0.38 ppm due to the uncertainty in the electron mass (A.1) and possibly more due to an uncertainty in the nuclear masses. Corrections for changes in these mass values may be made for each isotope as described in the preceding paragraph. For isotopes not given in the table, an approximate result may be obtained by adding the fraction (4.12) to each entry; this leaves only terms of order $(m/M)\alpha^3 Z^{4\text{or}5}\ln(Z\alpha)^{-2}E_n$, such as (2.9) or the recoil corrections in (2.10) and (2.11), improperly accounted for.

The QED uncertainties for most levels are independent, so the combined QED uncertainty for the difference between two levels is usually given by adding the squares of the individual uncertainties. This can be similarly combined with a ± 1.6 ppm uncertainty to approximately account for the fine structure constant used (3.3). The resulting over-estimate due to non-independence of these uncertainties is serious only for Fine Structure Intervals and certain isotope comparisons, as briefly discussed at the end of section 3. These differences may be known more precisely than given by the table. For example, the Fine Structure Intervals have uncertainties as small as (2.47),

$$\pm\frac{\alpha}{\pi}\frac{(Z\alpha)^2}{2} = \pm 0.062\, Z^2 \text{ ppm}, \tag{4.14}$$

for $l > 1$ (excluding the ± 1.6 ppm α^2 uncertainty). If such precision is required, section 2.4 gives the appropriate Fine Structure formulas. If a desired Lamb shift or fine structure separation has been measured, its calculated value (and correct uncertainty) may be found in table D.

The energy level table was produced by modifying a computer program normally used to print out calculation details (tabulations of the various contributing terms and uncertainties in desired units and dimensionlessly) for Lamb shifts and splittings and fine structure separations of an atomic state being studied. These have been calculated on request to suit the needs of various researchers, and may continue to be provided whenever possible. Those print-outs headed "using terms most recently revised by G.W.E. on 21 Jan 74" use the terms listed in this paper, but the (printed-out) atomic constants (Rydberg, m/M, nuclear size, etc.) are not necessarily those used here.

Appendix A. Nuclear Masses and Sizes

The nuclear masses used are exactly those listed in table A, and the electron mass is taken as exactly

$$m_e = 0.000\,548\,580\,26 \text{ u} \tag{A.1}$$

although this value [5] has an uncertainty of 21×10^{-11}u. These nuclear masses are derived from a tabulation of atomic masses [30] by subtracting Z electron masses. For $Z=1$ and $Z=2$, we also add the electron binding energies [5], 15×10^{-9}u and 85×10^{-9}u, respectively. The uncertainties (generally larger than the $\pm 11 \times 10^{-9}$ u for Hydrogen) are ignored since any use of the energy level table requiring such precision should consult the latest mass adjustment and take account of the uncertainty and the difference between that nuclear mass (or electron mass) and the value used here. The isotopes chosen are the most abundant or the most long-lived. For atoms for which no particular isotope is chosen ($Z > 83$), we use a relation (with $M = A$u)

$$A = Z(2 + 0.015A^{2/3}) \tag{A.2}$$

given by Elton [31]. However, for $Z > 100$, we use a relation

$$A = 0.00733Z^2 + 1.30Z + 63.6, \tag{A.3}$$

given by Pieper and Greiner [22].

The RMS nuclear radii used are those given by Hofstadter and Collard [31], mostly using their result

$$\sqrt{\langle r^2 \rangle} = [0.82\,A^{1/3} + 0.58 \pm 0.1] \text{ fm}. \tag{A.4}$$

The individually determined values used for certain isotopes with low Z are given in table A. For the proton, the size used [31], 0.80 ± 0.02 fm, is from measurements at Orsay [32] and is in good agreement with Hand, Miller, and Wilson's analysis [33] of earlier measurements, 0.805 ± 0.011 fm, and with recent measurements at Saskatchewan [34], 0.81 ± 0.04 fm, but not with recent measurements at Mainz [35], 0.87 ± 0.02 fm. For the deuteron, the size given is determined as in ref. [7], slightly modified [36]. For $Z > 100$, we use the relation

$$\sqrt{\langle r^2 \rangle} = \sqrt{\frac{3}{5}}\,1.2\,A^{1/3} \text{ fm } (\pm 10\%), \tag{A.5}$$

used by Pieper and Greiner [22]. It should be noted that the nuclear size effect (2.30) and its related experimental uncertainty vary roughly like $Z^{2/3}(Z\alpha)^4$, which is negligible compared to the $(Z\alpha)^6$ electron structure uncertainty (C.4) except for the smallest values of Z, where most sources of uncertainty are of the same order of magnitude.

TABLE A. Nuclear masses and sizes

^{Z}Element^A	Nuclear mass (units of u)	(using eq A.2)	RMS radius (units of fm)	(using eq A.4)
^{1}H^1	1.007 276 472 74	(2.02)	0.800 ± 0.020	(1.400 ± 0.1)
^{1}D^2	2.013 553 234 74	(2.02)	2.096 ± 0.014	(1.613 ± 0.1)
^{1}T^3	3.015 007 743 74	(2.02)	1.700 ± 0.050	(1.763 ± 0.1)
^{2}He^3	3.014 932 238 48	(4.08)	1.870 ± 0.050	(1.763 ± 0.1)
^{2}He^4	4.001 506 204 48	(4.08)	1.630 ± 0.040	(1.882 ± 0.1)
^{3}Li^6	6.013 477 159 22	(6.15)	2.500 ± 0.100	(2.070 ± 0.1)
^{3}Li^7	7.014 358 559 22	(6.15)	2.358 ± 0.100	(2.149 ± 0.1)
^{4}Be^9	9.009 988 178 96	(8.24)	2.286 ± 0.100	(2.286 ± 0.1)
^{5}B^{10}	10.010 195 598 70	(10.36)	2.347 ± 0.100	(2.347 ± 0.1)
^{5}B^{11}	11.006 562 478 70	(10.36)	2.404 ± 0.100	(2.404 ± 0.1)
^{6}C^{12}	11.996 708 518 44	(12.48)	2.420 ± 0.040	(2.457 ± 0.1)
^{7}N^{14}	13.999 233 935 18	(14.63)	2.556 ± 0.100	(2.556 ± 0.1)
^{8}O^{16}	15.990 525 837 92	(16.79)	2.646 ± 0.100	(2.646 ± 0.1)
^{9}F^{19}	18.993 466 577 66	(18.96)	2.768 ± 0.100	(2.768 ± 0.1)
^{10}Ne^{20}	19.986 953 247 40	(21.15)	2.806 ± 0.100	(2.806 ± 0.1)
^{11}Na^{23}	22.983 734 217 14	(23.35)	2.912 ± 0.100	(2.912 ± 0.1)
^{12}Mg^{24}	23.978 460 636 88	(25.56)	2.945 ± 0.100	(2.945 ± 0.1)
^{13}Al^{27}	26.974 408 856 62	(27.79)	3.040 ± 0.100	(3.040 ± 0.1)
^{14}Si^{28}	27.969 248 476 36	(30.03)	3.070 ± 0.100	(3.070 ± 0.1)
^{15}P^{31}	30.965 534 896 10	(32.28)	3.156 ± 0.100	(3.156 ± 0.1)
^{16}S^{32}	31.963 295 115 84	(34.55)	3.183 ± 0.100	(3.183 ± 0.1)
^{17}Cl^{35}	34.959 526 859 58	(36.82)	3.262 ± 0.100	(3.262 ± 0.1)
^{18}Ar^{40}	39.952 507 955 32	(39.11)	3.384 ± 0.100	(3.384 ± 0.1)
^{19}K^{39}	38.953 284 575 06	(41.41)	3.361 ± 0.100	(3.361 ± 0.1)
^{20}Ca^{40}	39.951 618 794 80	(43.72)	3.384 ± 0.100	(3.384 ± 0.1)
^{21}Sc^{45}	44.944 392 714 54	(46.05)	3.497 ± 0.100	(3.497 ± 0.1)

TABLE A. Nuclear masses and sizes

^{Z}Element^{A}	Nuclear mass (units of u)	(using eq A.2)	RMS radius (units of fm)	(using eq A.4)
$^{22}Ti^{48}$	47.935 878 434 28	(48.38)	3.560 ± 0.100	(3.560 ± 0.1)
$^{23}V^{51}$	50.931 346 754 02	(50.73)	3.621 ± 0.100	(3.621 ± 0.1)
$^{24}Cr^{52}$	51.927 343 173 76	(53.08)	3.641 ± 0.100	(3.641 ± 0.1)
$^{25}Mn^{55}$	54.924 328 293 50	(55.45)	3.698 ± 0.100	(3.698 ± 0.1)
$^{26}Fe^{56}$	55.920 670 813 24	(57.83)	3.717 ± 0.100	(3.717 ± 0.1)
$^{27}Co^{59}$	58.918 379 132 98	(60.22)	3.772 ± 0.100	(3.772 ± 0.1)
$^{28}Ni^{58}$	57.919 979 052 72	(62.62)	3.754 ± 0.100	(3.754 ± 0.1)
$^{29}Cu^{63}$	62.913 686 272 46	(65.03)	3.843 ± 0.100	(3.843 ± 0.1)
$^{30}Zn^{64}$	63.912 683 892 20	(67.46)	3.860 ± 0.100	(3.860 ± 0.1)
$^{31}Ga^{69}$	68.908 576 011 94	(69.89)	3.943 ± 0.100	(3.943 ± 0.1)
$^{32}Ge^{74}$	73.903 625 031 68	(72.33)	4.023 ± 0.100	(4.023 ± 0.1)
$^{33}As^{75}$	74.903 496 551 42	(74.79)	4.038 ± 0.100	(4.038 ± 0.1)
$^{34}Se^{80}$	79.897 871 271 16	(77.25)	4.113 ± 0.100	(4.113 ± 0.1)
$^{35}Br^{79}$	78.899 128 690 90	(79.72)	4.098 ± 0.100	(4.098 ± 0.1)
$^{36}Kr^{84}$	83.891 756 110 64	(82.21)	4.171 ± 0.100	(4.171 ± 0.1)
$^{37}Rb^{85}$	84.891 500 530 38	(84.70)	4.185 ± 0.100	(4.185 ± 0.1)
$^{38}Sr^{88}$	87.884 781 550 12	(87.21)	4.227 ± 0.100	(4.227 ± 0.1)
$^{39}Y^{89}$	88.884 481 369 86	(89.72)	4.241 ± 0.100	(4.241 ± 0.1)
$^{40}Zr^{90}$	89.882 767 289 60	(92.25)	4.255 ± 0.100	(4.255 ± 0.1)
$^{41}Nb^{93}$	92.883 888 509 34	(94.79)	4.295 ± 0.100	(4.295 ± 0.1)
$^{42}Mo^{98}$	97.882 369 629 08	(97.33)	4.361 ± 0.100	(4.361 ± 0.1)
$^{43}Tc^{99}$	98.882 661 048 82	(99.89)	4.373 ± 0.100	(4.373 ± 0.1)
$^{44}Ru^{102}$	101.880 210 568 56	(102.45)	4.411 ± 0.100	(4.411 ± 0.1)
$^{45}Rh^{103}$	102.880 825 888 30	(105.03)	4.424 ± 0.100	(4.424 ± 0.1)
$^{46}Pd^{106}$	105.878 252 308 04	(107.61)	4.461 ± 0.100	(4.461 ± 0.1)
$^{47}Ag^{107}$	106.879 307 727 78	(110.21)	4.473 ± 0.100	(4.473 ± 0.1)
$^{48}Cd^{114}$	113.877 034 947 52	(112.81)	4.556 ± 0.100	(4.556 ± 0.1)
$^{49}In^{115}$	114.876 994 567 26	(115.42)	4.568 ± 0.100	(4.568 ± 0.1)
$^{50}Sn^{120}$	119.874 778 287 00	(118.05)	4.625 ± 0.100	(4.625 ± 0.1)
$^{51}Sb^{121}$	120.875 844 706 74	(120.68)	4.636 ± 0.100	(4.636 ± 0.1)
$^{52}Te^{130}$	129.877 705 726 48	(123.33)	4.734 ± 0.100	(4.734 ± 0.1)
$^{53}I^{127}$	126.875 400 746 22	(125.98)	4.702 ± 0.100	(4.702 ± 0.1)
$^{54}Xe^{132}$	131.874 533 465 96	(128.64)	4.755 ± 0.100	(4.755 ± 0.1)
$^{55}Cs^{133}$	132.875 264 085 70	(131.31)	4.766 ± 0.100	(4.766 ± 0.1)
$^{56}Ba^{138}$	137.874 514 505 44	(134.00)	4.817 ± 0.100	(4.817 ± 0.1)
$^{57}La^{139}$	138.875 133 925 18	(136.69)	4.828 ± 0.100	(4.828 ± 0.1)
$^{58}Ce^{140}$	139.873 666 344 92	(139.39)	4.838 ± 0.100	(4.838 ± 0.1)
$^{59}Pr^{141}$	140.875 331 764 66	(142.10)	4.848 ± 0.100	(4.848 ± 0.1)
$^{60}Nd^{142}$	141.874 851 184 40	(144.82)	4.858 ± 0.100	(4.858 ± 0.1)
$^{61}Pm^{147}$	146.881 702 604 14	(147.55)	4.908 ± 0.100	(4.908 ± 0.1)
$^{62}Sm^{152}$	151.885 743 023 88	(150.29)	4.956 ± 0.100	(4.956 ± 0.1)
$^{63}Eu^{153}$	152.886 699 443 62	(153.04)	4.966 ± 0.100	(4.966 ± 0.1)
$^{64}Gd^{158}$	157.889 013 863 36	(155.80)	5.013 ± 0.100	(5.013 ± 0.1)
$^{65}Tb^{159}$	158.889 728 283 10	(158.56)	5.022 ± 0.100	(5.022 ± 0.1)
$^{66}Dy^{164}$	163.893 011 702 84	(161.34)	5.068 ± 0.100	(5.068 ± 0.1)
$^{67}Ho^{165}$	164.893 602 122 58	(164.13)	5.078 ± 0.100	(5.078 ± 0.1)
$^{68}Er^{166}$	165.893 020 542 32	(166.92)	5.087 ± 0.100	(5.087 ± 0.1)
$^{69}Tm^{169}$	168.896 392 962 06	(169.73)	5.114 ± 0.100	(5.114 ± 0.1)
$^{70}Yb^{174}$	173.900 480 381 80	(172.54)	5.158 ± 0.100	(5.158 ± 0.1)
$^{71}Lu^{175}$	174.901 846 801 54	(175.37)	5.167 ± 0.100	(5.167 ± 0.1)
$^{72}Hf^{180}$	179.907 077 221 28	(178.20)	5.210 ± 0.100	(5.210 ± 0.1)
$^{73}Ta^{181}$	180.907 981 641 02	(181.04)	5.218 ± 0.100	(5.218 ± 0.1)
$^{74}W^{184}$	183.910 380 060 76	(183.90)	5.244 ± 0.100	(5.244 ± 0.1)
$^{75}Re^{187}$	186.914 647 480 50	(186.76)	5.269 ± 0.100	(5.269 ± 0.1)
$^{76}Os^{192}$	191.919 821 900 24	(189.63)	5.311 ± 0.100	(5.311 ± 0.1)
$^{77}Ir^{193}$	192.920 723 319 98	(192.51)	5.319 ± 0.100	(5.319 ± 0.1)
$^{78}Pt^{195}$	194.922 014 739 72	(195.40)	5.335 ± 0.100	(5.335 ± 0.1)
$^{79}Au^{197}$	196.923 210 159 46	(198.30)	5.351 ± 0.100	(5.351 ± 0.1)
$^{80}Hg^{202}$	201.926 756 579 20	(201.20)	5.391 ± 0.100	(5.391 ± 0.1)
$^{81}Tl^{205}$	204.930 002 998 94	(204.12)	5.415 ± 0.100	(5.415 ± 0.1)
$^{82}Pb^{208}$	207.931 674 418 68	(207.05)	5.438 ± 0.100	(5.438 ± 0.1)
$^{83}Bi^{209}$	208.934 868 838 42	(209.98)	5.446 ± 0.100	(5.446 ± 0.1)
^{84}Po	212.926 453 704 95	(212.93)	5.477 ± 0.100	(5.477 ± 0.1)
^{85}At	215.880 753 270 35	(215.88)	5.499 ± 0.100	(5.499 ± 0.1)
^{86}Rn	218.844 307 837 08	(218.85)	5.522 ± 0.100	(5.522 ± 0.1)

^{Z}Element^{A}	Nuclear mass (units of u)	(using eq A.2)	RMS radius (units of fm)	(using eq A.4)
^{87}Fr	221.817 112 344 32	(221.82)	5.544 ± 0.100	(5.544 ± 0.1)
^{88}Ra^{226}	225.977 160 937 12	(224.80)	5.575 ± 0.100	(5.575 ± 0.1)
^{89}Ac	227.790 452 372 28	(227.79)	5.588 ± 0.100	(5.588 ± 0.1)
^{90}Th^{232}	231.988 701 776 60	(230.79)	5.619 ± 0.100	(5.619 ± 0.1)
^{91}Pa	233.800 738 532 10	(233.80)	5.632 ± 0.100	(5.632 ± 0.1)
^{92}U^{238}	238.000 346 616 08	(236.82)	5.662 ± 0.100	(5.662 ± 0.1)
^{93}Np	239.847 940 310 84	(239.85)	5.675 ± 0.100	(5.675 ± 0.1)
^{94}Pu	242.885 376 138 61	(242.89)	5.696 ± 0.100	(5.696 ± 0.1)
^{95}Am	245.932 031 256 15	(245.94)	5.718 ± 0.100	(5.718 ± 0.1)
^{96}Cm	248.987 902 960 74	(248.99)	5.739 ± 0.100	(5.739 ± 0.1)
^{97}Bk	252.052 988 775 07	(252.06)	5.760 ± 0.100	(5.760 ± 0.1)
^{98}Cf	255.127 286 447 21	(255.13)	5.781 ± 0.100	(5.781 ± 0.1)
^{99}Es	258.210 793 939 42	(258.22)	5.802 ± 0.100	(5.802 ± 0.1)
^{100}Fm	261.303 509 427 23	(261.31)	5.822 ± 0.100	(5.822 ± 0.1)
^{101}Md	269.673 330 000 00	(264.41)	6.005 ± 0.601	(5.878 ± 0.1)
^{102}No	272.461 320 000 00	(267.52)	6.026 ± 0.603	(5.896 ± 0.1)
^{103}Lr	275.263 970 000 00	(270.64)	6.047 ± 0.605	(5.914 ± 0.1)
^{104}Rf	278.081 280 000 00	(273.77)	6.067 ± 0.607	(5.932 ± 0.1)
^{105}Ha	280.913 250 000 00	(276.91)	6.088 ± 0.609	(5.950 ± 0.1)

Appendix B. Bethe Logarithms

The infra-red logarithm, originally defined and calculated by Bethe [37], is a nonrelativistic logarithmic average of excitation energies,

$$\sum_{n'}\left|\left\langle n\left|\frac{\mathbf{p}}{\mu}\right|n'\right\rangle\right|^2(E_n-E_{n'})\ln\frac{mc^2}{2|E_n-E_{n'}|}$$

$$\equiv\frac{2(Z\alpha)^4\mu c^2}{n^3}\left[\delta_{l0}\ln\frac{m}{(Z\alpha)^2\mu}+L_n\right],\quad \text{(B.1)}$$

in which the $Z\alpha$ and reduced mass dependences may be exactly separated (because the Coulomb wave functions and energies are nonrelativistic) to leave a $Z\alpha$- and μ-independent constant L_n depending only on the quantum numbers n and l of the state $|n\rangle$. This is the leading term in the self-energy contribution (2.10) and also occurs in the Bethe-Salpeter recoil correction (2.9). It has been calculated most precisely for the 1S, 2S, and 2P states by Huff [38] and for the greatest number of states (1S, 2S, 2P, 3S, 3P, 3D, 4S, and 4P) by Harriman [39]. Instead of attempting a similar calculation, we have simply assumed extrapolations of the form

$$L_n=(-2.71631\pm0.00075)-(0.02402\pm0.00020)\left(\frac{5}{n}\right)^{3/2}$$

$$\text{for } l=0\ (n\geqq5)\quad \text{(B.2)}$$

$$L_n=(+0.0525\pm0.0025)-(0.00838\pm0.00248)\left(\frac{5}{n}\right)^{3/2}$$

$$\text{for } l=1\ (n\geqq5)\quad \text{(B.3)}$$

$$L_n=0.01148224\left[1-\left(\frac{2}{n}\right)^{3/2}\right]\pm\left[0.002-0.001\left(\frac{4}{n}\right)^{3/2}\right]$$

$$\text{for } l=2\ (n\geqq4)\quad \text{(B.4)}$$

$$L_n=\frac{0.1623834}{(2l+1)}\left[\left(\frac{1}{l}\right)^{3/2}-\left(\frac{1}{n}\right)^{3/2}\right]\left[1\pm\left(0.5-0.25\left(\frac{l+1}{n}\right)^{3/2}\right)\right]$$

$$\text{for } l\geqq3\ (n\geqq4)\quad \text{(B.5)}$$

These extrapolations turn out to yield the largest uncertainty in the calculations for these states. They could probably be improved by calculations similar to those of Huff or Harriman, but as yet (see table D) no measurements seem to warrant any such precision. Table B lists the values and uncertainties used for all states considered in the energy level tabulations.

Added note: After preparation of the tables, we found Bethe logarithms for $n \leqq 8$ were accurately calculated by S. Klarsfeld and A. Maquet (Phys. Lett. **43B**, 201–203 (1973)), differing from Harriman's by several times his estimated errors. Our extrapolations turn out to be essentially correct (within the estimated uncertainties for roughly 68% of the cases), with errors largest for S-states (up to three times the uncertainties) but less than twice the uncertainties for all other states (and averaging only half the uncertainty for $l \geqq 2$). However, as noted above and by Klarsfeld and Maquet, even the largest differences are "hardly significant from the experimental point of view". The improved Bethe logarithms would change no calculated value in table D by more than the given uncertainty (although a few would change by about the size of the uncertainty). The effect is primarily to reduce the calculated uncertainties. For example, the calculated $4F_{5/2}$–$4D_{3/2}$ interval in $^4He^+$ increases by 0.23 MHz, the calculated uncertainty changes from ± 0.29 MHz to ± 0.01 MHz, and the measurement comparison changes from $+1.5\sigma$ to $+1.7\sigma$; this is the only case in which the measurement comparison change is larger than 0.1σ.

TABLE B. Bethe logarithms

State	Bethe logarithm L_n
1S	−2.984 128 555 9±0.000 000 000 3
2S	+2.811 768 893 2±0.000 000 000 3
2P	0.030 016 708 9±0.000 000 000 3
3S	−2.767 699 000 0±0.000 008 000 0
3P	0.038 188 520 0±0.000 000 010 0
3D	0.005 232 100 0±0.000 000 200 0
4S	−2.749 859 000 0±0.000 006 000 0
4P	0.041 954 000 0±0.000 000 300 0
4D	0.007 422 655 1±0.001 000 000 0
4F	0.001 564 682 1±0.000 391 170 5
5S	−2.740 330 000 0±0.000 550 000 0
5P	0.044 120 000 0±0.000 020 000 0
5D	0.008 577 437 5±0.001 284 458 2
5F	0.002 389 526 7±0.000 767 311 8
5G	0.000 641 545 8±0.000 160 386 4
6S	−2.734 582 633 1±0.000 597 854 8
6P	0.046 125 118 0±0.000 613 400 1
6D	0.009 272 481 9±0.001 455 668 9
6F	0.002 885 987 0±0.001 050 260 4
6G	0.001 027 681 6±0000 318 394 8
6H	0.000 315 929 3±0.000 078 932 3
7S	−2.730 810 432 3±0.000 629 263 7
7P	0.047 441 148 1±0.001 002 869 6
7D	0.009 728 664 2±0.001 568 040 6
7F	0.003 211 831 5±0.001 259 070 5
7G	0.001 281 116 2±0.000 447 211 5
7H	0.000 523 284 9±0.000 157 827 9
7I	0.000 175 454 7±0.000 043 863 7
8S	−2.728 178 423 4±0.000 651 178 8
8P	0.048 359 392 7±0.001 274 617 4
8D	0.010 046 960 0±0.001 646 446 6
8F	0.003 439 185 7±0.001 415 608 9
8G	0.001 457 947 2±0.000 548 878 5
8H	0.000 667 964 8±0.000 225 518 4
8I	0.000 297 876 2±0.000 087 986 1
8K	0.000 106 098 6±0.000 026 524 7
9S	−2.726 256 361 6±0.000 667 182 7
9P	0.049 029 953 8±0.001 473 065 1
9D	0.010 279 400 0±0.001 703 703 7
9F	0.003 605 214 3±0.001 535 554 2
9G	0.001 587 080 6±0.000 629 243 1
9H	0.000 773 619 3±0.000 281 533 4
9I	0.000 387 276 2±0.000 127 226 5
9K	0.000 183 578 6±0.000 053 327 2
9L	0.000 068 364 7±0.000 017 091 2
10S	−2.724 802 352 5±0.000 679 289 3
10P	0.049 537 222 6±0.001 623 187 6
10D	0.010 455 237 2±0.001 747 017 8
10F	0.003 730 812 3±0.001 629 448 9
10G	0.001 684 767 9±0.000 693 470 1
11H	0.000 853 545 4±0.000 327 599 7
10I	0.000 454 905 9±0.000 160 847 7
10K	0.000 242 191 0±0.000 077 771 1
10L	0.000 120 081 5±0.000 034 408 9
10M	0.000 046 272 9±0.000 011 568 2
11S	−2.723 671 036 7±0.000 688 709 1
11P	0.049 931 911 4±0.001 739 992 9
11D	0.010 592 050 3±0.001 780 719 0
11F	0.003 828 535 9±0.001 704 386 7
11G	0.001 760 775 1±0.000 745 488 2
11H	0.000 915 733 1±0.000 365 641 9
11I	0.000 507 526 3±0.000 189 352 7
11K	0.000 287 795 4±0.000 099 273 6
11L	0.000 160 320 7±0.000 050 498 1
11M	0.000 082 276 4±0.000 023 309 2
11N	0.000 032 574 5±0.000 008 143 6
12S	−2.722 770 351 4±0.000 696 208 6
12P	0.050 246 138 9±0.001 832 986 2
12O	0.000 023 678 8±0.000 005 919 7
13S	−2.722 039 450 6±0.000 702 294 3
13P	0.050 501 132 5±0.001 908 449 7
13Q	0.000 017 678 0±0.000 004 419 5
14S	−2.721 436 677 0±0.000 707 313 3
14P	0.050 711 425 8±0.001 970 684 5
14R	0.000 013 499 1±0.000 003 374 8
15S	−2.720 932 651 2±0.000 711 510 0
15P	0.050 887 268 2±0.002 022 723 8
15T	0.000 010 509 2±0.000 002 627 3
16S	−2.720 506 121 3±0.000 715 061 4
16P	0.051 036 074 2±0.002 066 761 8
16U	0.000 008 319 6±0.000 002 079 9

TABLE B. Bethe logarithms—Continued

State	Bethe logarithm L_n	State	Bethe logarithm L_n
17S	−2.720 141 374 7±0.000 718 098 5	19S	−2.719 552 631 6±0.000 723 000 6
17P	0.051 163 325 6±0.002 104 420 9	19P	0.051 368 723 9±0.002 165 207 1
17V	0.000 006 683 3±0.000 001 670 8	19X	0.000 004 476 8±0.000 001 119 2
18S	−2.719 826 569 9±0.000 720 719 7	20S	−2.719 312 500 0±0.000 725 000 0
18P	0.051 273 153 4±0.002 136 923 7	20P	0.051 452 500 0±0.002 190 000 0
18W	0.000 005 438 6±0.000 001 359 6	20Y	0.000 003 723 0±0.000 000 930 8

Appendix C. Electron Structure Corrections

The purpose of this appendix is to present the results used in calculating the energy levels in sufficient detail that any future developments may be placed in context. The notation is defined, and the small $Z\alpha$ coefficients are tabulated for the states for which the electron structure uncertainties are not dominated by the lowest order terms (the Bethe logarithms in Appendix B). The results are briefly compared with previous and independent calculations.

The terms (such as the Bethe logarithm L_n in Appendix B) in the lowest order electron structure have not only been calculated, but have been clearly separated [3] from the higher order correction terms, $H(Z\alpha)$ in eq (2.29). The leading small $Z\alpha$ contributions in

$$H(Z\alpha)=C_5Z\alpha+(Z\alpha)^2[C_{62}\ln^2(Z\alpha)^2 + C_{61}\ln(Z\alpha)^{-2}+G(Z\alpha)] \qquad (C.1)$$

have been calculated exactly [20], but have not been separated from the sixth order remainder $G(Z\alpha)$. However, this calculation of $H(Z\alpha)$ has been redone [7] so as to yield an approximation valid for all $Z\alpha$ while accounting for all terms contributing to the known coefficients C_5, C_{62}, and C_{61}. As a simple example, the magnetic moment correction (2.22) is briefly discussed in section 2.2. The original calculation [7] was only for $n=1$ (with only ad hoc modifications for $n>1$) but has now been extended to all states; this is discussed in detail in an article in preparation. The estimated uncertainty, representing roughly a 68 percent confidence level,

$$\delta H(Z\alpha)=(Z\alpha)^2\delta G(Z\alpha)$$

$$=(Z\alpha)^2\delta C_{60}\int_0^1 dz\frac{2z(1-z)(1+y)}{z+(Z\alpha)^2(1-z)}, \qquad (C.2a)$$

with

$$y^2=\frac{(1-z)(Z\alpha)^2}{z+(Z\alpha)^2(1-z)}, \qquad (C.2b)$$

uses an integral similar to others used in $H(Z\alpha)$.

The coefficients in the small $Z\alpha$ expansions

$$G(Z\alpha)=C_{60}+C_7Z\alpha+\ldots \qquad (C.3)$$

and

$$\delta H(Z\alpha)=(Z\alpha)^2\delta C_{60}\left[1+\frac{3\pi}{8}Z\alpha+\ldots\right] \qquad (C.4a)$$

$$=\delta C_{60}(Z\alpha)^2+\delta C_7(Z\alpha)^3+\ldots \qquad (C4.b)$$

are given in table C for $n\leq 4$ and their Fine Structure Interval differences are given in section 2.4. It should be noted that these coefficients are exact only for the *approximation* used for $H(Z\alpha)$. For example, $\Delta C_{60}=-11/128$ (given in (2.44) and [5]) is the exact coefficient for our approximation for the Fine Structure difference $\Delta H(Z\alpha)=H_{2P_{3/2}}-H_{2P_{1/2}}$ but is not expected to be the correct coefficient difference (in fact, the estimated uncertainty $\delta\Delta C_{60}=\pm 9/32$ is larger than the approximation). We might also note that there are known to be small C_{71} coefficients (of $Z\alpha\ln(Z\alpha)^{-2}$ in $G(Z\alpha)$) and that our approximation has none. Finally, we might repeat that neither our calculations nor the numerical calculations referred to below are direct determinations of $G(Z\alpha)$; they are all some form of remainder calculation. The numerical calculations of $G(Z\alpha)$ suffer from the usual uncertainty magnification encountered in numerical calculations of remainders. Our calculation of $G(Z\alpha)$ suffers from not using complete exact forms for $H(Z\alpha)$.

The validity of our $H(Z\alpha)$ approximation for large $Z\alpha$ has been confirmed by independent numerical calculations [24, 27, 26] which agree with our results within the estimated uncertainties. Those calculations are more accurate than ours for large $Z\alpha$ but are difficult for small $Z\alpha$ and have only been done for a few states. At small $Z\alpha$ the calculations confirm the general validity of the unique methods used (our $H(Z\alpha)$ approximation method and the numerical methods of Mohr [27]) and agree very well for the $2P_{1/2}$ state but differ by a few or many times the estimated uncertainties for S-states. For example, for the $1S_{1/2}$ state at $Z=10$, we get $G(Z\alpha)=-17.5\pm 0.5$ and Mohr gets -20.0 ± 0.4. The source of this small discrepancy is not yet known, and present experiments do not distinguish between them. (For the $Z = 1$ Lamb shift, with $G_{2S_{1/2}}-G_{2P_{1/2}} = -17.1 \pm 0.6$ (ours) vs. -23.4 ± 1.2 (Mohr's), the 6.3 ± 1.3 discrepancy is only 0.046 ± 0.009 MHz for hydrogen, where the other uncertainties are ± 0.009 MHz and the most precise measurement [40] has ± 0.020 MHz uncertainty and lies between the two calculated results, slightly closer to the larger (ours) but not much more than one standard deviation above the other (Mohr's). The hydrogen comparison is also clouded by questions of the proton size [32–35], as noted in Appendix A.) (Added note: Another very recent precision measurement [42] is below Mohr's calculation by only 0.1σ, of 1.1σ if a large proton size is used, is 2.1σ below ours, and is 1.1σ below the other measurement; obviously, more work is needed before these differences are clearly resolved.) In any event, the cited uncertainties do indeed represent the order of magnitude of the uncertainties, and any users of the energy level tables that require more precise results or details should consult the author directly.

TABLE C. Electron structure correction coefficients

State	Known coefficients C_5	C_{62}	C_{61}	Coeff. for approx. $H(Z\alpha) \pm \delta H$ C_{60}	C_7	δC_{60}
$1S_{1/2}$	6.968	−0.75	4.065	−19.343	30.04	±0.5
$2S_{1/2}$	6.968	−0.75	4.448	−17.598	18.48	±0.5
$2P_{1/2}$	0	0	0.429	−0.352	−1.28	±0.33
$2P_{3/2}$	0	0	0.242	−0.438	−0.32	±0.18
$3S_{1/2}$	6.968	−0.75	4.409	−17.339	17.18	±0.5
$3P_{1/2}$	0	0	0.496	−0.396	−1.58	±0.39
$3P_{3/2}$	0	0	0.274	−0.497	−0.36	±0.20
$3D_{3/2}$	0	0	0.00741	−0.020	0.09	±0.010
$3D_{5/2}$	0	0	0.00741	−0.020	−0.06	±0.010
$4S_{1/2}$	6.968	−0.75	4.359	−17.268	17.00	±0.5
$4P_{1/2}$	0	0	0.520	−0.412	−1.68	±0.41
$4P_{3/2}$	0	0	0.285	−0.519	−0.37	±0.21
$4D_{3/2}$	0	0	0.00833	−0.024	0.088	±0.012
$4D_{5/2}$	0	0	0.00833	−0.024	−0.059	±0.012
$4F_{5/2}$	0	0	0.00119	−0.0039	0.042	±0.0020
$4F_{7/2}$	0	0	0.00119	−0.0039	−0.031	±0.0020

Appendix D. Lamb Shift and Fine Structure Measurements

Table D is a complete list of measurements of Lamb shift differences (between the two states having the same value of n and j but different $l = j \mp 1/2$) and other fine structure differences (between states having the same value of n) and Lamb shifts of single states (difference from their non-QED values). The calculated value (and the associated uncertainty in the calculation) is also included, as well as a comparison expressed as a multiple of a "standard deviation". The uncertainties in the calculations and in most of the measurements are intended to be 68 percent confidence level estimates, so they are combined in quadrature and we might expect 32 percent of the measurements to differ from the calculations by more than the combined uncertainties. The references are only to the most recent published article when there were a series of related measurements or reports. Changes from the published values are noted, and reference is sometimes made to a detailed discussion by Taylor, Parker, and Langenberg [4] of the measurements or to a summary and discussion by G. W. Series [41] of older optical measurements. Table D is meant to include all measurements known as of September, 1976 and the author would appreciate knowing of any omissions or additions. The units indicate the type of measurement, being frequency units of MHz or GHz except for optical measurements given in units of cm^{-1}. The calculated values correspond to the tabulated energy levels except that more decimal places are sometimes given in table D, leading to slight differences (smaller than the uncertainties) with the rounded-off entries in the energy level table.

TABLE D. Lamb shift and fine structure measurements

Atom	Interval	Calculated values ($\pm 1\sigma$)	Measured value ($\pm 1\sigma$)	Calc-meas σ	Ref., notes
H	$1S_{1/2}$–$1S_{1/2}$	8149.43 ±0.080 MHz	8600 ±800 MHz	−0.6	c
	$2S_{1/2}$–$2P_{1/2}$	1057.910 ±0.010 MHz	1057.893 ± 0.020 MHz	+0.8	d
			1057.862 ± 0.020 MHz	+2.1	iii
			1057.90 ± 0.10 MHz	+0.1	e
			1057.859 ± 0.063 MHz	+0.8	f
			1057.758 ± 0.089 MHz	+1.7	g, a
			1057.960 ± 0.095 MHz	−0.5	g, a
			1058.05 ± 0.26 MHz	−0.5	jjj
			1058.3 ± 0.5 MHz	−0.8	kkk
			1057.3 ± 0.9 MHz	+0.7	h
	$2P_{3/2}$–$2S_{1/2}$	9911.122 ±0.018 MHz	9911.173 ± 0.042 MHz	−1.1	i
			9911.250 ± 0.063 MHz	−2.0	j
			9911.377 ± 0.026 MHz	−8.0	k

TABLE D. Lamb shift and fine structure measurements—Continued

Atom	Interval	Calculated values ($\pm 1\sigma$)	Measured value ($\pm 1\sigma$)	Calc-meas / σ	Ref., notes
	$2P_{3/2}-2P_{1/2}$	10969.032 ±0.018 MHz	10969.127 ± 0.095 MHz	−1.0	l, a
			10969.6 ± 0.7 MHz	−0.8	m, a
	$3S_{1/2}-3P_{1/2}$	314.896 ±0.003 MHz	314.819 ± 0.048 MHz	+1.6	n
			315.11 ± 0.89 MHz	−0.2	o
			313.6 ± 2.9 MHz	+0.4	p, a
	$3P_{3/2}-3S_{1/2}$	2935.190 ±0.005 MHz	2933.5 ± 1.2 MHz	+1.4	o
	$3D_{3/2}-3S_{1/2}$	2929.859 ±0.005 MHz	2929.9 ± 0.8 MHz	−0.1	q
	$3D_{3/2}-3P_{1/2}$	3244.755 ±0.005 MHz	3255.6 ± 8.4 MHz	−1.3	o
	$3D_{5/2}-3S_{1/2}$	4013.195 ±0.007 MHz	4013.8 ± 0.8 MHz	−0.8	q
	$3D_{5/2}-3P_{3/2}$	1078.0051±0.0018 MHz	1080.3 ± 2.9 MHz	−0.8	o
	$4S_{1/2}-4P_{1/2}$	133.0849±0.0013 MHz	132.53 $^{+0.58}_{-0.78}$ MHz	+1.0	r
			133.18 ± 0.59 MHz	−0.2	o
	$4P_{3/2}-4S_{1/2}$	1238.044 ±0.002 MHz	1237.79 $^{+0.29}_{-0.26}$ MHz	+0.9	r
			1235.9 ± 1.3 MHz	+1.6	o
	$4D_{3/2}-4P_{1/2}$	1368.853 ±0.017 MHz	1371.1 ± 1.2 MHz	−1.9	o
	$4D_{5/2}-4P_{3/2}$	454.757 ±0.017 MHz	455.7 ± 1.6 MHz	−0.6	o
	$4F_{5/2}-4D_{3/2}$	456.209 ±0.018 MHz	456.8 ± 1.6 MHz	−0.4	o
	$4F_{7/2}-4D_{5/2}$	227.692 ±0.018 MHz	227.96 ± 0.41 MHz	−0.7	o
	$5S_{1/2}-5P_{1/2}$	68.203 ±0.005 MHz	64.6 ± 5.0 MHz	+0.7	o
	$5P_{3/2}-5S_{1/2}$	633.814 ±0.005 MHz	622.4 ± 10.1 MHz	+1.1	o
	$5D_{3/2}-5P_{1/2}$	700.843 ±0.011 MHz	704.3 ± 7.1 MHz	−0.5	o
	$5D_{5/2}-5P_{3/2}$	232.827 ±0.011 MHz	232.2 ± 2.9 MHz	+0.2	o
	$5F_{7/2}-5D_{5/2}$	116.575 ±0.013 MHz	117.2 ± 1.5 MHz	−0.4	o
D	$1S_{1/2}-1S_{1/2}$	8172.22 ±0.12 MHz	8300 ±300 MHz	−0.4	c
		0.27260 cm^{-1}	0.262 ± 0.038 cm^{-1}	+0.3	s
	$2S_{1/2}-2S_{1/2}$	0.03488 cm^{-1}	0.0369 ± 0.0016 cm^{-1}	−1.3	t, b
	$2S_{1/2}-2P_{1/2}$	1059.272 ±0.015 MHz	1059.282 ± 0.064 MHz	−0.2	u, a
			1058.996 ± 0.064 MHz	+4.2	g, a
			1059.6 ± 1.1 MHz	−0.3	h
	$2P_{3/2}-2S_{1/2}$	9912.754 ±0.021 MHz	9912.607 ± 0.056 MHz	+2.5	v, a
			9912.803 ± 0.094 MHz	−0.5	v, a
	$3S_{1/2}-3S_{1/2}$	0.01040 cm^{-1}	0.0083 $^{+0.002}_{-0.003}$ cm^{-1}	+1.0	w, b
	$3S_{1/2}-3P_{1/2}$	315.300 ±0.004 MHz	315.3 ± 0.4 MHz	−0.0	x, a
	$3P_{3/2}-3P_{1/2}$	3250.973 ±0.005 MHz	3250.7 ± 1.0 MHz	+0.3	x, a
	$3P_{3/2}-3D_{3/2}$	5.3369 ±0.0005 MHz	5.0 ± 5.0 MHz	+0.1	x, aa
	$4S_{1/2}-4P_{1/2}$	133.2550 ±0.0019 MHz	133.0 ± 5.0 MHz	+0.1	x, a
T	$2S_{1/2}-2S_{1/2}$	0.03489 cm^{-1}	0.037 $^{+0.002}_{-0.004}$ cm^{-1}	−0.5	y, b
$^3He^+$	$3S_{1/2}-3P_{1/2}$	(139.593 ±0.007) × 10^{-3} cm^{-1}	(140.0 ± 1.0) × 10^{-3} cm^{-1}	−0.4	z, bb
	$4S_{1/2}-4P_{1/2}$	(59.017 ±0.003) × 10^{-3} cm^{-1}	(60.3 ± 2.0) × 10^{-3} cm^{-1}	−0.6	z, bb
			(57.4 ± 4.9) × 10^{-3} cm^{-1}	+0.3	ee, cc
			(57.4 ± 4.1) × 10^{-3} cm^{-1}	+0.4	ee, dd
	$5S_{1/2}-5P_{1/2}$	(30.251 ±0.003) × 10^{-3} cm^{-1}	(37.7 ± 9.5) × 10^{-3} cm^{-1}	−0.8	ee, cc
	$6S_{1/2}-6P_{1/2}$	(17.516 ±0.002) × 10^{-3} cm^{-1}	(31.5 ± 8.3) × 10^{-3} cm^{-1}	−1.7	ee, dd
$^4He^+$	$2S_{1/2}-2P_{1/2}$	14044.6 ±0.5 MHz	14046.2 ± 1.2 MHz	−1.2	ff
			14040.2 ± 1.8 MHz	+2.4	gg, a
		(468.48 ±0.02) × 10^{-3} cm^{-1}	(480 ± 20) × 10^{-3} cm^{-1}	−0.6	hh, b
	$3S_{1/2}-3P_{1/2}$	4184.39 ±0.16 MHz	4183.17 ± 0.54 MHz	+2.2	ii
			4184.2 ± 2.4 MHz	+0.1	ww
		(139.576 ±0.005) × 10^{-3} cm^{-1}	(138.5 ± 1.6) × 10^{-3} cm^{-1}	+0.7	jj, kk
			(139.1 ± 2.0) × 10^{-3} cm^{-1}	+0.2	z, bb
	$3P_{3/2}-3S_{1/2}$	47843.41 ±0.17 MHz	47844.05 ± 0.48 MHz	−1.3	ii
	$3D_{5/2}-3P_{3/2}$	17255.36 ±0.04 MHz	17259 ± 5 MHz	−0.7	ww
	$4S_{1/2}-4P_{1/2}$	1769.08 ±0.07 MHz	1769 ± 2 MHz	+0.0	lll
			1768 ± 5 MHz	+0.2	ll
			1766 ± 7.5 MHz	+0.4	mm
			1751 ± 13 MHz	+1.4	nn, a
			1755 ± 44 MHz	+0.3	oo
		(59.010 ±0.002) × 10^{-3} cm^{-1}	(60.0 ± 4.0) × 10^{-3} cm^{-1}	−0.2	jj
			(57.1 ± 2.0) × 10^{-3} cm^{-1}	+1.0	z, bb
			(60.3 ± 2.9) × 10^{-3} cm^{-1}	−0.4	ee, cc
			(58.9 ± 3.3) × 10^{-3} cm^{-1}	+0.0	ee, dd

TABLE D. Lamb shift and fine structure measurements—Continued

Atom	Interval	Calculated values ($\pm 1\sigma$)	Measured value ($\pm 1\sigma$)	$\frac{\text{Calc-meas}}{\sigma}$	Ref., notes
	$4P_{3/2}-4S_{1/2}$	20180.05 $\pm$0.07 MHz	20179.7 $\pm$ 1.2 MHz	+0.3	ll
			20180.6 $\pm$ 0.8 MHz	−0.7	tt
	$4D_{3/2}-4S_{1/2}$	20143.51 $\pm$0.28 MHz	20145.3 $\pm$ 3.9 MHz	−0.5	pp, aa
	$4D_{5/2}-4S_{1/2}$	27459.25 $\pm$0.28 MHz	27455.4 $\pm$ 3.7 MHz	+1.0	pp, aa
	$4D_{5/2}-4D_{3/2}$	7315.745 $\pm$0.012 MHz	7310.1 $\pm$ 7.6 MHz	+0.7	pp, aa, qq
	$4F_{5/2}-4S_{1/2}$	27446.06 $\pm$0.13 MHz	27435.7 $\pm$ 6.9 MHz	+1.5	rr, aa
	$4F_{5/2}-4D_{3/2}$	7302.55 $\pm$0.29 MHz	7300.2 $\pm$ 1.5 MHz	+1.5	ss
	$4F_{7/2}-4S_{1/2}$	31103.84 $\pm$0.13 MHz	31098.8 $\pm$ 5.2 MHz	+1.0	rr, aa
	$4F_{7/2}-4F_{5/2}$	3657.783 $\pm$0.006 MHz	3663.1 $\pm$ 12.1 MHz	−0.4	rr, aa, qq
	$5S_{1/2}-5P_{1/2}$	906.78 $\pm$0.08 MHz	902 $\pm$ 15 MHz	+0.3	oo
		(30.247 $\pm$0.003) $\times 10^{-3}$ cm^{-1}	(36.6 $\pm$ 4.9) $\times 10^{-3}$ cm^{-1}	−1.3	ee, cc, kk
	$5P_{3/2}-5S_{1/2}$	10331.12 $\pm$0.09 MHz	10332.9 $\pm$ 1.4 MHz	−1.3	tt
	$5D_{3/2}-5S_{1/2}$	10312.27 $\pm$0.20 MHz	10309.8 $\pm$ 14.6 MHz	+0.2	uu
	$5D_{5/2}-5S_{1/2}$	14057.93 $\pm$0.20 MHz	14055.4 $\pm$ 12.6 MHz	+0.2	uu
	$5F_{5/2}-5S_{1/2}$	14051.13 $\pm$0.14 MHz	14058.2 $\pm$ 13.6 MHz	−0.5	uu
	$5F_{7/2}-5S_{1/2}$	15923.92 $\pm$0.14 MHz	15924.5 $\pm$ 10.7 MHz	−0.1	uu
	$5G_{7/2}-5S_{1/2}$	15920.38 $\pm$0.09 MHz	15924.4 $\pm$ 5.9 MHz	−0.7	vv
	$5G_{9/2}-5S_{1/2}$	17044.04 $\pm$0.09 MHz	17040.5 $\pm$ 8.0 MHz	+0.4	vv
	$5G_{9/2}-5G_{7/2}$	1123.663 $\pm$0.002 MHz	1116.1 $\pm$ 14.0 MHz	+0.5	vv, qq
	$6S_{1/2}-6P_{1/2}$	525.05 $\pm$0.07 MHz	530 $\pm$ 10 MHz	−0.5	oo
			524 $\pm$ 13 MHz	+0.1	oo
		(17.514 $\pm$0.002) $\times 10^{-3}$ cm^{-1}	(20.7 $\pm$ 7.7) $\times 10^{-3}$ cm^{-1}	−0.4	ee, dd
	$6P_{3/2}-6S_{1/2}$	5978.33 $\pm$0.07 MHz	5979.1 $\pm$ 1.2 MHz	−0.6	tt
	$6G_{9/2}-6D_{5/2}$	1728.05 $\pm$0.12 MHz	1728.6 $\pm$ 1.4 MHz	−0.4	xx, yy
	$6G_{9/2}-6F_{7/2}$	648.21 $\pm$0.09 MHz	648.5 $\pm$ 5.0 MHz	−0.1	oo
	$6H_{9/2}-6F_{5/2}$	1730.73 $\pm$0.08 MHz	1728.6 $\pm$ 1.4 MHz	+1.5	xx, yy
	$6H_{9/2}-6G_{7/2}$	649.00 $\pm$0.03 MHz	648.5 $\pm$ 5.0 MHz	+0.1	oo
	$6H_{11/2}-6D_{5/2}$	2160.29 $\pm$0.12 MHz	2161.2 $\pm$ 3.0 MHz	−0.3	xx
	$6H_{11/2}-6G_{9/2}$	432.24 $\pm$0.03 MHz	437 $\pm$ 10 MHz	−0.5	oo
	$7S_{1/2}-7P_{1/2}$	330.77 $\pm$0.06 MHz	333 $\pm$ 8 MHz	−0.3	oo
	$7G_{9/2}-7P_{3/2}$	2446.26 $\pm$0.06 MHz	2452.8 $\pm$ 3.6 MHz	−1.8	xx, zz
	$7H_{9/2}-7D_{3/2}$	2452.44 $\pm$0.08 MHz	2452.8 $\pm$ 3.6 MHz	−0.1	xx, zz
	$9H_{9/2}-9P_{1/2}$	3077.49 $\pm$0.04 MHz	3097.2 $\pm$ 16.0 MHz	−1.2	xx
$^6Li^{++}$	$2S_{1/2}-2P_{1/2}$	62762 $\pm$9 MHz	62765 $\pm$ 21 MHz	−0.1	aaa
			62790 $\pm$ 70 MHz	−0.4	bbb
			63031 $\pm$327 MHz	−0.8	ccc
C^{5+}	$2S_{1/2}-2P_{1/2}$	783.64 $\pm$0.23 GHz	780.1 $\pm$ 8.0 GHz	+0.4	ddd
O^{7+}	$2S_{1/2}-2P_{1/2}$	2204.6 $\pm$1.3 GHz	2202.7 $\pm$ 11.0 GHz	+0.2	eee
			2215.6 $\pm$ 7.5 GHz	−1.4	fff
F^{8+}	$2P_{3/2}-2S_{1/2}$	68833.9 $\pm$2.4 GHz	68853.9 $\pm$ 35 GHz	−0.6	ggg
Ar^{17+}	$2S_{1/2}-2P_{1/2}$	39.01 $\pm$0.16 THz	38.3 $\pm$ 1.1 THz	+0.6	hhh

[a] See reference [4].

[b] See also reference [41], which includes references to earlier optical measurements.

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[cc] From analysis of the $5 \rightarrow 4$ optical transitions.
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Hydrogenic levels (QED uncertainties)

	$^{1}H^{1}$	$^{1}D^{2}$	$^{1}T^{3}$	$^{2}He^{3}$	$^{2}He^{4}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$1S_{1/2}$	−109678.773704(3)	−109708.616541(4)	−109718.545884(9)	−438889.2004(2)	−438908.8853(1)
$2S_{1/2}$	−27419.8178352(3)	−27427.2786891(5)	−27429.761055(1)	−109724.19675(2)	−109729.11842(2)
$2P_{1/2}$	−27419.85312328(9)	−27427.31402261(8)	−27429.79638632(8)	−109724.665283(5)	−109729.586898(5)
$2P_{3/2}$	−27419.48723576(5)	−27426.94803523(5)	−27429.43036572(5)	−109718.808389(3)	−109723.729740(3)
$3S_{1/2}$	−12186.5502372(1)	−12189.8661741(1)	−12190.9694460(3)	−48765.727780(7)	−48767.915169(5)
$3P_{1/2}$	−12186.56074102(3)	−12189.87669137(3)	−12190.97996250(3)	−48765.867373(2)	−48768.054745(2)
$3P_{3/2}$	−12186.45232982(2)	−12189.76825057(2)	−12190.87151186(2)	−48764.131991(1)	−48766.319284(1)
$3D_{3/2}$	−12186.452507652(2)	−12189.768428589(2)	−12190.871689936(2)	−48764.13484515(8)	−48766.32213927(8)
$3D_{5/2}$	−12186.416371435(2)	−12189.732282508(2)	−12190.835540575(2)	−48763.55643940(7)	−48765.74370750(6)
$4S_{1/2}$	−6854.91884539(4)	−6856.78405833(6)	−6857.4046475(1)	−27430.467164(3)	−27431.697559(2)
$4P_{1/2}$	−6854.92328462(1)	−6856.78850324(1)	−6857.40909211(1)	−27430.5261817(8)	−27431.7565691(8)
$4P_{3/2}$	−6854.877548700(7)	−6856.742754836(7)	−6857.363339547(7)	−27429.7940706(4)	−27431.0244251(4)
$4D_{3/2}$	−6854.8776246(6)	−6856.7428308(6)	−6857.3634156(6)	−27429.795289(9)	−27431.025644(9)
$4D_{5/2}$	−6854.8623796(6)	−6856.7275817(6)	−6857.3481650(6)	−27429.551273(9)	−27430.781617(9)
$4F_{5/2}$	−6854.8624071(2)	−6856.7276092(2)	−6857.3481925(2)	−27429.551713(4)	−27430.782057(4)
$4F_{7/2}$	−6854.8547847(2)	−6856.7199847(2)	−6857.3405673(2)	−27429.429708(4)	−27430.660047(4)
$5S_{1/2}$	−4387.1408809(2)	−4388.3346162(2)	−4388.7317927(2)	−17555.382518(3)	−17556.169965(3)
$5P_{1/2}$	−4387.143155962(9)	−4388.336894130(9)	−4388.734070421(9)	−17555.4127688(4)	−17556.2002122(4)
$5P_{3/2}$	−4387.119739197(7)	−4388.313470974(7)	−4388.710645139(7)	−17555.0379298(2)	−17555.8253563(2)
$5D_{3/2}$	−4387.1197784(4)	−4388.3135102(4)	−4388.7106844(4)	−17555.038558(6)	−17555.825985(6)
$5D_{5/2}$	−4387.1119729(4)	−4388.3057026(4)	−4388.7028761(4)	−17554.913622(6)	−17555.701043(6)
$5F_{5/2}$	−4387.1119871(2)	−4388.3057168(2)	−4388.7028903(2)	−17554.913849(4)	−17555.701270(4)
$5F_{7/2}$	−4387.1080844(2)	−4388.3018130(2)	−4388.6989861(2)	−17554.851382(4)	−17555.638801(4)
$5G_{7/2}$	−4387.10809178(5)	−4388.30182041(5)	−4388.69899353(5)	−17554.8515005(7)	−17555.6389187(7)
$5G_{9/2}$	−4387.10575016(5)	−4388.29947816(5)	−4388.69665107(5)	−17554.8140208(7)	−17555.6014374(7)
$6S_{1/2}$	−3046.6219504(1)	−3047.4509326(1)	−3047.7267492(1)	−12191.178541(2)	−12191.725376(2)
$6P_{1/2}$	−3046.6232675(1)	−3047.4522514(1)	−3047.7280680(1)	−12191.196057(2)	−12191.742890(2)
$6P_{3/2}$	−3046.6097162(1)	−3047.4386964(1)	−3047.7145117(1)	−12190.979137(2)	−12191.525960(2)
$6D_{3/2}$	−3046.6097391(2)	−3047.4387193(2)	−3047.7145346(2)	−12190.979504(4)	−12191.526328(4)
$6D_{5/2}$	−3046.6052220(2)	−3047.4342010(2)	−3047.7100160(2)	−12190.907203(4)	−12191.454023(4)
$6F_{5/2}$	−3046.6052303(2)	−3047.4342093(2)	−3047.7100242(2)	−12190.907335(3)	−12191.454155(3)
$6F_{7/2}$	−3046.6029718(2)	−3047.4319501(2)	−3047.7077649(2)	−12190.871185(3)	−12191.418004(3)
$6G_{7/2}$	−3046.60297605(5)	−3047.43195444(5)	−3047.70776916(5)	−12190.8712538(9)	−12191.4180725(9)
$6G_{9/2}$	−3046.60162095(5)	−3047.43059897(5)	−3047.70641357(5)	−12190.8495641(9)	−12191.3963818(9)
$6H_{9/2}$	−3046.60162360(1)	−3047.43060162(1)	−3047.70641622(1)	−12190.8496066(2)	−12191.3964243(2)
$6H_{11/2}$	−3046.60072020(1)	−3047.42969798(1)	−3047.70551250(1)	−12190.8351469(2)	−12191.3819639(2)
$7S_{1/2}$	−2238.33245130(7)	−2238.94149911(7)	−2239.14413973(7)	−8956.751161(1)	−8957.152916(1)
$7P_{1/2}$	−2238.3332810(1)	−2238.9423299(1)	−2239.1449705(1)	−8956.762196(2)	−8957.163949(2)
$7P_{3/2}$	−2238.3247472(1)	−2238.9337938(1)	−2239.1364336(1)	−8956.625593(2)	−8957.027341(2)
$7D_{3/2}$	−2238.3247617(2)	−2238.9338083(2)	−2239.1364481(2)	−8956.625826(3)	−8957.027574(3)
$7D_{5/2}$	−2238.3219172(2)	−2238.9309630(2)	−2239.1336025(2)	−8956.580296(3)	−8956.982041(3)
$7F_{5/2}$	−2238.3219224(1)	−2238.9309682(1)	−2239.1336077(1)	−8956.580379(2)	−8956.982124(2)
$7F_{7/2}$	−2238.3205001(1)	−2238.9295455(1)	−2239.1321849(1)	−8956.557614(2)	−8956.959358(2)
$7G_{7/2}$	−2238.32050284(5)	−2238.92954824(5)	−2239.13218764(5)	−8956.5576573(8)	−8956.9594018(8)
$7G_{9/2}$	−2238.31964948(5)	−2238.92869465(5)	−2239.13133397(5)	−8956.5439985(8)	−8956.9457424(8)
$7H_{9/2}$	−2238.31965116(2)	−2238.92869632(2)	−2239.13133565(2)	−8956.5440253(3)	−8956.9457692(3)
$7H_{11/2}$	−2238.31908225(2)	−2238.92812726(2)	−2239.13076654(2)	−8956.5349195(3)	−8956.9366629(3)
$7I_{11/2}$	−2238.319083395(5)	−2238.928128403(5)	−2239.130767679(5)	−8956.53493776(7)	−8956.93668121(7)
$7I_{13/2}$	−2238.318677035(5)	−2238.927721932(5)	−2239.130361171(5)	−8956.52843362(7)	−8956.93017677(7)
$8S_{1/2}$	−1713.72205915(5)	−1714.18836116(5)	−1714.34350777(5)	−6857.4928029(8)	−6857.8003955(8)
$8P_{1/2}$	−1713.72261513(9)	−1714.18891785(9)	−1714.34406442(9)	−6857.500197(1)	−6857.807789(1)
$8P_{3/2}$	−1713.71689816(9)	−1714.18319932(9)	−1714.33834537(9)	−6857.408685(1)	−6857.716272(1)
$8D_{3/2}$	−1713.7169079(1)	−1714.1832091(1)	−1714.3383551(1)	−6857.408841(2)	−6857.716429(2)
$8D_{5/2}$	−1713.7150023(1)	−1714.1813029(1)	−1714.3364488(1)	−6857.378339(2)	−6857.685926(2)
$8F_{5/2}$	−1713.7150058(1)	−1714.1813064(1)	−1714.3364523(1)	−6857.378395(2)	−6857.685981(2)
$8F_{7/2}$	−1713.7140530(1)	−1714.1803534(1)	−1714.3354992(1)	−6857.363144(2)	−6857.670730(2)
$8G_{7/2}$	−1713.71405480(4)	−1714.18035519(4)	−1714.33550098(4)	−6857.3631735(6)	−6857.6707591(6)

Hydrogenic levels (AED uncertainties)—Continued

	$^1H^1$	$^1D^2$	$^1T^3$	$^2He^3$	$^2He^4$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$8G_{9/2}$	−1713.71348312(4)	−1714.17978335(4)	−1714.33492909(4)	−6857.3540232(6)	−6857.6616084(6)
$8H_{9/2}$	−1713.71348424(2)	−1714.17978447(2)	−1714.33493022(2)	−6857.3540412(3)	−6857.6616264(3)
$8H_{11/2}$	−1713.71310312(2)	−1714.17940325(2)	−1714.33454896(2)	−6857.3479410(3)	−6857.6555259(3)
$8I_{11/2}$	−1713.713103884(6)	−1714.179404014(6)	−1714.334549724(6)	−6857.3479532(1)	−6857.6555382(1)
$8I_{13/2}$	−1713.712831655(6)	−1714.179131710(6)	−1714.334277396(6)	−6857.3435960(1)	−6857.6511807(1)
$8K_{13/2}$	−1713.712832211(2)	−1714.179132266(2)	−1714.334277952(2)	−6857.34360488(3)	−6857.65118962(3)
$8K_{15/2}$	−1713.712628039(2)	−1714.178928039(2)	−1714.334073706(2)	−6857.34033693(3)	−6857.64792153(3)
$9S_{1/2}$	−1354.05122143(3)	−1354.41965744(3)	−1354.54224234(4)	−5418.2533664(6)	−5418.4964020(6)
$9P_{1/2}$	−1354.05161198(7)	−1354.42004848(7)	−1354.54263336(7)	−5418.258561(1)	−5418.501595(1)
$9P_{3/2}$	−1354.04759677(7)	−1354.41603218(7)	−1354.53861669(7)	−5418.194289(1)	−5418.437321(1)
$9D_{3/2}$	−1354.04760364(8)	−1354.41603906(8)	−1354.53862357(8)	−5418.194399(1)	−5418.437431(1)
$9D_{5/2}$	−1354.04626527(8)	−1354.41470032(8)	−1354.53728471(8)	−5418.172976(1)	−5418.416008(1)
$9F_{5/2}$	−1354.04626772(8)	−1354.41470277(8)	−1354.53728716(8)	−5418.173016(1)	−5418.416047(1)
$9F_{7/2}$	−1354.04559853(8)	−1354.41403340(8)	−1354.53661773(8)	−5418.162305(1)	−5418.405335(1)
$9G_{7/2}$	−1354.04559981(3)	−1354.41403468(3)	−1354.53661901(3)	−5418.1623252(5)	−5418.4053558(5)
$9G_{9/2}$	−1354.04519830(3)	−1354.41363306(3)	−1354.53621736(3)	−5418.1558986(5)	−5418.3989289(5)
$9H_{9/2}$	−1354.04519909(1)	−1354.41363385(1)	−1354.53621815(1)	−5418.1559113(2)	−5418.3989416(2)
$9H_{11/2}$	−1354.04493142(1)	−1354.41336611(1)	−1354.53595038(1)	−5418.1516269(2)	−5418.3946570(2)
$9I_{11/2}$	−1354.044931955(6)	−1354.413366644(6)	−1354.535950915(6)	−5418.1516355(1)	−5418.3946656(1)
$9I_{13/2}$	−1354.044740759(6)	−1354.413175396(6)	−1354.535759650(6)	−5418.1485753(1)	−5418.3916053(1)
$9K_{13/2}$	−1354.044741150(3)	−1354.413175788(3)	−1354.535760042(3)	−5418.14858152(4)	−5418.39161151(4)
$9K_{15/2}$	−1354.044597754(3)	−1354.413032352(3)	−1354.535616593(3)	−5418.14628634(4)	−5418.38931622(4)
$9L_{15/2}$	−1354.0445980509(8)	−1354.4130326492(9)	−1354.5356168903(8)	−5418.14629109(1)	−5418.38932098(1)
$9L_{17/2}$	−1354.0444865204(8)	−1354.4129210881(9)	−1354.5355053192(8)	−5418.14450595(1)	−5418.38753576(1)
$10S_{1/2}$	−1096.78097442(2)	−1097.07940748(2)	−1097.17870121(3)	−4388.7768987(4)	−4388.9737571(4)
$10P_{1/2}$	−1096.78125916(6)	−1097.07969259(6)	−1097.17898629(6)	−4388.7806858(9)	−4388.9775437(9)
$10P_{3/2}$	−1096.77833207(6)	−1097.07676470(6)	−1097.17605814(6)	−4388.7338316(9)	−4388.9306874(9)
$10D_{3/2}$	−1096.77833709(6)	−1097.07676973(6)	−1097.17606317(6)	−4388.733912(1)	−4388.930768(1)
$10D_{5/2}$	−1096.77736142(6)	−1097.07579379(6)	−1097.17508714(6)	−4388.718295(1)	−4388.915150(1)
$10F_{5/2}$	−1096.77736321(6)	−1097.07579558(6)	−1097.17508893(6)	−4388.7183239(9)	−4388.9151790(9)
$10F_{7/2}$	−1096.77687537(6)	−1097.07530761(6)	−1097.17460091(6)	−4388.7105156(9)	−4388.9073703(9)
$10G_{7/2}$	−1096.77687630(3)	−1097.07530854(3)	−1097.17460185(3)	−4388.7105305(4)	−4388.9073853(4)
$10G_{9/2}$	−1096.77658360(3)	−1097.07501576(3)	−1097.17430904(3)	−4388.7058456(4)	−4388.9027001(4)
$10H_{9/2}$	−1096.77658418(1)	−1097.07501634(1)	−1097.17430962(1)	−4388.7058548(2)	−4388.9027094(2)
$10H_{11/2}$	−1096.77638904(1)	−1097.07482115(1)	−1097.17411441(1)	−4388.7027315(2)	−4388.8995859(2)
$10I_{11/2}$	−1096.776389437(6)	−1097.074821543(6)	−1097.174114805(6)	−4388.70273779(9)	−4388.89959221(9)
$10I_{13/2}$	−1096.776250055(6)	−1097.074682123(6)	−1097.173975373(6)	−4388.70050686(9)	−4388.89736118(9)
$10K_{13/2}$	−1096.776250341(3)	−1097.074682409(3)	−1097.173975659(3)	−4388.70051143(5)	−4388.89736575(5)
$10K_{15/2}$	−1096.776145805(3)	−1097.074577844(3)	−1097.173871085(3)	−4388.69883824(5)	−4388.89569249(5)
$10L_{15/2}$	−1096.776146021(1)	−1097.074578061(1)	−1097.173871302(1)	−4388.69884171(2)	−4388.89569596(2)
$10L_{17/2}$	−1096.776064716(1)	−1097.074496733(1)	−1097.173789966(1)	−4388.69754034(2)	−4388.89439453(2)
$10M_{17/2}$	−1096.7760648859(4)	−1097.0744969035(4)	−1097.1737901366(4)	−4388.697543071(7)	−4388.894397261(7)
$10M_{19/2}$	−1096.7759998413(4)	−1097.0744318412(4)	−1097.1737250683(4)	−4388.696501977(7)	−4388.893356120(7)
$11S_{1/2}$	−906.43020253(2)	−906.67684136(2)	−906.75890226(2)	−3627.0826085(3)	−3627.2453011(3)
$11P_{1/2}$	−906.43041648(5)	−906.67705558(5)	−906.75911646(5)	−3627.0854541(8)	−3627.2481463(8)
$11P_{3/2}$	−906.42821732(5)	−906.67485582(5)	−906.75691650(5)	−3627.0502519(8)	−3627.2129425(8)
$11D_{3/2}$	−906.42822110(5)	−906.67485961(5)	−906.75692029(5)	−3627.0503126(8)	−3627.2130032(8)
$11D_{5/2}$	−906.42748806(5)	−906.67412637(5)	−906.75618698(5)	−3627.0385793(8)	−3627.2012695(8)
$11F_{5/2}$	−906.42748941(5)	−906.67412771(5)	−906.75618833(5)	−3627.0386009(7)	−3627.2012910(7)
$11F_{7/2}$	−906.42712289(5)	−906.67376109(5)	−906.75582167(5)	−3627.0327344(7)	−3627.1954242(7)
$11G_{7/2}$	−906.42712359(2)	−906.67376180(2)	−906.75582238(2)	−3627.0327456(3)	−3627.1954355(3)
$11G_{9/2}$	−906.42690368(2)	−906.67354182(2)	−906.75560239(2)	−3627.0292257(3)	−3627.1919154(3)
$11H_{9/2}$	−906.42690411(1)	−906.67354226(1)	−906.75560282(1)	−3627.0292327(2)	−3627.1919224(2)
$11H_{11/2}$	−906.42675750(1)	−906.67339561(1)	−906.75545616(1)	−3627.0268861(2)	−3627.1895757(2)
$11I_{11/2}$	−906.426757800(5)	−906.673395907(5)	−906.755456455(5)	−3627.02689080(8)	−3627.18958042(8)
$11I_{13/2}$	−906.426653080(5)	−906.673291159(5)	−906.755351698(5)	−3627.02521468(8)	−3627.18790422(8)
$11K_{13/2}$	−906.426653295(3)	−906.673291373(3)	−906.755351912(3)	−3627.02521811(4)	−3627.18790765(4)
$11K_{15/2}$	−906.426574755(3)	−906.673212813(3)	−906.755273344(3)	−3627.02396102(4)	−3627.18665050(4)
$11L_{15/2}$	−906.426574918(1)	−906.673212976(1)	−906.755273508(1)	−3627.02396363(2)	−3627.18665311(2)
$11L_{17/2}$	−906.426513832(1)	−906.673151873(1)	−906.755212399(1)	−3627.02298589(2)	−3627.18567533(2)
$11M_{17/2}$	−906.4265139602(6)	−906.6731520009(6)	−906.7552125272(6)	−3627.02298794(1)	−3627.18567738(1)

Hydrogenic levels (QED uncertainties) — Continued

	$^{1}H^{1}$	$^{1}D^{2}$	$^{1}T^{3}$	$^{2}He^{3}$	$^{2}He^{4}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$11M_{19/2}$	−906.4264650912(6)	−906.6731031186(6)	−906.7551636405(6)	−3627.02220576(1)	−3627.18489516(1)
$11N_{19/2}$	−906.4264651946(2)	−906.6731032219(2)	−906.7551637438(2)	−3627.022207409(4)	−3627.184896812(4)
$11N_{21/2}$	−906.4264252108(2)	−906.6730632273(2)	−906.7551237456(2)	−3627.021567437(4)	−3627.184256811(4)
$12S_{1/2}$	−761.65290399(1)	−761.86014907(1)	−761.92910299(2)	−3047.7528351(3)	−3047.8895418(2)
$12P_{1/2}$	−761.65306880(4)	−761.86031409(4)	−761.92926800(4)	−3047.7550271(6)	−3047.8917336(6)
$12O_{23/2}$	−761.6499689122(1)	−761.8572133566(1)	−761.9261669865(1)	−3047.705408509(2)	−3047.842112775(2)
$13S_{1/2}$	−648.98217184(1)	−649.15875932(1)	−649.21751294(1)	−2596.8988459(2)	−2597.0153296(2)
$13P_{1/2}$	−648.98230147(3)	−649.15888912(3)	−649.21764273(3)	−2596.9005701(5)	−2597.0170536(5)
$13Q_{25/2}$	−648.97984629280(7)	−649.15643327500(7)	−649.21518666384(7)	−2596.861271168(1)	−2596.977752909(1)
$14S_{1/2}$	−559.581428918(9)	−559.733690547(9)	−559.78435054(1)	−2239.1605561(2)	−2239.2609934(2)
$14P_{1/2}$	−559.58153271(3)	−559.73379448(3)	−559.78445447(3)	−2239.1619366(4)	−2239.2623738(4)
$14R_{27/2}$	−559.57955527207(4)	−559.73181649652(5)	−559.78247630628(5)	−2239.1302846163(7)	−2239.2307204034(7)
$15S_{1/2}$	−487.457495457(8)	−487.590132233(8)	−487.634262709(8)	−1950.5559455(1)	−1950.6434375(1)
$15P_{1/2}$	−487.45757985(2)	−487.59021674(2)	−487.63434720(2)	−1950.5570680(3)	−1950.6445599(3)
$15T_{29/2}$	−487.45596388056(3)	−487.58860032497(3)	−487.63273064814(3)	−1950.5312018737(5)	−1950.6186926039(5)
$16S_{1/2}$	−428.429358101(6)	−428.545933376(6)	−428.584719919(7)	−1714.3544981(1)	−1714.4313954(1)
$16P_{1/2}$	−428.42942764(2)	−428.54600301(2)	−428.58478954(2)	−1714.3554231(3)	−1714.4323202(3)
$16U_{31/2}$	−428.42809018455(2)	−428.54466518452(2)	−428.58345160137(2)	−1714.3340150293(3)	−1714.4109111788(3)
$17S_{1/2}$	−379.508294780(5)	−379.611558678(5)	−379.645916301(6)	−1518.59672266(9)	−1518.66483917(9)
$17P_{1/2}$	−379.50835276(2)	−379.61161673(2)	−379.64597435(2)	−1518.5974938(2)	−1518.6656102(2)
$17V_{33/2}$	−379.50723334224(1)	−379.61049701003(1)	−379.64485452807(1)	−1518.5795758688(2)	−1518.6476914891(2)
$18S_{1/2}$	−338.511977355(5)	−338.604086192(5)	−338.634732339(5)	−1354.54995608(8)	−1354.61071429(8)
$18P_{1/2}$	−338.51202620(1)	−338.60413510(1)	−338.63478124(1)	−1354.5506057(2)	−1354.6113639(2)
$18W_{35/2}$	−338.511079907608(8)	−338.60318854956(1)	−338.633834607315(9)	−1354.5354589277(2)	−1354.5962163792(1)
$19S_{1/2}$	−303.816802757(4)	−303.899471068(4)	−303.926976193(4)	−1215.71730066(7)	−1215.77183154(6)
$19P_{1/2}$	−303.81684429(1)	−303.89951265(1)	−303.92701777(1)	−1215.7178531(2)	−1215.7723839(2)
$19X_{37/2}$	−303.816037196971(6)	−303.898705340992(8)	−303.926210390061(7)	−1215.7049343525(1)	−1215.7594645860(1)
$20S_{1/2}$	−274.194630876(3)	−274.269239020(3)	−274.294062392(3)	−1097.18432077(6)	−1097.23353486(6)
$20P_{1/2}$	−274.19466648(1)	−274.26927467(1)	−274.29409804(1)	−1097.1847944(2)	−1097.2340084(2)
$20Y_{39/2}$	−274.193972584215(4)	−274.268580584194(6)	−274.293403890979(5)	−1097.17368747628(9)	−1097.22290101200(8)

Hydrogenic levels (QED uncertainties)

	$^{3}Li^{6}$	$^{3}Li^{7}$	$^{4}Be^{9}$	$^{5}B^{10}$	$^{5}B^{11}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$1S_{1/2}$	−987648.171(2)	−987661.027(2)	−1756018.824(8)	−2744094.27(2)	−2744107.87(2)
$2S_{1/2}$	−246921.3693(3)	−246924.5835(3)	−439033.573(1)	−686092.918(3)	−686096.321(3)
$2P_{1/2}$	−246923.46283(6)	−246926.67667(6)	−439039.5736(3)	−686106.432(1)	−686109.835(1)
$2P_{3/2}$	−246893.80481(3)	−246897.01826(3)	−438945.8156(2)	−685877.4621(7)	−685880.8643(7)
$3S_{1/2}$	−109739.84303(9)	−109741.27149(8)	−195116.4956(3)	−304906.7258(8)	−304908.2380(9)
$3P_{1/2}$	−109740.46721(2)	−109741.89556(2)	−195118.2857(1)	−304910.7594(5)	−304912.2719(5)
$3P_{3/2}$	−109731.67957(1)	−109733.10781(1)	−195090.50514(6)	−304842.9149(2)	−304844.4271(2)
$3D_{3/2}$	−109731.6940667(7)	−109733.1223076(7)	−195090.551103(4)	−304843.02753(1)	−304844.53971(1)
$3D_{5/2}$	−109728.7654863(6)	−109730.1936889(6)	−195081.294464(3)	−304820.42631(1)	−304821.93837(1)
$4S_{1/2}$	−61727.36031(4)	−61728.16379(3)	−109748.8889(1)	−171499.8756(4)	−171500.7263(4)
$4P_{1/2}$	−61727.624263(9)	−61728.427699(9)	−109749.64613(5)	−171501.5821(2)	−171502.4329(2)
$4P_{3/2}$	−61723.917019(5)	−61724.720407(5)	−109737.92641(3)	−171472.9610(1)	−171473.8117(1)
$4D_{3/2}$	−61723.92321(5)	−61724.72660(5)	−109737.9460(1)	−171473.0092(4)	−171473.8598(4)

Hydrogenic levels (QED uncertainties)—Continued

	$^{3}Li^{6}$	$^{3}Li^{7}$	$^{4}Be^{9}$	$^{5}B^{10}$	$^{5}B^{11}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$4D_{5/2}$	−61722.68770(5)	−61723.49107(5)	−109734.0408(1)	−171463.4740(4)	−171464.3246(4)
$4F_{5/2}$	−61722.68993(2)	−61723.49330(2)	−109734.04787(6)	−171463.4912(1)	−171464.3418(1)
$4F_{7/2}$	−61722.07221(2)	−61722.87557(2)	−109732.09545(6)	−171458.7243(1)	−171459.5749(1)
$5S_{1/2}$	−39504.91639(2)	−39505.43060(2)	−70237.40060(7)	−109755.2908(2)	−109755.8352(2)
$5P_{1/2}$	−39505.051702(5)	−39505.565894(5)	−70237.78882(3)	−109756.1658(1)	−109756.7102(1)
$5P_{3/2}$	−39503.153614(3)	−39503.667781(3)	−70231.78844(1)	−109741.51221(6)	−109742.05660(6)
$5D_{3/2}$	−39503.15681(3)	−39503.67097(3)	−70231.7986(1)	−109741.5370(2)	−109742.0814(2)
$5D_{5/2}$	−39502.52423(3)	−39503.03839(3)	−70229.7991(1)	−109736.6550(2)	−109737.1994(2)
$5F_{5/2}$	−39502.52537(2)	−39503.03953(2)	−70229.80272(6)	−109736.6639(1)	−109737.2082(1)
$5F_{7/2}$	−39502.20910(2)	−39502.72326(2)	−70228.80307(6)	−109734.2232(1)	−109734.7676(1)
$5G_{7/2}$	−39502.209699(4)	−39502.723854(4)	−70228.80497(1)	−109734.22783(3)	−109734.77218(3)
$5G_{9/2}$	−39502.019938(4)	−39502.534091(4)	−70228.20520(1)	−109732.76352(3)	−109733.30786(3)
$6S_{1/2}$	−27433.66728(1)	−27434.02437(1)	−48775.01201(4)	−76216.5976(1)	−76216.9757(1)
$6P_{1/2}$	−27433.745635(9)	−27434.102709(9)	−48775.23683(3)	−76217.10433(9)	−76217.48242(9)
$6P_{3/2}$	−27432.647214(8)	−27433.004274(8)	−48771.76444(3)	−76208.62449(7)	−76209.00253(7)
$6D_{3/2}$	−27432.64908(2)	−27433.00614(2)	−48771.77036(6)	−76208.6390(2)	−76209.0170(2)
$6D_{5/2}$	−27432.28300(2)	−27432.64006(2)	−48770.61326(6)	−76205.8138(2)	−76206.1918(2)
$6F_{5/2}$	−27432.28367(1)	−27432.64073(1)	−48770.61537(5)	−76205.8189(1)	−76206.1969(1)
$6F_{7/2}$	−27432.10064(1)	−27432.45769(1)	−48770.03687(5)	−76204.4065(1)	−76204.7845(1)
$6G_{7/2}$	−27432.100989(4)	−27432.458042(4)	−48770.03797(1)	−76204.40917(3)	−76204.78719(3)
$6G_{9/2}$	−27431.991174(4)	−27432.348226(4)	−48769.69088(1)	−76203.56176(3)	−76203.93978(3)
$6H_{9/2}$	−27431.991389(1)	−27432.348441(1)	−48769.691560(3)	−76203.563414(8)	−76203.941434(8)
$6H_{11/2}$	−27431.918179(1)	−27432.275230(1)	−48769.460173(3)	−76202.998492(8)	−76203.376509(8)
$7S_{1/2}$	−20155.178837(9)	−20155.441184(8)	−35834.16749(3)	−55994.55623(8)	−55994.83398(8)
$7P_{1/2}$	−20155.228201(9)	−20155.490540(9)	−35834.30913(3)	−55994.87547(8)	−55995.15324(8)
$7P_{3/2}$	−20154.536490(9)	−20154.798819(9)	−35832.12247(3)	−55989.53551(7)	−55989.81325(7)
$7D_{3/2}$	−20154.53767(1)	−20154.80000(1)	−35832.12622(4)	−55989.5447(1)	−55989.8224(1)
$7D_{5/2}$	−20154.30714(1)	−20154.56947(1)	−35831.39755(4)	−55987.7656(1)	−55988.0433(1)
$7F_{5/2}$	−20154.30756(1)	−20154.56989(1)	−35831.39888(3)	−55987.76880(8)	−55988.04654(8)
$7F_{7/2}$	−20154.19230(1)	−20154.45463(1)	−35831.03458(3)	−55986.87934(8)	−55987.15708(8)
$7G_{7/2}$	−20154.192521(4)	−20154.454846(4)	−35831.03527(1)	−55986.88104(3)	−55987.15877(3)
$7G_{9/2}$	−20154.123366(4)	−20154.385690(4)	−35830.81670(1)	−55986.34739(3)	−55986.62512(3)
$7H_{9/2}$	−20154.123502(1)	−20154.385826(1)	−35830.817125(4)	−55986.34844(1)	−55986.62617(1)
$7H_{11/2}$	−20154.077399(1)	−20154.339723(1)	−35830.671411(4)	−55985.99268(1)	−55986.27041(1)
$7I_{11/2}$	−20154.0774916(4)	−20154.3398151(4)	−35830.671703(1)	−55985.993396(3)	−55986.271125(3)
$7I_{13/2}$	−20154.0445612(4)	−20154.3068842(4)	−35830.567623(1)	−55985.739290(3)	−55986.017017(3)
$8S_{1/2}$	−15431.207893(6)	−15431.408750(6)	−27435.21414(2)	−42870.04729(5)	−42870.25994(5)
$8P_{1/2}$	−15431.240972(7)	−15431.441824(7)	−27435.30905(2)	−42870.26123(6)	−42870.47389(6)
$8P_{3/2}$	−15430.777582(7)	−15430.978428(7)	−27433.84417(2)	−42866.68393(6)	−42866.89658(6)
$8D_{3/2}$	−15430.778378(9)	−15430.979223(9)	−27433.84670(3)	−42866.69011(7)	−42866.90276(7)
$8D_{5/2}$	−15430.623940(9)	−15430.824783(9)	−27433.35855(3)	−42865.49823(7)	−42865.71087(7)
$8F_{5/2}$	−15430.624223(8)	−15430.825066(8)	−27433.35944(3)	−42865.50041(6)	−42865.71306(6)
$8F_{7/2}$	−15430.547007(8)	−15430.747850(8)	−27433.11539(3)	−42864.90454(6)	−42865.11718(6)
$8G_{7/2}$	−15430.547155(3)	−15430.747997(3)	−27433.11585(1)	−42864.90568(2)	−42865.11832(2)
$8G_{9/2}$	−15430.500826(3)	−15430.701668(3)	−27432.96943(1)	−42864.54818(2)	−42864.76082(2)
$8H_{9/2}$	−15430.500917(1)	−15430.701759(1)	−27432.969714(4)	−42864.54888(1)	−42864.76152(1)
$8H_{11/2}$	−15430.470032(1)	−15430.670874(1)	−27432.872096(4)	−42864.31055(1)	−42864.52319(1)
$8I_{11/2}$	−15430.4700941(5)	−15430.6709357(5)	−27432.872293(2)	−42864.311033(4)	−42864.523669(4)
$8I_{13/2}$	−15430.4480333(5)	−15430.6488745(5)	−27432.802567(2)	−42864.140801(4)	−42864.353436(4)
$8K_{13/2}$	−15430.4480784(2)	−15430.6489196(2)	−27432.8027092(5)	−42864.141149(1)	−42864.353784(1)
$8K_{15/2}$	−15430.4315328(2)	−15430.6323738(2)	−27432.7504152(5)	−42864.013476(1)	−42864.226111(1)
$9S_{1/2}$	−12192.495383(4)	−12192.654084(4)	−21677.00327(1)	−33872.13277(4)	−33872.30079(4)
$9P_{1/2}$	−12192.518621(6)	−12192.677318(6)	−21677.06994(2)	−33872.28306(5)	−33872.45109(5)
$9P_{3/2}$	−12192.193169(6)	−12192.351862(6)	−21676.04112(2)	−33869.77065(5)	−33869.93866(5)
$9D_{3/2}$	−12192.193730(7)	−12192.352422(7)	−21676.04290(2)	−33869.77500(5)	−33869.94302(5)
$9D_{5/2}$	−12192.085263(7)	−12192.243954(7)	−21675.70006(2)	−33868.93791(5)	−33869.10592(5)
$9F_{5/2}$	−12192.085462(6)	−12192.244153(6)	−21675.70069(2)	−33868.93944(5)	−33869.10746(5)
$9F_{7/2}$	−12192.031231(6)	−12192.189922(6)	−21675.52928(2)	−33868.52095(5)	−33868.68896(5)

Hydrogenic levels (QED uncertainties)—Continued

	$^{3}Li^{6}$	$^{3}Li^{7}$	$^{4}Be^{9}$	$^{5}B^{10}$	$^{5}B^{11}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$9G_{7/2}$	−12192.031335(3)	−12192.190025(3)	−21675.529609(8)	−33868.52175(2)	−33868.68976(2)
$9G_{9/2}$	−12191.998797(3)	−12192.157487(3)	−21675.426767(8)	−33868.27066(2)	−33868.43867(2)
$9H_{9/2}$	−12191.998861(1)	−12192.157551(1)	−21675.426970(4)	−33868.271157(9)	−33868.439167(9)
$9H_{11/2}$	−12191.977169(1)	−12192.135859(1)	−21675.358410(4)	−33868.103770(9)	−33868.271780(9)
$9I_{11/2}$	−12191.9772130(5)	−12192.1359027(5)	−21675.358548(2)	−33868.104107(4)	−33868.272116(4)
$9I_{13/2}$	−12191.9617189(5)	−12192.1204084(5)	−21675.309577(2)	−33867.984548(4)	−33868.152556(4)
$9K_{13/2}$	−12191.9617506(2)	−12192.1204401(2)	−21675.3096776(7)	−33867.984792(2)	−33868.152801(2)
$9K_{15/2}$	−12191.9501301(2)	−12192.1088195(2)	−21675.2729498(7)	−33867.895124(2)	−33868.063132(2)
$9L_{15/2}$	−12191.95015423(7)	−12192.10884356(7)	−21675.2730259(2)	−33867.8953094(5)	−33868.0633174(5)
$9L_{17/2}$	−12191.94111610(7)	−12192.09980532(7)	−21675.2444600(2)	−33867.8255675(5)	−33867.9935752(5)
$10S_{1/2}$	−9875.878844(3)	−9876.007390(3)	−17558.23801(1)	−27436.09777(3)	−27436.23387(3)
$10P_{1/2}$	−9875.895786(5)	−9876.024330(5)	−17558.28663(2)	−27436.20735(4)	−27436.34345(4)
$10P_{3/2}$	−9875.658533(5)	−9875.787074(5)	−17557.53663(2)	−27434.37582(4)	−27434.51192(4)
$10D_{3/2}$	−9875.658943(5)	−9875.787483(5)	−17557.53792(2)	−27434.37901(4)	−27434.51510(4)
$10D_{5/2}$	−9875.579871(5)	−9875.708410(5)	−17557.28799(2)	−27433.76877(4)	−27433.90486(4)
$10F_{5/2}$	−9875.580016(5)	−9875.708556(5)	−17557.28845(2)	−27433.76989(4)	−27433.90598(4)
$10F_{7/2}$	−9875.540482(5)	−9875.669021(5)	−17557.16350(2)	−27433.46481(4)	−27433.60090(4)
$10G_{7/2}$	−9875.540557(2)	−9875.669096(2)	−17557.163736(6)	−27433.46539(2)	−27433.60148(2)
$10G_{9/2}$	−9875.516837(2)	−9875.645376(2)	−17557.088765(6)	−27433.28235(2)	−27433.41844(2)
$10H_{9/2}$	−9875.516884(1)	−9875.645423(1)	−17557.088913(3)	−27433.282710(7)	−27433.418798(7)
$10H_{11/2}$	−9875.501071(1)	−9875.629609(1)	−17557.038933(3)	−27433.160686(7)	−27433.296773(7)
$10I_{11/2}$	−9875.5011024(5)	−9875.6296411(5)	−17557.039033(1)	−27433.160932(4)	−27433.297019(4)
$10I_{13/2}$	−9875.4898072(5)	−9875.6183457(5)	−17557.003334(1)	−27433.073772(4)	−27433.209859(4)
$10K_{13/2}$	−9875.4898304(2)	−9875.6183689(2)	−17557.0034067(7)	−27433.073951(2)	−27433.210038(2)
$10K_{15/2}$	−9875.4813590(2)	−9875.6098974(2)	−17556.9766320(7)	−27433.008582(2)	−27433.144669(2)
$10L_{15/2}$	−9875.4813766(1)	−9875.6099150(1)	−17556.9766876(3)	−27433.0087179(8)	−27433.1448046(8)
$10L_{17/2}$	−9875.4747878(1)	−9875.6033261(1)	−17556.9558630(3)	−27432.9578760(8)	−27433.0939624(8)
$10M_{17/2}$	−9875.47480160(3)	−9875.60333993(3)	−17556.9559066(1)	−27432.9579826(3)	−27433.0940690(3)
$10M_{19/2}$	−9875.46953057(3)	−9875.59806884(3)	−17556.9392470(1)	−27432.9173092(3)	−27433.0533954(3)
$11S_{1/2}$	−8161.854122(2)	−8161.960358(2)	−14510.847772(7)	−22674.23383(2)	−22674.34631(2)
$11P_{1/2}$	−8161.866853(4)	−8161.973087(4)	−14510.88430(1)	−22674.31617(3)	−22674.42865(3)
$11P_{3/2}$	−8161.688602(4)	−8161.794834(4)	−14510.32082(1)	−22672.94013(3)	−22673.05261(3)
$11D_{3/2}$	−8161.688910(4)	−8161.795142(4)	−14510.32179(1)	−22672.94253(3)	−22673.05500(3)
$11D_{5/2}$	−8161.629502(4)	−8161.735733(4)	−14510.13402(1)	−22672.48405(3)	−22672.59652(3)
$11F_{5/2}$	−8161.629611(4)	−8161.735842(4)	−14510.13436(1)	−22672.48489(3)	−22672.59736(3)
$11F_{7/2}$	−8161.599909(4)	−8161.706139(4)	−14510.04048(1)	−22672.25568(3)	−22672.36815(3)
$11G_{7/2}$	−8161.599966(2)	−8161.706196(2)	−14510.040662(5)	−22672.25612(1)	−22672.36859(1)
$11G_{9/2}$	−8161.582144(2)	−8161.688375(2)	−14509.984335(5)	−22672.11860(1)	−22672.23107(1)
$11H_{9/2}$	−8161.5821795(8)	−8161.6884099(8)	−14509.984446(3)	−22672.118869(6)	−22672.231338(6)
$11H_{11/2}$	−8161.5702987(8)	−8161.6765290(8)	−14509.946895(3)	−22672.027190(6)	−22672.139659(6)
$11I_{11/2}$	−8161.5703226(4)	−8161.6765530(4)	−14509.946971(1)	−22672.027375(3)	−22672.139844(3)
$11I_{13/2}$	−8161.5618364(4)	−8161.6680666(4)	−14509.920149(1)	−22671.961891(3)	−22672.074359(3)
$11K_{13/2}$	−8161.5618538(2)	−8161.6680840(2)	−14509.9202043(7)	−22671.962025(2)	−22672.074494(2)
$11K_{15/2}$	−8161.5554892(2)	−8161.6617193(2)	−14509.9000882(7)	−22671.912912(2)	−22672.025381(2)
$11L_{15/2}$	−8161.5555024(1)	−8161.6617325(1)	−14509.9001299(4)	−22671.9130144(9)	−22672.0254828(9)
$11L_{17/2}$	−8161.5505521(1)	−8161.6567822(1)	−14509.8844841(4)	−22671.8748161(9)	−22671.9872843(9)
$11M_{17/2}$	−8161.55056252(5)	−8161.65679258(5)	−14509.8845169(2)	−22671.8748962(4)	−22671.9873644(4)
$11M_{19/2}$	−8161.54660231(5)	−8161.65283233(5)	−14509.8720003(2)	−22671.8443377(4)	−22671.9568057(4)
$11N_{19/2}$	−8161.54661069(2)	−8161.65284070(2)	−14509.87202674(6)	−22671.8444023(1)	−22671.9568704(1)
$11N_{21/2}$	−8161.54337052(2)	−8161.64960050(2)	−14509.86178589(6)	−22671.8194000(1)	−22671.9318679(1)
$12S_{1/2}$	−6858.203873(2)	−6858.293141(2)	−12193.077001(6)	−19052.49337(2)	−19052.58788(2)
$12P_{1/2}$	−6858.213680(3)	−6858.302946(3)	−12193.10514(1)	−19052.55680(3)	−19052.65132(3)
$12O_{23/2}$	−6857.96244259(1)	−6858.05170528(1)	−12192.31099123(3)	−19050.61765911(8)	−19050.71216342(8)
$13S_{1/2}$	−5843.661509(1)	−5843.737571(1)	−10389.319265(5)	−16233.95935(1)	−16234.03987(1)
$13P_{1/2}$	−5843.669223(3)	−5843.745284(3)	−10389.341400(8)	−16234.00924(2)	−16234.08977(2)
$13Q_{25/2}$	−5843.470238127(6)	−5843.546296273(6)	−10388.71241875(2)	−16232.47340524(5)	−16232.55392962(5)
$14S_{1/2}$	−5038.655996(1)	−5038.721580(1)	−8958.101592(4)	−13997.56061(1)	−13997.63004(1)
$14P_{1/2}$	−5038.662173(2)	−5038.727756(2)	−8958.119315(7)	−13997.60056(2)	−13997.67000(2)
$14R_{27/2}$	−5038.501907349(4)	−5038.567488097(4)	−8957.61272493(1)	−13996.36357975(3)	−13996.43301149(3)

Hydrogenic levels (QED uncertainties)—Continued

	$^{3}Li^{6}$	$^{3}Li^{7}$	$^{4}Be^{9}$	$^{5}B^{10}$	$^{5}B^{11}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$15S_{1/2}$	−4389.2205962(9)	−4389.2777269(9)	−7803.474456(3)	−12193.363536(8)	−12193.424022(8)
$15P_{1/2}$	−4389.225618(2)	−4389.282748(2)	−7803.488867(6)	−12193.39602(1)	−12193.45651(1)
$15T_{29/2}$	−4389.094649174(3)	−4389.151777292(3)	−7803.074881220(8)	−12192.38516224(2)	−12192.44564500(2)
$16S_{1/2}$	−3857.7067285(8)	−3857.7569408(7)	−6858.501256(2)	−10716.771468(7)	−10716.824629(7)
$16P_{1/2}$	−3857.710867(1)	−3857.761078(1)	−6858.513130(5)	−10716.79823(1)	−10716.85140(1)
$16U_{31/2}$	−3857.602470343(2)	−3857.652680603(2)	−6858.170496440(5)	−10715.96160283(1)	−10716.01476151(1)
$17S_{1/2}$	−3417.2020830(6)	−3417.2465616(6)	−6075.333900(2)	−9493.016200(6)	−9493.063290(6)
$17P_{1/2}$	−3417.205533(1)	−3417.250011(1)	−6075.343800(4)	−9493.03852(1)	−9493.08561(1)
$17V_{33/2}$	−3417.114808475(1)	−3417.159285383(1)	−6075.057025416(4)	−9492.338281097(9)	−9492.38536975(1)
$18S_{1/2}$	−3048.0556863(5)	−3048.0953601(5)	−5419.034516(2)	−8467.503452(5)	−8467.545455(5)
$18P_{1/2}$	−3048.058593(1)	−3048.098266(1)	−5419.042856(3)	−8467.522251(9)	−8467.564256(9)
$18W_{35/2}$	−3047.981899494(1)	−3048.0215717977(9)	−5418.800433979(3)	−8466.930315074(7)	−8466.972316988(7)
$19S_{1/2}$	−2735.6476993(5)	−2735.6833068(4)	−4863.610258(1)	−7599.616668(4)	−7599.654367(4)
$19P_{1/2}$	−2735.6501708(9)	−2735.6857779(9)	−4863.617350(3)	−7599.632654(7)	−7599.670353(7)
$19X_{37/2}$	−2735.5847590506(7)	−2735.6203652183(7)	−4863.410588221(2)	−7599.127793238(5)	−7599.165490247(5)
$20S_{1/2}$	−2468.9192841(4)	−2468.9514198(4)	−4389.399487(1)	−6858.632566(4)	−6858.666589(4)
$20P_{1/2}$	−2468.9214032(8)	−2468.9535385(8)	−4389.405567(3)	−6858.646272(6)	−6858.680296(6)
$20Y_{39/2}$	−2468.8651651729(6)	−2468.8972997393(6)	−4389.227803439(2)	−6858.212217113(4)	−6858.246238664(4)

Hydrogenic levels (QED uncertainties)

	$^{6}C^{12}$	$^{7}N^{14}$	$^{8}O^{16}$	$^{9}F^{19}$	$^{10}Ne^{20}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$1S_{1/2}$	−3952061.52(5)	−5380089.3(1)	−7028393.6(3)	−8897240.3(6)	−10986873.(1)
$2S_{1/2}$	−988157.335(6)	−1345282.50(2)	−1757538.21(4)	−2225009.09(7)	−2747776.5(1)
$2P_{1/2}$	−988183.474(4)	−1345328.05(1)	−1757611.75(2)	−2225121.13(5)	−2747939.56(9)
$2P_{3/2}$	−987708.508(2)	−1344447.735(6)	−1756109.23(1)	−2222713.04(3)	−2744266.95(5)
$3S_{1/2}$	−439132.090(2)	−597812.178(5)	−780971.88(1)	−988641.68(2)	−1220849.61(4)
$3P_{1/2}$	−439139.896(1)	−597825.786(4)	−780993.858(8)	−988675.18(2)	−1220898.39(3)
$3P_{3/2}$	−438999.1607(7)	−597564.941(2)	−780548.640(4)	−987961.614(8)	−1219810.11(2)
$3D_{3/2}$	−438999.39532(4)	−597565.3771(1)	−780549.3878(2)	−987962.8182(4)	−1219811.9517(8)
$3D_{5/2}$	−438952.52424(4)	−597478.53167(9)	−780401.2121(2)	−987725.4309(4)	−1219450.0722(8)
$4S_{1/2}$	−246990.6513(8)	−336230.030(2)	−439229.394(5)	−556002.859(9)	−686562.74(2)
$4P_{1/2}$	−246993.9543(6)	−336235.789(2)	−439238.697(4)	−556017.040(7)	−686583.39(1)
$4P_{3/2}$	−246934.5839(3)	−336125.7510(8)	−439050.884(2)	−555716.032(4)	−686124.320(7)
$4D_{3/2}$	−246934.6841(7)	−336125.937(1)	−439051.204(2)	−555716.547(4)	−686125.109(6)
$4D_{5/2}$	−246914.9096(7)	−336089.298(1)	−438988.688(2)	−555616.390(4)	−685972.424(6)
$4F_{5/2}$	−246914.9453(3)	−336089.3637(5)	−438988.8008(9)	−555616.571(1)	−685972.699(2)
$4F_{7/2}$	−246905.0602(3)	−336071.0492(5)	−438957.5550(9)	−555566.517(1)	−685896.403(2)
$5S_{1/2}$	−158064.3833(5)	−215169.319(1)	−281076.193(2)	−355792.666(5)	−439325.051(9)
$5P_{1/2}$	−158066.0770(3)	−215172.2718(8)	−281080.964(2)	−355799.938(4)	−439335.642(7)
$5P_{3/2}$	−158035.6807(2)	−215115.9356(4)	−280984.812(1)	−355645.837(2)	−439100.627(4)
$5D_{3/2}$	−158035.7324(5)	−215116.0319(9)	−280984.976(2)	−355646.103(2)	−439101.034(4)
$5D_{5/2}$	−158025.6078(5)	−215097.2721(9)	−280952.968(2)	355594.822(2)	−439022.858(4)
$5F_{5/2}$	−158025.6262(3)	−215097.3062(5)	−280953.0261(9)	−355594.915(1)	−439023.000(2)
$5F_{7/2}$	−158020.5649(3)	−215087.9289(5)	−280937.0276(9)	−355569.287(1)	−438983.934(2)
$5G_{7/2}$	−158020.57448(6)	−215087.9466(1)	−280937.0579(2)	−355569.3351(3)	−438984.0084(5)
$5G_{9/2}$	−158017.53798(6)	−215082.3209(1)	−280927.4603(2)	−355553.9609(3)	−438960.5745(5)
$6S_{1/2}$	−109762.0354(3)	−149414.0401(7)	−195176.243(1)	−247053.264(3)	−305048.696(5)
$6P_{1/2}$	−109763.0163(2)	−149415.7504(5)	−195179.006(1)	−247057.476(2)	−305054.829(4)

Hydrogenic levels (QED uncertainties)—Continued

	$^{6}C^{12}$	$^{7}N^{14}$	$^{8}O^{16}$	$^{9}F^{19}$	$^{10}Ne^{20}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$6P_{3/2}$	−109745.4265(2)	−149383.1501(4)	−195123.3656(7)	−246968.305(1)	−304918.840(2)
$6D_{3/2}$	−109745.4567(3)	−149383.2063(6)	−195123.462(1)	−246968.460(2)	−304919.077(2)
$6D_{5/2}$	−109739.5976(3)	−149372.3500(6)	−195104.939(1)	−246938.784(2)	−304873.837(2)
$6F_{5/2}$	−109739.6083(2)	−149372.3698(4)	−195104.9725(7)	−246938.838(1)	−304873.920(2)
$6F_{7/2}$	−109736.6793(2)	−149366.9431(4)	−195095.7139(7)	−246924.006(1)	−304851.312(2)
$6G_{7/2}$	−109736.68484(7)	−149366.9534(1)	−195095.7315(2)	−246924.0344(4)	−304851.3548(5)
$6G_{9/2}$	−109734.92758(7)	−149363.6977(1)	−195090.1772(2)	−246915.1370(4)	−304837.7930(5)
$6H_{9/2}$	−109734.93102(2)	−149363.70412(3)	−195090.18810(5)	−246915.15440(9)	−304837.8195(1)
$6H_{11/2}$	−109733.75956(2)	−149361.53380(3)	−195086.48553(5)	−246909.22341(9)	−304828.7795(1)
$7S_{1/2}$	−80638.7679(2)	−109768.5145(4)	−143386.1286(9)	−181494.617(2)	−224096.177(3)
$7P_{1/2}$	−80639.3859(2)	−109769.5920(4)	−143387.8694(8)	−181497.271(2)	−224100.041(3)
$7P_{3/2}$	−80628.3093(2)	−109749.0633(3)	−143352.8326(6)	−181441.120(1)	−224014.411(2)
$7D_{3/2}$	−80628.3284(2)	−109749.0989(4)	−143352.8936(7)	−181441.218(1)	−224014.561(2)
$7D_{5/2}$	−80624.6388(2)	−109742.2624(4)	−143341.2290(7)	−181422.530(1)	−223986.073(2)
$7F_{5/2}$	−80624.6455(2)	−109742.2749(3)	−143341.2503(5)	−181422.5646(9)	−223986.125(1)
$7F_{7/2}$	−80622.8010(2)	−109738.8574(3)	−143335.4199(5)	−181413.2245(9)	−223971.888(1)
$7G_{7/2}$	−80622.80451(6)	−109738.8640(1)	−143335.4310(2)	−181413.2423(3)	−223971.9148(5)
$7G_{9/2}$	−80621.69790(6)	−109736.8137(1)	−143331.9332(2)	−181407.6392(3)	−223963.3743(5)
$7H_{9/2}$	−80621.70007(2)	−109736.81774(4)	−143331.94005(7)	−181407.6502(1)	−223963.3911(2)
$7H_{11/2}$	−80620.96235(2)	−109735.45099(4)	−143329.60836(7)	−181403.9151(1)	−223957.6981(2)
$7I_{11/2}$	−80620.963833(6)	−109735.45373(1)	−143329.61304(2)	−181403.92260(3)	−223957.70948(5)
$7I_{13/2}$	−80620.436907(6)	−109734.47752(1)	−143327.94762(2)	−181401.25487(3)	−223953.64334(5)
$8S_{1/2}$	−61737.4246(1)	−84038.4879(3)	−109774.8226(6)	−138948.499(1)	−171560.936(2)
$8P_{1/2}$	−61737.8388(1)	−84039.2100(3)	−109775.9891(6)	−138950.277(1)	−171563.525(2)
$8P_{3/2}$	−61730.4185(1)	−84025.4578(2)	−109752.5183(4)	−138912.6627(8)	−171506.163(1)
$8D_{3/2}$	−61730.4314(2)	−84025.4817(3)	−109752.5594(5)	−138912.7289(8)	−171506.265(1)
$8D_{5/2}$	−61727.9596(2)	−84020.9018(3)	−109744.7451(5)	−138900.2097(8)	−171487.180(1)
$8F_{5/2}$	−61727.9641(1)	−84020.9102(2)	−109744.7594(4)	−138900.2327(7)	−171487.215(1)
$8F_{7/2}$	−61726.7284(1)	−84018.6208(2)	−109740.8534(4)	−138893.9755(7)	−171477.677(1)
$8G_{7/2}$	−61726.73079(5)	−84018.62520(9)	−109740.8609(2)	−138893.9875(3)	−171477.6953(4)
$8G_{9/2}$	−61725.98944(5)	−84017.25169(9)	−109738.5176(2)	−138890.2338(3)	−171471.9738(4)
$8H_{9/2}$	−61725.99090(2)	−84017.25439(4)	−109738.52225(7)	−138890.2412(1)	−171471.9851(2)
$8H_{11/2}$	−61725.49669(2)	−84016.33877(4)	−109736.96019(7)	−138887.7390(1)	−171468.1712(2)
$8I_{11/2}$	−61725.497679(8)	−84016.34061(1)	−109736.96333(3)	−138887.74402(4)	−171468.17883(6)
$8I_{13/2}$	−61725.144678(8)	−84015.68662(1)	−109735.84762(3)	−138885.95682(4)	−171465.45479(6)
$8K_{13/2}$	−61725.145399(2)	−84015.687956(5)	−109735.849895(8)	−138885.96047(1)	−171465.46035(2)
$8K_{15/2}$	−61724.880653(2)	−84015.197472(5)	−109735.013137(8)	−138884.62012(1)	−171463.41742(2)
$9S_{1/2}$	−48779.15353(8)	−66398.8607(2)	−86732.3802(4)	−109781.2008(8)	−135546.277(2)
$9P_{1/2}$	−48779.4445(1)	−66399.3680(2)	−86733.1997(4)	−109782.4501(8)	−135548.096(1)
$9P_{3/2}$	−48774.2331(1)	−66389.7096(2)	−86716.7159(3)	−109756.0335(6)	−135507.811(1)
$9D_{3/2}$	−48774.2421(1)	−66389.7265(2)	−86716.7449(3)	−109756.0801(6)	−135507.8828(8)
$9D_{5/2}$	−48772.5061(1)	−66386.5099(2)	−86711.2567(3)	−109747.2877(6)	−135494.4792(8)
$9F_{5/2}$	−48772.5093(1)	−66386.5158(2)	−86711.2668(3)	−109747.3038(5)	−135494.5038(8)
$9F_{7/2}$	−48771.6415(1)	−66384.9079(2)	−86708.5236(3)	−109742.9093(5)	−135487.8052(8)
$9G_{7/2}$	−48771.64313(4)	−66384.91098(8)	−86708.5288(1)	−109742.9177(2)	−135487.8180(3)
$9G_{9/2}$	−48771.12246(4)	−66383.94633(8)	−86706.8831(1)	−109740.2813(2)	−135483.7996(3)
$9H_{9/2}$	−48771.12348(2)	−66383.94823(3)	−86706.88629(6)	−109740.28653(9)	−135483.8075(1)
$9H_{11/2}$	−48770.77638(2)	−66383.30516(3)	−86705.78921(6)	−109738.52914(9)	−135481.1288(1)
$9I_{11/2}$	−48770.777080(8)	−66383.30645(2)	−86705.79141(3)	−109738.53267(4)	−135481.13423(6)
$9I_{13/2}$	−48770.529155(8)	−66382.84713(2)	−86705.00781(3)	−109737.27746(4)	−135479.22104(6)
$9K_{13/2}$	−48770.529663(3)	−66382.848067(6)	−86705.00941(1)	−109737.28002(2)	−135479.22495(3)
$9K_{15/2}$	−48770.343722(3)	−66382.503583(6)	−86704.42173(1)	−109736.33865(2)	−135477.79011(3)
$9L_{15/2}$	−48770.344107(1)	−66382.504296(2)	−86704.422945(3)	−109736.340595(6)	−135477.793084(9)
$9L_{17/2}$	−48770.199488(1)	−66382.236368(2)	−86703.965864(3)	−109735.608429(6)	−135476.677133(9)
$10S_{1/2}$	−39510.42869(6)	−53781.8039(2)	−70251.0514(3)	−88919.2798(6)	−109787.151(1)
$10P_{1/2}$	−39510.64082(9)	−53782.1738(2)	−70251.6489(3)	−88920.1907(6)	−109788.478(1)
$10P_{3/2}$	−39506.84178(8)	−53775.1330(2)	−70239.6327(3)	−88900.9337(5)	−109759.1117(8)
$10D_{3/2}$	−39506.84840(8)	−53775.1454(2)	−70239.6538(3)	−88900.9678(4)	−109759.1639(6)
$10D_{5/2}$	−39505.58287(8)	−53772.8005(2)	−70235.6530(3)	−88894.5582(4)	−109749.3928(6)
$10F_{5/2}$	−39505.58520(8)	−53772.8048(1)	−70235.6603(2)	−88894.5700(4)	−109749.4108(6)

Hydrogenic levels (QED uncertainties) – Continued

	$^{6}C^{12}$	$^{7}N^{14}$	$^{8}O^{16}$	$^{9}F^{19}$	$^{10}Ne^{20}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$10F_{7/2}$	39504.95253(8)	−53771.6326(1)	−70233.6605(2)	88891.3663(4)	−109744.5275(6)
$10G_{7/2}$	−39504.95375(3)	−53771.63488(6)	−70233.6643(1)	−88891.3725(2)	−109744.5368(3)
$10G_{9/2}$	−39504.57417(3)	−53770.93165(6)	−70232.4646(1)	−88889.4506(2)	−109741.6074(3)
$10H_{9/2}$	−39504.57492(2)	−53770.93304(3)	−70232.46696(5)	−88889.45439(8)	−109741.6132(1)
$10H_{11/2}$	−39504.32188(2)	−53770.46424(3)	−70231.66718(5)	−88888.17324(8)	−109739.6605(1)
$10I_{11/2}$	−39504.322394(8)	−53770.46518(1)	−70231.66879(2)	−88888.17582(4)	−109739.66441(6)
$10I_{13/2}$	−39504.141657(8)	−53770.13033(1)	−70231.09755(2)	−88887.26077(4)	−109738.26968(6)
$10K_{13/2}$	−39504.142027(4)	−53770.131020(7)	−70231.09872(1)	−88887.26264(2)	−109738.27254(3)
$10K_{15/2}$	−39504.006476(4)	−53769.879890(7)	−70230.67029(1)	−88886.57637(2)	−109737.22653(3)
$10L_{15/2}$	−39504.006757(2)	−53769.880411(3)	−70230.671179(5)	−88886.577796(8)	−109737.22870(1)
$10L_{17/2}$	−39503.901330(2)	−53769.685090(3)	−70230.337966(5)	−88886.044044(8)	−109736.41517(1)
$10M_{17/2}$	−39503.9015505(5)	−53769.685500(1)	−70230.338664(2)	−88886.045162(3)	−109736.416869(4)
$10M_{19/2}$	−39503.8172090(5)	−53769.529245(1)	−70230.072097(2)	−88885.618167(3)	−109735.766055(4)
$11S_{1/2}$	−32652.77461(5)	−44446.8950(1)	−58057.2213(2)	−73484.6028(5)	−90729.5107(9)
$11P_{1/2}$	−32652.93401(7)	−44447.1729(1)	−58057.6703(3)	−73485.2872(5)	−90730.5074(8)
$11P_{3/2}$	−32650.07977(6)	−44441.8832(1)	−58048.6425(2)	−73470.8197(4)	−90708.4451(6)
$11D_{3/2}$	−32650.08476(6)	−44441.8925(1)	−58048.6584(2)	−73470.8453(3)	−90708.4843(5)
$11D_{5/2}$	−32649.13396(6)	−44440.1307(1)	−58045.6526(2)	−73466.0297(3)	−90701.1433(5)
$11F_{5/2}$	−32649.13570(6)	−44440.1340(1)	−58045.6581(2)	−73466.0386(3)	−90701.1568(5)
$11F_{7/2}$	−32648.66038(6)	−44439.2533(1)	−58044.1556(2)	−73463.6317(3)	−90697.4880(5)
$11G_{7/2}$	−32648.66129(3)	−44439.25500(5)	−58044.15847(8)	−73463.6363(1)	−90697.4950(2)
$11G_{9/2}$	−32648.37611(3)	−44438.72665(5)	−58043.25709(8)	−73462.1923(1)	−90695.2941(2)
$11H_{9/2}$	−32648.37667(1)	−44438.72769(2)	−58043.25887(4)	−73462.19519(7)	−90695.2984(1)
$11H_{11/2}$	−32648.18656(1)	−44438.37548(2)	−58042.65798(4)	−73461.23265(7)	−90693.8313(1)
$11I_{11/2}$	−32648.186945(7)	−44438.37619(1)	−58042.65919(2)	−73461.23459(3)	−90693.83427(5)
$11I_{13/2}$	−32648.051154(7)	−44438.12461(1)	−58042.23000(2)	−73460.54709(3)	−90692.78639(5)
$11K_{13/2}$	−32648.051432(4)	−44438.125126(6)	−58042.23088(1)	−73460.54850(2)	−90692.78854(3)
$11K_{15/2}$	−32647.949591(4)	−44437.936448(6)	−58041.90900(1)	−73460.03290(2)	−90692.00266(3)
$11L_{15/2}$	−32647.949802(2)	−44437.936839(3)	−58041.909668(6)	−73460.033967(9)	−90692.00429(1)
$11L_{17/2}$	−32647.870593(2)	−44437.790092(3)	−58041.659319(6)	−73459.632949(9)	−90691.39306(1)
$11M_{17/2}$	−32647.8707586(8)	−44437.790399(2)	−58041.659844(3)	−73459.633789(4)	−90691.394344(6)
$11M_{19/2}$	−32647.8073916(8)	−44437.673002(2)	−58041.459567(3)	−73459.312980(4)	−90690.905375(6)
$11N_{19/2}$	−32647.8075256(3)	−44437.6732507(5)	−58041.4599906(9)	−73459.313659(1)	−90690.906408(2)
$11N_{21/2}$	−32647.7556800(3)	−44437.5771994(5)	−58041.2961295(9)	−73459.051182(1)	−90690.506349(2)
$12S_{1/2}$	−27437.06525(4)	−37347.11491(9)	−48783.1276(2)	−61745.7693(4)	−76235.3819(7)
$12P_{1/2}$	−27437.18803(6)	−37347.3290(1)	−48783.4735(2)	−61746.2965(4)	−76236.1497(7)
$12O_{23/2}$	−27433.1662476(2)	−37339.8764856(3)	−48770.7565127(5)	−61725.9204757(8)	−76205.083304(1)
$13S_{1/2}$	−23378.08278(3)	−31821.93887(7)	−41565.9111(1)	−52610.5321(3)	−64956.0538(5)
$13P_{1/2}$	−23378.17936(5)	−31822.10728(9)	−41566.1831(2)	−52610.9468(3)	−64956.6577(5)
$13Q_{25/2}$	−23374.9940580(1)	−31816.2048088(2)	−41556.1112085(3)	−52594.8089207(5)	−64932.0532344(8)
$14S_{1/2}$	−20157.44964(2)	−27437.96474(6)	−35839.4141(1)	−45362.2312(2)	−56006.6037(4)
$14P_{1/2}$	−20157.52697(4)	−27438.09958(7)	−35839.6319(1)	−45362.5633(2)	−56007.0873(4)
$14R_{27/2}$	−20154.96149663(6)	−27433.3456892(1)	−35831.5199962(2)	−45349.5658101(3)	−55987.2708912(5)
$15S_{1/2}$	−17559.23899(2)	−23901.25721(5)	−31219.66960(9)	−39514.8340(2)	−48786.8915(3)
$15P_{1/2}$	−17559.30187(3)	−23901.36685(6)	−31219.8467(1)	−39515.1040(2)	−48787.2847(3)
$15T_{29/2}$	−17557.20537293(4)	−23897.48199826(8)	−31213.2177165(1)	−39504.4826613(2)	−48771.0910775(3)
$16S_{1/2}$	−15432.81690(1)	−21006.76383(4)	−27438.82004(8)	−34729.2847(2)	−42878.2645(3)
$16P_{1/2}$	−15432.86871(3)	−21006.85418(5)	−27438.96599(9)	−34729.5072(2)	−42878.5884(3)
$16U_{31/2}$	−15431.13355625(3)	−21003.63891676(5)	−27433.47955940(9)	−34720.7165724(2)	−42865.1860682(2)
$17S_{1/2}$	−13670.50715(1)	−18607.90985(3)	−24305.39793(7)	−30763.2244(1)	−37981.4700(2)
$17P_{1/2}$	−13670.55035(2)	−18607.98518(4)	−24305.51962(8)	−30763.4099(1)	−37981.7401(2)
$17V_{33/2}$	−13669.09808394(2)	−18605.29411353(4)	−24300.92767921(7)	−30756.0524880(1)	−37970.5229064(2)
$18S_{1/2}$	−12193.68706(1)	−16597.67031(3)	−21679.5998(3)	27439.6915(1)	33878.0067(2)
$18P_{1/2}$	−12193.72346(2)	−16597.73377(4)	−21679.70229(7)	−27439.8478(1)	−33878.2343(2)
$18W_{35/2}$	−12192.49579890(1)	−16595.45891429(3)	−21675.82056634(5)	−27433.62835710(9)	−33868.7520414(1)

Hydrogenic levels (QED uncertainties)—Continued

	$^{6}C^{12}$	$^{7}N^{14}$	$^{8}O^{16}$	$^{9}F^{19}$	$^{10}Ne^{20}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}
$19S_{1/2}$	−10943.863876(9)	−14896.42249(2)	−19457.41736(5)	−24627.03467(9)	−30405.3205(2)
$19P_{1/2}$	−10943.89482(2)	−14896.47644(3)	−19457.50452(6)	−24627.1675(1)	−30405.5140(2)
$19X_{37/2}$	−10942.84775663(1)	−14894.53623009(2)	−19454.19382630(4)	−24621.86303811(7)	−30397.4267085(1)
$20S_{1/2}$	−9876.792504(8)	−13443.93841(2)	−17560.17752(4)	−22225.67153(8)	−27440.4549(1)
$20P_{1/2}$	−9876.81904(1)	−13443.98468(3)	−17560.25225(5)	−22225.78545(9)	−27440.6207(2)
$20Y_{39/2}$	−9875.918822427(9)	−13442.31658013(2)	−17557.40588933(3)	−22221.22492233(6)	−27433.66774377(9)

Hydrogenic levels (QED uncertainties)

	$^{11}Na^{23}$	$^{12}Mg^{24}$	$^{13}Al^{27}$	$^{14}Si^{28}$	$^{15}P^{31}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	10^{3} cm^{-1}
$1S_{1/2}$	−13297672.(2)	−15829938.(3)	−18584123.(5)	−21560601.(8)	−24759.90(1)
$2S_{1/2}$	−3325958.0(2)	−3959654.1(4)	−4649005.1(7)	−5394134.(1)	−6195.206(2)
$2P_{1/2}$	−3326186.9(2)	−3959965.6(3)	−4649418.6(4)	−5394671.5(7)	−6195.891(1)
$2P_{3/2}$	−3320806.08(9)	−3952339.0(1)	−4638905.4(2)	−5380518.1(4)	−6177.2212(6)
$3S_{1/2}$	1477638.86(7)	1759043.9(1)	−2065115.9(2)	−2395897.5(3)	−2751.4482(5)
$3P_{1/2}$	−1477707.34(5)	−1759137.11(9)	−2065239.7(1)	−2396058.4(2)	−2751.6535(4)
$3P_{3/2}$	−1476112.84(3)	−1756877.06(5)	−2062124.16(8)	−2391864.0(1)	−2746.1205(2)
$3D_{3/2}$	−1476115.555(1)	−1756880.927(2)	−2062129.511(4)	−2391871.195(6)	−2746.130108(9)
$3D_{5/2}$	−1475585.623(1)	−1756130.227(2)	−2061095.283(4)	−2390479.758(6)	−2744.295965(9)
$4S_{1/2}$	−830929.45(3)	−989118.04(5)	−1161152.64(8)	−1347052.0(1)	−1546.8439(2)
$4P_{1/2}$	−830958.44(2)	−989157.52(4)	−1161205.06(7)	−1347120.1(1)	−1546.9308(2)
$4P_{3/2}$	−830285.85(1)	−988204.22(2)	−1159890.94(3)	−1345350.96(5)	−1544.59721(8)
$4D_{3/2}$	−830287.009(8)	−988205.87(1)	−1159893.22(2)	−1345354.05(2)	−1544.60131(3)
$4D_{5/2}$	−830063.414(8)	−987889.12(1)	−1159456.83(2)	−1344766.91(2)	−1543.82734(3)
$4F_{5/2}$	−830063.818(3)	−987889.689(5)	−1159457.613(6)	−1344767.971(9)	−1543.82873(1)
$4F_{7/2}$	−829952.103(3)	−987731.452(5)	−1159239.640(6)	1344474.752(9)	−1543.44227(1)
$5S_{1/2}$	−531684.66(2)	−632879.17(3)	−742921.88(4)	−861822.44(7)	−989.5961(1)
$5P_{1/2}$	−531699.52(1)	−632899.42(2)	−742948.77(3)	−861857.38(5)	−989.64072(8)
$5P_{3/2}$	−531355.212(7)	−632411.42(1)	−742276.08(2)	−860951.80(3)	−988.44623(4)
$5D_{3/2}$	−531355.810(5)	−632412.268(8)	−742277.26(1)	−860953.40(1)	−988.44835(2)
$5D_{5/2}$	−531241.328(5)	−632250.088(8)	−742053.82(1)	−860652.77(1)	−988.05207(2)
$5F_{5/2}$	−531241.536(3)	−632250.382(5)	−742054.224(6)	−860653.320(9)	−988.05278(1)
$5F_{7/2}$	−531184.333(3)	632169.357(5)	−741942.610(6)	−860503.173(9)	−987.85489(1)
$5G_{7/2}$	−531184.4415(7)	−632169.510(1)	−741942.821(1)	−860503.457(2)	−987.855265(2)
$5G_{9/2}$	−531150.1301(7)	−632120.913(1)	−741875.880(1)	−860413.412(2)	−987.736594(2)
$6S_{1/2}$	−369169.500(9)	−439420.03(2)	−515808.42(2)	−598340.20(4)	−687.02469(6)
$6P_{1/2}$	−369178.111(8)	−439431.75(1)	−515823.99(2)	−598360.43(3)	−687.05050(5)
$6P_{3/2}$	−368978.880(4)	−439149.387(7)	−515434.77(1)	−597836.48(2)	−686.35941(3)
$6D_{3/2}$	−368979.230(4)	−439149.885(5)	−515435.462(7)	−597837.408(9)	−686.36065(1)
$6D_{5/2}$	−368912.980(4)	−439056.033(5)	−515306.159(7)	−597663.441(9)	−686.13132(1)
$6F_{5/2}$	−368913.101(3)	−439056.204(4)	−515306.395(5)	−597663.758(7)	−686.131742(9)
$6F_{7/2}$	−368879.997(3)	−439009.313(4)	−515241.801(5)	−597576.864(7)	−686.017215(9)
$6G_{7/2}$	−368880.0595(8)	−439009.402(1)	−515241.924(2)	−597577.029(2)	−686.017432(3)
$6G_{9/2}$	−368860.2025(8)	−438981.277(1)	−515203.182(2)	−597524.916(2)	−685.948751(3)
$6H_{9/2}$	−368860.2414(2)	−438981.3319(3)	−515203.2579(4)	−597525.0174(5)	−685.9488856(7)
$6H_{11/2}$	−368847.0054(2)	−438962.5851(3)	−515177.4356(4)	−597490.2837(5)	−685.9031111(7)
$7S_{1/2}$	−271195.430(6)	−322795.02(1)	−378900.33(2)	−439514.76(2)	−504.64444(4)
$7P_{1/2}$	−271200.855(5)	−322802.410(8)	−378910.14(1)	−439527.50(2)	−504.66070(3)
$7P_{3/2}$	−271075.404(3)	−322624.614(5)	−378665.070(7)	−439197.60(1)	−504.22558(2)
$7D_{3/2}$	−271075.626(2)	−322624.930(3)	−378665.508(5)	−439198.194(6)	−504.226359(8)
$7D_{5/2}$	−271033.907(2)	−322565.829(3)	−378584.084(5)	−439088.645(6)	−504.081951(8)
$7F_{5/2}$	−271033.983(2)	−322565.937(3)	−378584.232(4)	−439088.845(5)	−504.082215(7)
$7F_{7/2}$	−271013.136(2)	−322536.408(3)	−378543.555(4)	−439034.124(5)	−504.010092(7)

Hydrogenic levels (QED uncertainties)—Continued

	$^{11}Na^{23}$	$^{12}Mg^{24}$	$^{13}Al^{27}$	$^{14}Si^{28}$	$^{15}P^{31}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	$10^3\ cm^{-1}$
$7G_{7/2}$	−271013.1759(7)	−322536.464(1)	−378543.632(1)	−439034.228(2)	−504.010229(2)
$7G_{9/2}$	−271000.6709(7)	−322518.752(1)	−378519.235(1)	−439001.410(2)	−503.966977(2)
$7H_{9/2}$	−271000.6954(2)	−322518.7869(3)	−378519.2827(5)	−439001.4739(6)	−503.9670616(8)
$7H_{11/2}$	−270992.3600(2)	−322506.9809(3)	−378503.0208(5)	−438979.5997(6)	−503.9382341(8)
$7I_{11/2}$	−270992.37669(7)	−322507.0046(1)	−378503.0533(1)	−438979.6435(2)	−503.9382918(2)
$7I_{13/2}$	−270986.42330(7)	−322498.5726(1)	−378491.4391(1)	−438964.0213(2)	−503.9177040(2)
$8S_{1/2}$	−207615.375(4)	−247113.516(6)	−290059.12(1)	−336454.38(2)	−386.30359(2)
$8P_{1/2}$	−207619.010(3)	−247118.466(6)	−290065.688(9)	−336462.92(1)	−386.31448(2)
$8P_{3/2}$	−207534.975(2)	−246999.368(3)	−289901.528(5)	−336241.942(8)	−386.02303(1)
$8D_{3/2}$	−207535.125(2)	−246999.581(2)	−289901.822(3)	−336242.340(4)	−386.023555(6)
$8D_{5/2}$	−207507.177(2)	−246959.989(2)	−289847.277(3)	−336168.954(4)	−385.926818(6)
$8F_{5/2}$	−207507.228(1)	−246960.062(2)	−289847.376(3)	−336169.088(4)	−385.926995(5)
$8F_{7/2}$	−207493.262(1)	−246940.280(2)	−289820.126(3)	−336132.430(4)	−385.878679(5)
$8G_{7/2}$	−207493.2886(6)	−246940.3173(8)	−289820.178(1)	−336132.500(1)	−385.878771(2)
$8G_{9/2}$	−207484.9112(6)	−246928.4516(8)	−289803.833(1)	−336110.513(1)	−385.849795(2)
$8H_{9/2}$	−207484.9277(2)	−246928.4749(3)	−289803.8652(5)	−336110.5565(6)	−385.8498515(8)
$8H_{11/2}$	−207479.3435(2)	−246920.5657(3)	−289792.9708(5)	−336095.9022(6)	−385.8305388(8)
$8I_{11/2}$	−207479.35469(9)	−246920.5815(1)	−289792.9927(2)	−336095.9316(2)	−385.8305776(3)
$8I_{13/2}$	−207475.36631(9)	−246914.9326(1)	−289785.2118(2)	−336085.4656(2)	−385.8167849(3)
$8K_{13/2}$	−207475.37445(3)	−246914.94416(4)	−289785.22771(5)	−336085.48695(7)	−385.8168130(1)
$8K_{15/2}$	−207472.38332(3)	−246910.70776(4)	−289779.39252(5)	−336077.63815(7)	−385.8064695(1)
$9S_{1/2}$	−164029.983(3)	−195233.454(5)	−229159.430(7)	−265809.41(1)	−305.18649(2)
$9P_{1/2}$	−164032.537(2)	−195236.931(4)	−229164.048(6)	−265815.41(1)	−305.19415(1)
$9P_{3/2}$	−163973.521(2)	−195153.292(2)	−229048.764(4)	−265660.224(6)	−304.989476(8)
$9D_{3/2}$	−163973.626(1)	−195153.442(2)	−229048.971(2)	−265660.504(3)	−304.989847(4)
$9D_{5/2}$	−163953.997(1)	−195125.636(2)	−229010.663(2)	−265608.964(3)	−304.921909(4)
$9F_{5/2}$	−163954.033(1)	−195125.687(2)	−229010.734(2)	−265609.059(3)	−304.922033(4)
$9F_{7/2}$	−163944.225(1)	−195111.794(2)	−228991.595(2)	−265583.313(3)	−304.888100(4)
$9G_{7/2}$	−163944.2435(5)	−195111.8203(6)	−228991.6314(9)	−265583.362(1)	−304.888164(2)
$9G_{9/2}$	−163938.3598(5)	−195103.4866(6)	−228980.1520(9)	−265567.920(1)	−304.867813(2)
$9H_{9/2}$	−163938.3714(2)	−195103.5030(3)	−228980.1745(4)	−265567.9507(5)	−304.8678534(7)
$9H_{11/2}$	−163934.4494(2)	−195097.9480(3)	−228972.5230(4)	−265557.6584(5)	−304.8542893(7)
$9I_{11/2}$	−163934.45728(9)	−195097.9592(1)	−228972.5383(2)	−265557.6791(2)	−304.8543165(3)
$9I_{13/2}$	−163931.65608(9)	−195093.9917(1)	−228967.0735(2)	−265550.3283(2)	−304.8446293(3)
$9K_{13/2}$	−163931.66181(4)	−195093.99986(5)	−228967.08469(8)	−265550.3433(1)	−304.8446491(1)
$9K_{15/2}$	−163929.56102(4)	−195091.02445(5)	−228962.98637(8)	−265544.8308(1)	−304.8373843(1)
$9L_{15/2}$	−163929.56537(1)	−195091.03061(2)	−228962.99485(2)	−265544.84216(3)	−304.83739934(4)
$9L_{17/2}$	−163927.93148(1)	−195088.71650(2)	−228959.80743(2)	−265540.55485(3)	−304.83174936(4)
$10S_{1/2}$	−132856.466(2)	−158128.004(3)	−185603.836(5)	−215285.007(8)	−247.17386(1)
$10P_{1/2}$	−132858.328(2)	−158130.540(3)	−185607.203(5)	−215289.381(7)	−247.17944(1)
$10P_{3/2}$	−132815.307(1)	−158069.571(2)	−185523.168(3)	−215176.264(4)	−247.030249(6)
$10D_{3/2}$	−132815.3841(9)	−158069.680(1)	−185523.319(2)	−215176.469(2)	−247.030521(3)
$10D_{5/2}$	−132801.0754(9)	−158049.411(1)	−185495.394(2)	−215138.898(2)	−246.980995(3)
$10F_{5/2}$	−132801.1017(9)	−158049.448(1)	−185495.445(2)	−215138.967(2)	−246.981086(3)
$10F_{7/2}$	−132793.9513(9)	−158039.320(1)	−185481.493(2)	−215120.198(2)	−246.956349(3)
$10G_{7/2}$	−132793.9650(4)	−158039.3390(5)	−185481.5197(7)	−215120.234(1)	−246.956396(1)
$10G_{9/2}$	−132789.6758(4)	−158033.2638(5)	−185473.1512(7)	−215108.977(1)	−246.941561(1)
$10H_{9/2}$	−132789.6842(2)	−158033.2757(2)	−185473.1677(3)	−215108.9994(5)	−246.9415898(6)
$10H_{11/2}$	−132786.8251(2)	−158029.2262(2)	−185467.5897(3)	−215101.4963(5)	−246.9317015(6)
$10I_{11/2}$	−132786.83086(9)	−158029.2343(1)	−185467.6009(2)	−215101.5113(2)	−246.9317214(3)
$10I_{13/2}$	−132784.78878(9)	−158026.3420(1)	−185463.6170(2)	−215096.1526(2)	−246.9246593(3)
$10K_{13/2}$	−132784.79296(4)	−158026.34793(6)	−185463.62516(8)	−215096.1636(1)	−246.9246737(1)
$10K_{15/2}$	−132783.26147(4)	−158024.17884(6)	−185460.63745(8)	−215092.1448(1)	−246.9193776(1)
$10L_{15/2}$	−132783.26464(2)	−158024.18333(3)	−185460.64364(4)	−215092.15316(5)	−246.91938860(6)
$10L_{17/2}$	−132782.07352(2)	−158022.49633(3)	−185458.31998(4)	−215089.02766(5)	−246.91526970(6)
$10M_{17/2}$	−132782.076018(6)	−158022.499861(9)	−185458.32484(1)	−215089.03420(2)	−246.91527831(2)
$10M_{19/2}$	−132781.123148(6)	−158021.150299(9)	−185456.46598(1)	−215086.53390(2)	−246.91198334(2)
$11S_{1/2}$	−109793.347(1)	−130676.663(2)	−153381.064(4)	−177907.302(6)	−204.257188(9)
$11P_{1/2}$	−109794.746(1)	−130678.568(2)	−153383.594(4)	−177910.589(6)	−204.261381(8)
$11P_{3/2}$	−109762.4257(9)	−130632.764(1)	−153320.461(2)	−177825.609(3)	−204.149303(5)

Hydrogenic levels (QED uncertainties)—Continued

	$^{11}Na^{23}$	$^{12}Mg^{24}$	$^{13}Al^{27}$	$^{14}Si^{28}$	$^{15}P^{31}$
Units:	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	$10^3\ cm^{-1}$
$11D_{3/2}$	−109762.4836(7)	−130632.846(1)	−153320.575(1)	−177825.763(2)	−204.149508(2)
$11D_{5/2}$	−109751.7334(7)	−130617.617(1)	−153299.595(1)	−177797.536(2)	−204.112300(2)
$11F_{5/2}$	−109751.7532(7)	−130617.645(1)	−153299.634(1)	−177797.588(2)	−204.112368(2)
$11F_{7/2}$	−109746.3811(7)	−130610.036(1)	−153289.151(1)	−177783.487(2)	−204.093783(2)
$11G_{7/2}$	−109746.3913(3)	−130610.0506(4)	−153289.1714(6)	−177783.5141(8)	−204.093819(1)
$11G_{9/2}$	−109743.1688(3)	−130605.4862(4)	−153282.8841(6)	−177775.0566(8)	−204.082672(1)
$11H_{9/2}$	−109743.1751(1)	−130605.4952(2)	−153282.8965(3)	−177775.0733(4)	−204.0826943(5)
$11H_{11/2}$	−109741.0270(1)	−130602.4527(2)	−153278.7056(3)	−177769.4360(4)	−204.0752650(5)
$11I_{11/2}$	−109741.03135(8)	−130602.4589(1)	−153278.7141(1)	−177769.4474(2)	−204.0752800(3)
$11I_{13/2}$	−109739.49710(8)	−130600.2858(1)	−153275.7209(1)	−177765.4213(2)	−204.0699741(3)
$11K_{13/2}$	−109739.50024(4)	−130600.29028(6)	−153275.72705(8)	−177765.4295(1)	−204.0699850(1)
$11K_{15/2}$	−109738.34961(4)	−130598.66060(6)	−153273.48232(8)	−177762.4102(1)	−204.0660059(1)
$11L_{15/2}$	−109738.35199(2)	−130598.66398(3)	−153273.48697(4)	−177762.41641(5)	−204.06601415(7)
$11L_{17/2}$	−109737.45708(2)	−130597.39650(3)	−153271.74116(4)	−177760.06815(5)	−204.06291952(7)
$11M_{17/2}$	−109737.458958(9)	−130597.39916(1)	−153271.74481(2)	−177760.07307(2)	−204.06292599(3)
$11M_{19/2}$	−109736.743047(9)	−130596.38520(1)	−153270.34821(2)	−177758.19454(2)	−204.06045040(3)
$11N_{19/2}$	−109736.744560(3)	−130596.387343(5)	−153270.351157(7)	−177758.198503(9)	−204.06045563(1)
$11N_{21/2}$	−109736.158826(3)	−130595.557762(5)	−153269.208509(7)	−177756.661568(9)	−204.05843021(1)
$12S_{1/2}$	−92253.083(1)	−109799.268(2)	−128875.213(3)	−149481.468(5)	−171.619469(7)
$12P_{1/2}$	−92254.160(1)	−109800.735(2)	−128877.161(3)	−149484.000(4)	−171.622699(6)
$12O_{23/2}$	−92208.659730(2)	−109736.267537(3)	−128788.327669(4)	−149364.458403(5)	−171.465086696(7)
$13S_{1/2}$	−78603.3840(9)	−93552.811(2)	−109805.368(2)	−127361.466(4)	−146.222263(6)
$13P_{1/2}$	−78604.2317(9)	−93553.965(1)	−109806.901(2)	−127363.457(3)	−146.224803(5)
$13Q_{25/2}$	−78568.195506(1)	−93502.907825(2)	−109736.546414(2)	−127268.783356(3)	−146.099979571(4)
$14S_{1/2}$	−67773.2813(7)	−80662.476(1)	−94675.039(2)	−109811.279(3)	−126.072150(5)
$14P_{1/2}$	−67773.9601(7)	−80663.400(1)	−94676.266(2)	−109812.874(3)	−126.074183(4)
$14R_{27/2}$	−67744.9368105(7)	−80622.279297(1)	−94619.603828(1)	−109736.625834(2)	−125.973654549(3)
$15S_{1/2}$	−59036.4700(6)	−70263.727(1)	−82469.371(2)	−95653.641(2)	−109.817329(4)
$15P_{1/2}$	−59037.0219(6)	−70264.4780(9)	−82470.369(1)	−95654.937(2)	−109.818982(3)
$15T_{29/2}$	−59013.3046940(5)	−70230.8748039(7)	−82424.066332(1)	−95592.630109(1)	−109.736834128(2)
$16S_{1/2}$	−51886.2917(5)	−61753.4828(8)	−72480.438(1)	−84067.340(2)	−96.514858(3)
$16P_{1/2}$	−51886.7464(5)	−61754.1020(8)	−72481.261(1)	−84068.409(2)	−96.516221(3)
$16U_{31/2}$	−51867.1173819(4)	−61726.2911497(5)	−72442.9393687(7)	−84016.842120(1)	−96.448233951(1)
$17S_{1/2}$	−45960.5911(4)	−54700.6737(7)	−64202.231(1)	−74465.405(2)	−85.490765(3)
$17P_{1/2}$	−45960.9702(4)	−54701.1899(6)	−64202.917(1)	−74466.295(2)	−85.491900(2)
$17V_{33/2}$	−45944.5415685(3)	−54677.9135910(4)	−64170.8438577(5)	−74423.1368858(7)	−85.434999709(1)
$18S_{1/2}$	−40994.9401(3)	−48790.5550(6)	−57265.2945(9)	−66419.268(1)	−76.252967(2)
$18P_{1/2}$	−40995.2595(3)	−48790.9900(5)	−57265.8719(9)	−66420.018(1)	−76.253924(2)
$18W_{35/2}$	−40981.3719797(2)	−48771.3139164(3)	−57238.7601376(4)	−66383.5357721(6)	−76.2058249432(8)
$19S_{1/2}$	−36792.6193(3)	−43788.9770(5)	−51394.7792(8)	−59610.111(1)	−68.435399(2)
$19P_{1/2}$	−36792.8909(3)	−43789.3468(5)	−51395.2701(7)	−59610.749(1)	−68.436213(2)
$19X_{37/2}$	−36781.0464204(2)	−43772.5654557(2)	−51372.1470644(3)	−59579.6339142(5)	−68.3951908425(6)
$20S_{1/2}$	−33204.8303(2)	−39518.8304(4)	−46382.7933(7)	−53796.785(1)	−61.761180(2)
$20P_{1/2}$	−33205.0632(3)	−39519.1474(4)	−46383.2143(6)	−53797.332(1)	−61.761878(1)
$20Y_{39/2}$	−33194.8799574(1)	−39504.7198766(2)	−46363.3345623(3)	−53770.5817264(4)	−61.7266098149(5)

Hydrogenic levels (QED uncertainties)—Continued

	$^{16}S^{32}$	$^{17}Cl^{35}$	$^{18}Ar^{40}$	$^{19}K^{39}$	$^{20}Ca^{40}$	$^{21}Sc^{45}$
Units:	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}
$1S_{1/2}$	−28182.46(2)	−31828.90(3)	−35699.78(4)	−39795.63(5)	−44117.21(7)	−48665.24(9)
$2S_{1/2}$	−7052.365(2)	−7965.801(3)	−8935.697(5)	−9962.225(6)	−11045.621(9)	−12186.11(1)
$2P_{1/2}$	−7053.225(2)	−7966.866(2)	−8936.998(3)	−9963.798(4)	−11047.504(6)	−12188.348(8)
$2P_{3/2}$	−7029.0320(8)	−7935.999(1)	−8898.158(2)	−9915.521(2)	−10988.154(3)	−12116.110(4)
$3S_{1/2}$	−3131.8188(7)	−3537.077(1)	−3967.289(1)	−4422.512(2)	−4902.832(3)	−5408.333(3)
$3P_{1/2}$	−3132.0765(5)	−3537.3962(7)	−3967.679(1)	−4422.983(1)	−4903.397(2)	−5409.003(3)
$3P_{3/2}$	−3124.9063(3)	−3528.2480(4)	−3956.1667(6)	−4408.6737(8)	−4885.805(1)	−5387.589(1)
$3D_{3/2}$	−3124.91875(1)	−3528.26394(2)	−3956.18686(3)	−4408.69891(4)	−4885.83601(5)	−5387.62741(7)
$3D_{5/2}$	−3122.54371(1)	−3525.23619(2)	−3952.38009(3)	−4403.97144(4)	−4880.02983(5)	−5380.56728(7)
$4S_{1/2}$	−1760.5510(3)	−1988.2048(4)	−2229.8351(6)	−2485.4673(8)	−2755.142(1)	−3038.897(1)
$4P_{1/2}$	−1760.6601(2)	−1988.3398(3)	−2230.0002(5)	−2485.6670(6)	−2755.3810(9)	−3039.180(1)
$4P_{3/2}$	−1757.6360(1)	−1984.4817(2)	−2225.1454(2)	−2479.6327(3)	−2747.9628(5)	−3030.1512(6)
$4D_{3/2}$	−1757.64135(4)	−1984.48854(5)	−2225.15403(6)	−2479.64343(8)	−2747.97607(9)	−3030.1675(1)
$4D_{5/2}$	−1756.63909(4)	−1983.21080(5)	−2223.54748(6)	−2477.64823(8)	−2745.52550(9)	−3027.1875(1)
$4F_{5/2}$	−1756.64090(1)	−1983.21310(2)	−2223.55037(2)	−2477.65182(3)	−2745.52990(4)	−3027.19289(4)
$4F_{7/2}$	−1756.14055(1)	−1982.57535(2)	−2222.74868(2)	−2476.65641(3)	−2744.30760(4)	−3025.70692(4)
$5S_{1/2}$	−1126.2546(1)	−1271.8153(2)	−1426.2938(3)	−1589.7034(4)	−1762.0660(6)	−1943.4023(7)
$5P_{1/2}$	−1126.3106(1)	−1271.8845(2)	−1426.3786(2)	−1589.8058(3)	−1762.1886(5)	−1943.5478(6)
$5P_{3/2}$	−1124.76277(6)	−1269.90986(9)	−1423.8939(1)	−1586.7175(2)	−1758.3923(2)	−1938.9274(3)
$5D_{3/2}$	−1124.76552(2)	−1269.91339(3)	−1423.89831(4)	−1586.72308(5)	−1758.39921(6)	−1938.93575(7)
$5D_{5/2}$	−1124.25235(2)	−1269.25916(3)	−1423.07572(4)	−1585.70149(5)	−1757.14446(6)	−1937.40992(7)
$5F_{5/2}$	−1124.25328(1)	−1269.26034(2)	−1423.07721(2)	−1585.70334(3)	−1757.14673(4)	−1937.41268(4)
$5F_{7/2}$	−1123.99706(1)	−1268.93376(2)	−1422.66666(2)	−1585.19358(3)	−1756.52075(4)	−1936.65165(4)
$5G_{7/2}$	−1123.997543(3)	−1268.934371(4)	−1422.667435(5)	−1585.194540(6)	−1756.521931(7)	−1936.653084(9)
$5G_{9/2}$	−1123.843908(3)	−1268.738558(4)	−1422.421300(5)	−1584.888953(6)	−1756.146715(7)	−1936.196961(9)
$6S_{1/2}$	−781.86861(8)	−882.8825(1)	−990.0757(2)	−1103.4557(2)	−1223.0359(3)	−1348.8287(4)
$6P_{1/2}$	−781.90101(7)	−882.9226(1)	−990.1248(1)	−1103.5150(2)	−1223.1069(3)	−1348.9130(4)
$6H_{11/2}$	−780.4146901(9)	−881.027062(1)	−987.740556(1)	−1100.552752(2)	−1219.467041(2)	−1344.484840(3)
$7S_{1/2}$	−574.29353(5)	−648.46896(8)	−727.1767(1)	−810.4213(1)	−898.2117(2)	−990.5558(3)
$7P_{1/2}$	−574.31394(5)	−648.49422(7)	−727.20764(9)	−810.4587(1)	−898.2564(2)	−990.6089(2)
$7I_{13/2}$	−573.3515090(3)	−647.2668542(4)	−725.6638937(5)	−808.5407581(6)	−895.8998408(7)	−987.7420795(9)
$8S_{1/2}$	−439.60946(4)	−496.37680(5)	−556.60969(7)	−620.3110(1)	−687.4869(1)	−758.1430(2)
$8P_{1/2}$	−439.62313(3)	−496.39372(4)	−556.63040(6)	−620.33599(9)	−687.5169(1)	−758.1786(2)
$8K_{15/2}$	−438.9650995(1)	−495.5545734(2)	−555.5749657(2)	−619.0247993(2)	−685.9058583(3)	−756.2188093(4)
$9S_{1/2}$	−347.29255(2)	−392.13106(4)	−439.70492(5)	−490.01601(7)	−543.06892(9)	−598.8676(1)
$9P_{1/2}$	−347.30216(2)	−392.14295(3)	−439.71946(4)	−490.03358(6)	−543.08996(8)	−598.8926(1)
$9L_{17/2}$	−346.83274855(6)	−391.54435622(7)	−438.96660781(9)	−489.0983115(1)	−541.9408505(1)	−597.4947239(2)
$10S_{1/2}$	−281.27170(2)	−317.58116(3)	−356.10435(4)	−396.84255(5)	−439.79925(7)	−484.97736(9)
$10P_{1/2}$	−281.27871(2)	−317.58983(2)	−356.11495(3)	−396.85536(4)	−439.81459(6)	−484.99557(8)
$10M_{19/2}$	−280.93228202(3)	−317.14806826(4)	−355.55935722(5)	−396.16516910(6)	−438.96660912(7)	−483.96406548(8)
$11S_{1/2}$	−232.43168(1)	−262.43278(2)	−294.26212(3)	−327.92058(4)	−363.41087(5)	−400.73521(7)
$11P_{1/2}$	−232.43694(1)	−262.43930(2)	−294.27009(2)	−327.93020(3)	−363.42239(5)	−400.74888(6)
$11N_{21/2}$	−232.17406725(2)	−262.10409245(2)	−293.84851001(2)	−327.40650139(3)	−362.77897069(4)	−399.96622893(5)
$12S_{1/2}$	−195.28992(1)	−220.49443(2)	−247.23422(2)	−275.50994(3)	−305.32374(4)	−336.67736(5)
$12P_{1/2}$	−195.293976(9)	−220.49944(1)	−247.24035(2)	−275.51735(3)	−305.33261(3)	−336.68789(5)
$12O_{23/2}$	−195.089832127(9)	−220.23912645(1)	−246.91296791(1)	−275.11066307(2)	−304.83296568(2)	−336.08013082(3)
$13S_{1/2}$	−166.388293(8)	−187.86084(1)	−210.64089(2)	−234.72890(2)	−260.12661(3)	−286.83542(4)
$13P_{1/2}$	−166.391481(7)	−187.86479(1)	−210.64572(1)	−234.73473(2)	−260.13359(3)	−286.84370(4)
$13Q_{25/2}$	−166.229807824(5)	−187.658632697(7)	−210.386449177(9)	−234.41266268(1)	−259.73791151(1)	−286.36240885(2)

Hydrogenic levels (QED uncertainties) — Continued

	$^{22}Ti^{48}$	$^{23}V^{51}$	$^{24}Cr^{52}$	$^{25}Mn^{55}$	$^{26}Fe^{56}$	$^{27}Co^{59}$
Units:	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}
$1S_{1/2}$	−53440.4(1)	−58443.5(2)	−63675.3(2)	−69136.8(3)	−74828.7(3)	−80752.2(4)
$2S_{1/2}$	−13383.92(2)	−14639.29(2)	−15952.48(3)	−17323.79(3)	−18753.48(4)	−20241.87(5)
$2P_{1/2}$	−13386.55(1)	−14642.36(1)	−15956.05(2)	−17327.91(2)	−18758.20(3)	−20247.26(3)
$2P_{3/2}$	−13299.411(5)	−14538.112(7)	−15832.258(9)	−17181.91(1)	−18587.12(1)	−20047.94(2)
$3S_{1/2}$	−5939.087(4)	−6495.185(6)	−7076.719(8)	−7683.79(1)	−8316.49(1)	−8974.94(2)
$3P_{1/2}$	−5939.876(3)	−6496.108(5)	−7077.790(6)	−7685.026(7)	−8317.913(9)	−8976.56(1)
$3P_{3/2}$	−5914.045(2)	−6465.202(2)	−7041.088(3)	−7641.741(4)	−8267.185(5)	−8917.462(6)
$3D_{3/2}$	−5914.09058(9)	−6465.2571(1)	−7041.1541(2)	−7641.8182(2)	−8267.2764(2)	−8917.5683(3)
$3D_{5/2}$	−5905.58313(9)	−6455.0900(1)	−7029.0948(2)	−7627.6135(2)	−8250.6510(2)	−8898.2242(3)
$4S_{1/2}$	−3336.764(2)	−3648.786(2)	−3975.004(3)	−4315.464(4)	−4670.209(5)	−5039.292(6)
$4P_{1/2}$	−3337.098(2)	−3649.177(2)	−3975.457(3)	−4315.987(3)	−4670.810(4)	−5039.978(5)
$4P_{3/2}$	−3326.2069(8)	−3636.146(1)	−3959.984(1)	−4297.739(2)	−4649.426(2)	−5015.065(3)
$4D_{3/2}$	−3326.2266(1)	−3636.1701(2)	−3960.0122(2)	−4297.7724(2)	−4649.4648(3)	−5015.1103(3)
$4D_{5/2}$	−3322.6356(1)	−3631.8783(2)	−3954.9214(2)	−4291.7756(2)	−4642.4457(3)	−5006.9429(3)
$4F_{5/2}$	−3322.64201(5)	−3631.88598(6)	−3954.93052(7)	−4291.78634(9)	−4642.4582(1)	−5006.9575(1)
$4F_{7/2}$	−3320.85181(5)	−3629.74701(6)	−3952.39408(7)	−4288.79938(9)	−4638.9631(1)	−5002.8920(1)
$5S_{1/2}$	−2133.729(1)	−2333.069(1)	−2541.443(2)	−2758.877(2)	−2985.394(3)	−3221.022(3)
$5P_{1/2}$	−2133.9004(8)	−2333.269(1)	−2541.675(1)	−2759.145(2)	−2985.702(2)	−3221.373(3)
$5P_{3/2}$	−2128.3272(4)	−2326.6017(5)	−2533.7587(7)	−2749.8097(9)	−2974.762(1)	−3208.629(1)
$5D_{3/2}$	−2128.33740(9)	−2326.6139(1)	−2533.7733(1)	−2749.8269(2)	−2974.7826(2)	−3208.6527(2)
$5D_{5/2}$	−2126.49869(9)	−2324.4164(1)	−2531.1666(1)	−2746.7563(2)	−2971.1885(2)	−3204.4706(2)
$5F_{5/2}$	−2126.50201(5)	−2324.42032(6)	−2531.17130(7)	−2746.76183(9)	−2971.1950(1)	−3204.4781(1)
$5F_{7/2}$	−2125.58514(5)	−2323.32480(6)	−2529.87217(7)	−2745.23190(9)	−2969.4047(1)	−3202.3956(1)
$5G_{7/2}$	−2125.58687(1)	−2323.32686(1)	−2529.87460(2)	−2745.23477(2)	−2969.40809(2)	−3202.39948(3)
$5G_{9/2}$	−2125.03740(1)	−2322.67040(1)	−2529.09622(2)	−2744.31822(2)	−2968.33572(2)	−3201.15221(3)
$6S_{1/2}$	−1480.8438(6)	−1619.0947(7)	−1763.5939(9)	−1914.357(1)	−2071.397(2)	−2234.731(2)
$6P_{1/2}$	−1480.9430(5)	−1619.2106(6)	−1763.7285(8)	−1914.512(1)	−2071.575(1)	−2234.935(2)
$6H_{11/2}$	−1475.604313(3)	−1612.826845(4)	−1756.152357(4)	−1905.582884(5)	−2061.117680(6)	−2222.758855(7)
$7S_{1/2}$	−1087.4597(4)	−1188.9320(5)	−1294.9809(6)	−1405.6163(8)	−1520.847(1)	−1640.683(1)
$7P_{1/2}$	−1087.5221(3)	−1189.0050(4)	−1295.0656(5)	−1405.7141(6)	−1520.9588(8)	−1640.811(1)
$7I_{13/2}$	−1084.066018(1)	−1184.872559(1)	−1290.161526(2)	−1399.934290(2)	−1514.190175(2)	−1632.930593(2)
$8S_{1/2}$	−832.2832(2)	−909.9134(3)	−991.0391(4)	−1075.6671(5)	−1163.8031(6)	−1255.4544(8)
$8P_{1/2}$	−832.3250(2)	−909.9623(3)	−991.0958(3)	−1075.7326(4)	−1163.8782(6)	−1255.5402(7)
$8K_{15/2}$	−829.9624834(4)	−907.1375160(5)	−987.7437130(6)	−1071.7820612(7)	−1159.2519791(9)	−1250.154482(1)
$9S_{1/2}$	−657.4147(2)	−718.7144(2)	−782.7707(3)	−849.5885(4)	−919.1716(4)	−991.5254(6)
$9P_{1/2}$	−657.4441(1)	−718.7488(2)	−782.8106(2)	−849.6345(3)	−919.2243(4)	−991.5856(5)
$9L_{17/2}$	−655.7589790(2)	−716.7340874(2)	−780.4198636(3)	−846.8170540(3)	−915.9251643(4)	−987.7449604(5)
$10S_{1/2}$	−532.3788(1)	−582.0066(2)	−633.8637(2)	−687.9538(3)	−744.2795(3)	−802.8451(4)
$10P_{1/2}$	−532.4002(1)	−582.0317(1)	−633.8928(2)	−687.9873(2)	−744.3180(3)	−802.8889(4)
$10M_{19/2}$	−531.1567497(1)	−580.5450259(1)	−632.1287254(1)	−685.9084336(2)	−741.8837297(2)	−800.0552132(2)
$11S_{1/2}$	−439.89496(9)	−480.8926(1)	−523.7301(2)	−568.4103(2)	−614.9354(2)	−663.3084(3)
$11P_{1/2}$	−439.91106(8)	−480.9114(1)	−523.7519(1)	−568.4355(2)	−614.9643(2)	−663.3413(3)
$11N_{21/2}$	−438.96761415(6)	−479.78341648(7)	−522.41348523(8)	−566.85829222(9)	−613.1174775(1)	−661.1915237(1)
$12S_{1/2}$	−369.57181(7)	−404.00898(9)	−439.9905(1)	−477.5185(2)	−516.5946(2)	−557.2213(2)
$12P_{1/2}$	−369.58421(6)	−404.02346(8)	−440.0073(1)	−477.5379(1)	−516.6169(2)	−557.2467(2)
$12O_{23/2}$	−368.85159565(3)	−403.14759707(4)	−438.96800122(5)	−476.31319700(5)	−515.18287410(6)	−555.57742978(8)
$13S_{1/2}$	−314.85608(6)	−344.19009(7)	−374.83872(9)	−406.8037(1)	−440.0864(1)	−474.6885(2)
$13P_{1/2}$	−314.86583(5)	−344.20149(6)	−374.85194(8)	−406.8190(1)	−440.1038(1)	−474.7085(2)
$13Q_{25/2}$	−314.28567078(2)	−343.50789453(2)	−374.02896127(3)	−405.84919736(3)	−438.96833316(4)	−473.38670173(5)

Hydrogenic levels (QED uncertainties)—Continued

	$^{28}Ni^{58}$	$^{29}Cu^{63}$	$^{30}Zn^{64}$	$^{31}Ga^{69}$	$^{32}Ge^{74}$	$^{33}As^{75}$
Units:	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}
$1S_{1/2}$	−86908.2(5)	−93297.7(6)	−99921.8(8)	−106781.8(9)	−113879.(1)	−121214.(1)
$2S_{1/2}$	−21789.26(6)	−23396.02(8)	−25062.5(1)	−26789.0(1)	−28575.9(1)	−30423.6(2)
$2P_{1/2}$	−21795.39(4)	−23402.95(5)	−25070.26(6)	−26797.71(8)	−28585.68(9)	−30434.5(1)
$2P_{3/2}$	−21564.43(2)	−23136.66(3)	−24764.69(3)	−26448.59(4)	−28188.42(5)	−29984.24(6)
$3S_{1/2}$	−9659.25(2)	−10369.53(2)	−11105.90(3)	−11868.49(3)	−12657.45(4)	−13472.89(5)
$3P_{1/2}$	−9661.09(1)	−10371.61(2)	−11108.24(2)	−11871.13(3)	−12660.39(3)	−13476.17(4)
$3P_{3/2}$	−9592.596(8)	−10292.641(9)	−11017.62(1)	−11767.58(1)	−12542.56(2)	−13342.60(2)
$3D_{3/2}$	−9592.7203(4)	−10292.7842(5)	−11017.7831(6)	−11767.7682(7)	−12542.7746(8)	−13342.840(1)
$3D_{5/2}$	−9570.3359(4)	−10267.0130(5)	−10988.2530(6)	−11734.0804(7)	−12504.5027(8)	−13299.529(1)
$4S_{1/2}$	−5422.759(8)	−5820.67(1)	−6233.07(1)	−6660.03(1)	−7101.60(2)	−7557.85(2)
$4P_{1/2}$	−5423.537(6)	−5821.550(8)	−6234.06(1)	−6661.14(1)	−7102.85(1)	−7559.23(2)
$4P_{3/2}$	−5394.670(3)	−5788.268(4)	−6195.873(5)	−6617.509(6)	−7053.197(7)	−7502.955(9)
$4D_{3/2}$	−5394.7227(4)	−5788.3297(5)	−6195.9430(5)	−6617.5900(6)	−7053.2889(7)	−7503.0592(8)
$4D_{5/2}$	−5385.2711(4)	−5777.4473(5)	−6183.4725(5)	−6603.3628(6)	−7037.1245(7)	−7484.7653(8)
$4F_{5/2}$	−5385.2879(1)	−5777.4667(2)	−6183.4947(2)	−6603.3880(2)	−7037.1532(2)	−7484.7977(3)
$4F_{7/2}$	−5380.5847(1)	−5772.0534(2)	−6177.2937(2)	−6596.3162(2)	−7029.1216(2)	−7475.7116(3)
$5S_{1/2}$	−3465.784(4)	−3719.714(5)	−3982.837(6)	−4255.187(7)	−4536.794(9)	−4827.69(1)
$5P_{1/2}$	−3466.183(3)	−3720.166(4)	−3983.346(5)	−4255.758(6)	−4537.432(7)	−4828.400(9)
$5P_{3/2}$	−3451.417(2)	−3703.143(2)	−3963.813(3)	−4233.443(3)	−4512.043(4)	−4799.624(5)
$5D_{3/2}$	−3451.4446(3)	−3703.1748(3)	−3963.8494(3)	−4233.4846(4)	−4512.0906(5)	−4799.6783(5)
$5D_{5/2}$	−3446.6049(3)	−3697.6024(3)	−3957.4638(3)	−4226.1994(4)	−4503.8133(5)	−4790.3105(5)
$5F_{5/2}$	−3446.6135(1)	−3697.6124(2)	−3957.4752(2)	−4226.2124(2)	−4503.8281(2)	−4790.3271(3)
$5F_{7/2}$	−3444.2043(1)	−3694.8393(2)	−3954.2985(2)	−4222.5894(2)	−4499.7132(2)	−4785.6719(3)
$5G_{7/2}$	−3444.20881(3)	−3694.84449(3)	−3954.30443(4)	−4222.59612(4)	−4499.72087(5)	−4785.68049(6)
$5G_{9/2}$	−3442.76605(3)	−3693.18409(3)	−3952.40261(4)	−4220.42745(4)	−4497.25817(5)	−4782.89479(6)
$6S_{1/2}$	−2404.373(2)	−2580.343(3)	−2762.655(4)	−2951.331(4)	−3146.386(5)	−3347.841(6)
$6P_{1/2}$	−2404.604(2)	−2580.605(2)	−2762.950(3)	−2951.661(4)	−3146.755(4)	−3348.252(5)
$6H_{11/2}$	−2390.504946(8)	−2564.36040(1)	−2744.32227(1)	−2930.39427(1)	−3122.57573(1)	−3320.86628(2)
$7S_{1/2}$	−1765.133(1)	−1894.212(2)	−2027.927(2)	−2166.292(3)	−2309.318(3)	−2457.018(4)
$7P_{1/2}$	−1765.279(1)	−1894.376(2)	−2028.112(2)	−2166.500(2)	−2309.551(3)	−2457.277(3)
$7I_{13/2}$	−1756.154332(3)	−1883.864508(3)	−2016.058819(4)	−2152.739823(4)	−2293.906869(5)	−2439.559521(6)
$8S_{1/2}$	−1350.627(1)	−1449.330(1)	−1551.569(1)	−1657.354(2)	−1766.692(2)	−1879.590(3)
$8P_{1/2}$	−1350.7244(9)	−1449.440(1)	−1551.693(1)	−1657.493(2)	−1766.847(2)	−1879.763(2)
$8K_{15/2}$	−1344.488571(1)	−1442.256561(1)	−1543.456612(2)	−1648.090608(2)	−1756.157968(2)	−1867.658278(2)
$9S_{1/2}$	−1066.6540(7)	−1144.5641(9)	−1225.259(1)	−1308.747(1)	−1395.032(2)	−1484.119(2)
$9P_{1/2}$	−1066.7224(6)	−1144.6414(7)	−1225.3465(9)	−1308.845(1)	−1395.141(1)	−1484.241(2)
$9L_{17/2}$	−1062.2756155(5)	−1139.5189190(6)	−1219.4733778(7)	−1302.1404374(8)	−1387.5195954(9)	−1475.610480(1)
$10S_{1/2}$	−863.6532(5)	−926.7090(6)	−992.0151(8)	−1059.5767(9)	−1129.398(1)	−1201.482(1)
$10P_{1/2}$	−863.7030(4)	−926.7653(5)	−992.0786(7)	−1059.6479(8)	−1129.477(1)	−1201.570(1)
$10M_{19/2}$	−860.4221926(3)	−922.9860945(3)	−987.7456863(4)	−1054.7021143(4)	−1123.8549465(5)	−1195.2038555(5)
$11S_{1/2}$	−713.5313(4)	−765.6083(5)	−819.5410(6)	−875.3336(7)	−932.9887(8)	−992.509(1)
$11P_{1/2}$	−713.5688(3)	−765.6506(4)	−819.5887(5)	−875.3870(6)	−933.0485(7)	−992.5760(9)
$11N_{21/2}$	−711.0798459(1)	−762.7836093(2)	−816.3017809(2)	−871.6352931(2)	−928.7837736(3)	−987.7469361(3)
$12S_{1/2}$	−599.4001(3)	−643.1340(4)	−688.4247(4)	−735.2751(5)	−783.6873(6)	−833.6638(8)
$12P_{1/2}$	−599.4289(3)	−643.1666(3)	−688.4614(4)	−735.3162(5)	−783.7333(6)	−833.7150(7)
$12O_{23/2}$	−597.49636411(9)	−640.9406473(1)	−685.9094021(1)	−732.4034027(1)	−780.4223262(2)	−829.9659220(2)
$13S_{1/2}$	−510.6114(2)	−547.8576(3)	−586.4281(3)	−626.3255(4)	−667.5513(5)	−710.1074(6)
$13P_{1/2}$	−510.6341(2)	−547.8832(2)	−586.4570(3)	−626.3578(4)	−667.5875(4)	−710.1477(5)
$13Q_{25/2}$	−509.10387144(6)	−546.12066314(6)	−584.43632324(7)	−624.05150507(9)	−664.9659271(1)	−707.1793690(1)

Hydrogenic levels (QED uncertainties)—Continued

	$^{34}Se^{80}$	$^{35}Br^{79}$	$^{36}Kr^{84}$	$^{37}Rb^{85}$	$^{38}Sr^{88}$	$^{39}Y^{89}$
Units:	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}
$1S_{1/2}$	−128788.(2)	−136604.(2)	−144661.(2)	−152963.(3)	−161509.(3)	−170303.(4)
$2S_{1/2}$	−32332.6(2)	−34303.2(2)	−36336.0(3)	−38431.3(3)	−40589.7(4)	−42811.6(4)
$2P_{1/2}$	−32344.7(1)	−34316.7(2)	−36350.8(2)	−38447.7(2)	−40607.6(3)	−42831.3(3)
$2P_{3/2}$	−31836.16(7)	−33744.20(9)	−35708.5(1)	−37729.1(1)	−39806.1(1)	−41939.6(2)
$3S_{1/2}$	−14314.97(6)	−15183.83(7)	−16079.64(8)	−17002.6(1)	−17952.7(1)	−18930.4(1)
$3P_{1/2}$	−14318.61(5)	−15187.87(5)	−16084.10(6)	−17007.46(8)	−17958.13(9)	−18936.3(1)
$3P_{3/2}$	−14167.74(2)	−15018.03(3)	−15893.52(3)	−16794.26(4)	−17720.29(5)	−18671.66(5)
$3D_{3/2}$	−14168.018(1)	−15018.341(1)	−15893.872(2)	−16794.646(2)	−17720.721(2)	−18672.143(3)
$3D_{5/2}$	−14119.183(1)	−14963.468(1)	−15832.413(2)	−16726.022(2)	−17644.318(2)	−18587.313(3)
$4S_{1/2}$	−8028.83(2)	−8514.62(3)	−9015.30(3)	−9530.92(4)	−10061.57(5)	−10607.33(6)
$4P_{1/2}$	−8030.37(2)	−8516.33(2)	−9017.18(3)	−9533.00(3)	−10063.85(4)	−10609.83(5)
$4P_{3/2}$	−7966.81(1)	−8444.78(1)	−8936.91(1)	−9443.20(2)	−9963.68(2)	−10498.39(2)
$4D_{3/2}$	−7966.930(1)	−8444.918(1)	−8937.056(1)	−9443.363(1)	−9963.869(2)	−10498.598(2)
$4D_{5/2}$	−7946.301(1)	−8421.737(1)	−8911.091(1)	−9414.368(1)	−9931.585(2)	−10462.750(2)
$4F_{5/2}$	−7946.3378(3)	−8421.7775(4)	−8911.1363(4)	−9414.4189(4)	−9931.6413(5)	−10462.8130(6)
$4F_{7/2}$	−7936.0964(3)	−8410.2736(4)	−8898.2564(4)	−9400.0427(4)	−9915.6417(5)	−10445.0559(6)
$5S_{1/2}$	−5127.91(1)	−5437.49(2)	−5756.47(2)	−6084.89(2)	−6422.78(2)	−6770.20(3)
$5P_{1/2}$	−5128.70(1)	−5438.37(1)	−5757.44(1)	−6085.95(2)	−6423.95(2)	−6771.48(2)
$5P_{3/2}$	−5096.204(6)	−5401.790(7)	−5716.403(8)	−6040.053(9)	−6372.76(1)	−6714.53(1)
$5D_{3/2}$	−5096.2647(6)	−5401.8590(7)	−5716.4807(8)	−6040.1397(9)	−6372.854(1)	−6714.637(1)
$5D_{5/2}$	−5085.7013(6)	−5389.9883(7)	−5703.1841(8)	−6025.2916(9)	−6356.321(1)	−6696.279(1)
$5F_{5/2}$	−5085.7201(3)	−5390.0093(3)	−5703.2076(4)	−6025.3178(4)	−6356.3503(5)	−6696.3113(5)
$5F_{7/2}$	−5080.4726(3)	−5384.1148(3)	−5696.6077(4)	−6017.9508(4)	−6348.1511(5)	−6687.2108(5)
$5G_{7/2}$	−5080.48234(6)	−5384.12568(7)	−5696.61989(8)	−6017.96442(9)	−6348.1662(1)	−6687.2276(1)
$5G_{9/2}$	−5077.34281(6)	−5380.59959(7)	−5692.67254(8)	−6013.55910(9)	−6343.2640(1)	−6681.7877(1)
$6S_{1/2}$	−3555.718(7)	−3770.035(9)	−3990.82(1)	−4218.09(1)	−4451.87(1)	−4692.18(2)
$6P_{1/2}$	−3556.175(6)	−3770.541(7)	−3991.377(9)	−4218.70(1)	−4452.54(1)	−4692.92(1)
$6H_{11/2}$	−3525.26936(2)	−3735.78273(2)	−3952.41111(2)	−4175.15231(3)	−4404.00920(3)	−4638.98155(3)
$7S_{1/2}$	−2609.405(5)	−2766.492(5)	−2928.294(6)	−3094.825(8)	−3266.101(9)	−3442.14(1)
$7P_{1/2}$	−2609.693(4)	−2766.810(5)	−2928.646(6)	−3095.212(7)	−3266.526(8)	−3442.603(9)
$7I_{13/2}$	−2589.700132(6)	−2744.326881(7)	−2903.443053(8)	−3067.046844(9)	−3235.14018(1)	−3407.72268(1)
$8S_{1/2}$	−1996.060(3)	−2116.108(4)	−2239.746(4)	−2366.983(5)	−2497.830(6)	−2632.296(7)
$8P_{1/2}$	−1996.252(3)	−2116.321(3)	−2239.982(4)	−2367.242(4)	−2498.114(5)	−2632.608(6)
$8K_{15/2}$	−1982.593251(3)	−2100.961405(3)	−2222.765166(3)	−2348.003060(4)	−2476.676460(4)	−2608.784984(5)
$9S_{1/2}$	−1576.017(2)	−1670.730(3)	−1768.267(3)	−1868.634(4)	−1971.838(4)	−2077.886(5)
$9P_{1/2}$	−1576.152(2)	−1670.880(2)	−1768.432(3)	−1868.815(3)	−1972.037(4)	−2078.105(4)
$9L_{17/2}$	−1566.414398(1)	−1659.930133(1)	−1756.159550(2)	−1855.101434(2)	−1956.756819(2)	−2061.125349(2)
$10S_{1/2}$	−1275.834(2)	−1352.460(2)	−1431.364(2)	−1512.550(3)	−1596.026(3)	−1681.795(4)
$10P_{1/2}$	−1275.933(1)	−1352.569(2)	−1431.484(2)	−1512.683(2)	−1596.171(3)	−1681.955(3)
$10M_{19/2}$	−1268.7498742(6)	−1344.4919877(7)	−1422.4316796(8)	−1502.5679361(9)	−1584.901565(1)	−1669.432247(1)
$11S_{1/2}$	−1053.900(1)	−1117.163(1)	−1182.304(2)	−1249.325(2)	−1318.230(2)	−1389.025(3)
$11P_{1/2}$	−1053.974(1)	−1117.245(1)	−1182.394(1)	−1249.424(2)	−1318.339(2)	−1389.144(2)
$11N_{21/2}$	−1048.5256175(3)	−1111.1189624(4)	−1175.5281794(4)	−1241.7524130(5)	−1309.7923125(5)	−1379.6475940(6)
$12S_{1/2}$	−885.2077(9)	−938.321(1)	−993.008(1)	−1049.270(2)	−1107.112(2)	−1166.536(2)
$12P_{1/2}$	−885.2646(8)	−938.3842(9)	−993.078(1)	−1049.347(1)	−1107.196(2)	−1166.628(2)
$12O_{23/2}$	−881.0348830(2)	−933.6284794(2)	−987.7477159(3)	−1043.3918619(3)	−1100.5614515(3)	−1159.2562340(4)
$13S_{1/2}$	−753.9965(7)	−799.2201(9)	−845.781(1)	−893.682(1)	−942.924(1)	−993.511(2)
$13P_{1/2}$	−754.0412(6)	−799.2696(7)	−845.8359(9)	−893.742(1)	−942.990(1)	−993.584(1)
$13Q_{25/2}$	−750.6924142(1)	−795.5044339(1)	−841.6162765(2)	−889.0273122(2)	−937.7379881(2)	−987.7480827(2)

Hydrogenic levels (QED uncertainties)

	^{40}Zr^{90}	^{41}Nb^{93}	^{42}Mo^{98}	^{43}Tc^{99}	^{44}Ru^{102}	^{45}Rh^{103}	^{46}Pd^{106}	^{47}Ag^{107}
Units:	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^3 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−179345.(4)	−188636.(5)	−198180.(5)	−207978.(6)	−218031.(7)	−228341.(8)	−238912.(9)	−249.74(1)
$2S_{1/2}$	−45097.6(5)	−47448.2(6)	−49863.9(7)	−52345.4(8)	−54893.2(9)	−57508.(1)	−60190.(1)	−62.941(1)
$2P_{1/2}$	−45119.1(3)	−47471.6(4)	−49889.5(5)	−52373.2(5)	−54923.4(6)	−57540.7(7)	−60225.7(8)	−62.9792(9)
$2P_{3/2}$	−44129.6(2)	−46376.3(2)	−48679.8(2)	−51040.1(3)	−53457.3(3)	−55931.6(4)	−58463.1(4)	−61.0518(5)
$3S_{1/2}$	−19935.6(2)	−20968.7(2)	−22029.8(2)	−23119.1(2)	−24236.8(3)	−25383.1(3)	−26558.3(3)	−27.7626(4)
$3P_{1/2}$	−19942.1(1)	−20975.8(1)	−22037.5(2)	−23127.4(2)	−24245.8(2)	−25392.9(2)	−26569.0(3)	−27.7741(3)
$3P_{3/2}$	−19648.43(6)	−20650.65(7)	−21678.39(8)	−22731.68(9)	−23810.6(1)	−24915.2(1)	−26045.6(1)	−27.2017(2)
$3D_{3/2}$	−19648.966(3)	−20651.248(4)	−21679.045(4)	−22732.407(5)	−23811.399(5)	−24916.078(6)	−26046.511(7)	−27.202756(8)
$3D_{5/2}$	−19555.027(3)	−20547.478(4)	−21564.686(4)	−22606.662(5)	−23673.430(5)	−24765.006(6)	−25881.416(7)	−27.022672(8)

	^{48}Cd^{114}	^{49}In^{115}	^{50}Sn^{120}	^{51}Sb^{121}	^{52}Te^{130}	^{53}I^{127}	^{54}Xe^{132}	^{55}Cs^{133}
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−260.84(1)	−272.20(1)	−283.83(1)	−295.73(2)	−307.91(2)	−320.36(2)	−333.09(2)	−346.10(3)
$2S_{1/2}$	−65.761(1)	−68.650(2)	−71.610(2)	−74.641(2)	−77.744(2)	−80.920(3)	−84.170(3)	−87.494(3)
$2P_{1/2}$	−65.802(1)	−68.694(1)	−71.658(1)	−74.692(1)	−77.799(2)	−80.979(2)	−84.232(2)	−87.561(2)
$2P_{3/2}$	−63.6978(5)	−66.4014(6)	−69.1625(7)	−71.9814(8)	−74.8581(8)	−77.7927(9)	−80.785(1)	−83.836(1)
$3S_{1/2}$	−28.9963(4)	−30.2595(5)	−31.5525(5)	−32.8757(6)	−34.2293(7)	−35.6135(8)	−37.0287(9)	−38.4751(9)
$3P_{1/2}$	−29.0086(3)	−30.2728(4)	−31.5668(4)	−32.8910(5)	−34.2457(5)	−35.6311(6)	−37.0475(7)	−38.4952(7)
$3P_{3/2}$	−28.3838(2)	−29.5917(2)	−30.8257(2)	−32.0858(3)	−33.3720(3)	−34.6844(3)	−36.0232(3)	−37.3883(4)
$3D_{3/2}$	−28.384889(9)	−29.59296(1)	−30.82705(1)	−32.08723(1)	−33.37357(1)	−34.68613(2)	−36.02501(2)	−37.39026(2)
$3D_{5/2}$	−28.188808(9)	−29.37983(1)	−30.59577(1)	−31.83664(1)	−33.10248(1)	−34.39328(2)	−35.70911(2)	−37.04995(2)

	^{56}Ba^{138}	^{57}La^{139}	^{58}Ce^{140}	^{59}Pr^{141}	^{60}Nd^{142}	^{61}Pm^{147}	^{62}Sm^{152}	^{63}Eu^{153}
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−359.39(3)	−372.98(3)	−386.85(3)	−401.02(4)	−415.48(4)	−430.25(5)	−445.32(5)	−460.70(5)
$2S_{1/2}$	−90.894(4)	−94.370(4)	−97.924(4)	−101.557(5)	−105.269(5)	−109.062(6)	−112.937(6)	−116.894(7)
$2P_{1/2}$	−90.965(2)	−94.446(3)	−98.005(3)	−101.643(3)	−105.360(3)	−109.159(4)	−113.039(4)	−117.003(5)
$2P_{3/2}$	−86.946(1)	−90.113(1)	−93.340(2)	−96.625(2)	−99.969(2)	−103.372(2)	−106.834(2)	−110.355(2)
$3S_{1/2}$	−39.953(1)	−41.463(1)	−43.005(1)	−44.580(1)	−46.188(2)	−47.829(2)	−49.504(2)	−51.213(2)
$3P_{1/2}$	−39.9746(8)	−41.4860(9)	−43.030(1)	−44.606(1)	−46.215(1)	−47.858(1)	−49.535(1)	−51.246(2)
$3P_{3/2}$	−38.7799(4)	−40.1980(5)	−41.6427(5)	−43.1141(6)	−44.6123(6)	−46.1374(7)	−47.6895(8)	−49.2686(8)
$3D_{3/2}$	−38.78199(2)	−40.20025(2)	−41.64514(3)	−43.11674(3)	−44.61514(3)	−46.14042(3)	−47.69269(4)	−49.27201(4)
$3D_{5/2}$	−38.41586(2)	−39.80684(2)	−41.22293(3)	−42.66415(3)	−44.13052(3)	−45.62209(3)	−47.13887(4)	−48.68088(4)

	^{64}Gd^{158}	^{65}Tb^{159}	^{66}Dy^{164}	^{67}Ho^{165}	^{68}Er^{166}	^{69}Tm^{169}	^{70}Yb^{174}	^{71}Lu^{175}
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−476.39(6)	−492.39(7)	−508.72(7)	−525.37(8)	−542.35(8)	−559.66(9)	−577.3(1)	−595.3(1)
$2S_{1/2}$	−120.937(7)	−125.065(8)	−129.280(9)	−133.58(1)	−137.98(1)	−142.46(1)	−147.04(1)	−151.71(1)
$2P_{1/2}$	−121.052(5)	−125.187(5)	−129.408(6)	−133.719(6)	−138.120(7)	−142.612(8)	−147.198(8)	−151.879(9)
$2P_{3/2}$	−113.937(3)	−117.577(3)	−121.278(3)	−125.038(4)	−128.859(4)	−132.740(4)	−136.681(4)	−140.684(5)
$3S_{1/2}$	−52.957(2)	−54.736(2)	−56.550(3)	−58.400(3)	−60.286(3)	−62.210(3)	−64.171(4)	−66.170(4)
$3P_{1/2}$	−52.992(2)	−54.772(2)	−56.588(2)	−58.441(2)	−60.329(2)	−62.255(3)	−64.219(3)	−66.220(3)
$3P_{3/2}$	−50.8748(9)	−52.508(1)	−54.169(1)	−55.857(1)	−57.573(1)	−59.317(1)	−61.088(1)	−62.887(2)
$3D_{3/2}$	−50.87850(4)	−52.51224(5)	−54.17333(5)	−55.86188(6)	−57.57798(6)	−59.32174(7)	−61.09327(7)	−62.89267(8)
$3D_{5/2}$	−50.24817(4)	−51.84075(5)	−53.45866(5)	−55.10193(6)	−56.77059(6)	−58.46467(7)	−60.18420(7)	−61.92922(8)

Hydrogenic levels (QED uncertainties)—Continued

	^{72}Hf^{180}	^{73}Ta^{181}	^{74}W^{184}	^{75}Re^{187}	^{76}Os^{192}	^{77}Ir^{193}	^{78}Pt^{195}	^{79}Au^{197}	^{80}Hg^{202}
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−613.7(1)	−632.4(1)	−651.4(1)	−670.8(1)	−690.6(2)	−710.8(2)	−731.4(2)	−752.3(2)	−773.7(2)
$2S_{1/2}$	−156.48(1)	−161.35(2)	−166.31(2)	−171.38(2)	−176.56(2)	−181.83(2)	−187.22(2)	−192.72(2)	−198.33(3)
$2P_{1/2}$	−156.66(1)	−161.53(1)	−166.51(1)	−171.59(1)	−176.77(1)	−182.06(1)	−187.46(2)	−192.97(2)	−198.59(2)
$2P_{3/2}$	−144.747(5)	−148.871(6)	−153.056(6)	−157.303(7)	−161.611(7)	−165.981(8)	−170.413(8)	−174.908(9)	−179.465(9)
$3S_{1/2}$	−68.207(4)	−70.284(5)	−72.400(5)	−74.557(5)	−76.755(6)	−78.995(6)	−81.277(7)	−83.603(7)	−85.973(8)
$3P_{1/2}$	−68.260(3)	−70.340(4)	−72.459(4)	−74.619(4)	−76.820(5)	−79.063(5)	−81.348(5)	−83.677(6)	−86.050(6)
$3P_{3/2}$	−64.714(2)	−66.569(2)	−68.453(2)	−70.364(2)	−72.304(2)	−74.273(3)	−76.270(3)	−78.296(3)	−80.351(3)
$3D_{3/2}$	−64.72005(9)	−66.57552(9)	−68.4592(1)	−70.3712(1)	−72.3117(1)	−74.2807(1)	−76.2784(1)	−78.3049(1)	−80.3604(2)
$3D_{5/2}$	−63.69976(9)	−65.49584(9)	−67.3175(1)	−69.1648(1)	−71.0377(1)	−72.9364(1)	−74.8607(1)	−76.8108(1)	−78.7868(2)

	^{81}Tl^{205}	^{82}Pb^{208}	^{83}Bi^{209}	^{84}Po	^{85}At	^{86}Rn	^{87}Fr	^{88}Ra^{226}	^{89}Ac
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−795.5(2)	−817.6(2)	−840.2(3)	−863.3(3)	−886.8(3)	−910.7(3)	−935.1(3)	−960.0(3)	−985.3(4)
$2S_{1/2}$	−204.06(3)	−209.90(3)	−215.87(3)	−221.96(3)	−228.17(4)	−234.52(4)	−241.00(4)	−247.61(4)	−254.37(5)
$2P_{1/2}$	−204.33(2)	−210.18(2)	−216.16(2)	−222.26(2)	−228.49(2)	−234.85(3)	−241.35(3)	−247.98(3)	−254.75(3)
$2P_{3/2}$	−184.08(1)	−188.77(1)	−193.51(1)	−198.32(1)	−203.19(1)	−208.13(1)	−213.13(1)	−218.20(2)	−223.32(2)
$3S_{1/2}$	−88.387(8)	−90.847(9)	−93.354(9)	−95.91(1)	−98.51(1)	−101.16(1)	−103.87(1)	−106.62(1)	−109.43(1)
$3P_{1/2}$	−88.468(6)	−90.932(7)	−93.443(7)	−96.001(8)	−98.608(8)	−101.265(9)	−103.972(9)	−106.73(1)	−109.54(1)
$3P_{3/2}$	−82.435(3)	−84.549(4)	−86.691(4)	−88.863(4)	−91.065(4)	−93.296(5)	−95.558(5)	−97.849(5)	−100.171(6)
$3D_{3/2}$	−82.4449(2)	−84.5587(2)	−86.7017(2)	−88.8743(2)	−91.0765(2)	−93.3085(2)	−95.5703(2)	−97.8623(3)	−100.1844(3)
$3D_{5/2}$	−80.7885(2)	−82.8162(2)	−84.8697(2)	−86.9492(2)	−89.0547(2)	−91.1862(2)	−93.3438(2)	−95.5276(3)	−97.7374(3)

	^{90}Th^{232}	^{91}Pa	^{92}U^{238}	^{93}Np	^{94}Pu	^{95}Am	^{96}Cm	^{97}Bk
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−1011.2(4)	−1037.5(4)	−1064.4(4)	−1091.8(5)	−1119.8(5)	−1148.3(5)	−1177.5(6)	−1207.2(6)
$2S_{1/2}$	−261.27(5)	−268.33(5)	−275.53(6)	−282.89(6)	−290.42(6)	−298.11(7)	−305.98(7)	−314.02(7)
$2P_{1/2}$	−261.67(3)	−268.74(3)	−275.96(4)	−283.34(4)	−290.88(4)	−298.59(4)	−306.48(5)	−314.54(5)
$2P_{3/2}$	−228.52(2)	−233.78(2)	−239.10(2)	−244.49(2)	−249.95(2)	−255.47(2)	−261.06(3)	−266.71(3)
$3S_{1/2}$	−112.29(1)	−115.21(2)	−118.18(2)	−121.22(2)	−124.31(2)	−127.46(2)	−130.68(2)	−133.96(2)
$3P_{1/2}$	−112.41(1)	−115.33(1)	−118.31(1)	−121.35(1)	−124.45(1)	−127.61(2)	130.83(2)	−134.12(2)
$3P_{3/2}$	−102.523(6)	−104.905(6)	−107.318(7)	−109.762(7)	−112.237(7)	−114.743(8)	−117.280(8)	−119.849(9)
$3D_{3/2}$	−102.5369(3)	−104.9199(3)	−107.3336(3)	−109.7782(3)	−112.2537(4)	−114.7605(4)	−117.2986(4)	−119.8682(4)
$3D_{5/2}$	−99.9735(3)	−102.2359(3)	−104.5245(3)	−106.8395(3)	−109.1808(4)	−111.5486(4)	−113.9428(4)	−116.3636(4)

	^{98}Cf	^{99}Es	^{100}Fm	^{101}Md	^{102}No	^{103}Lr	^{104}Rf	^{105}Ha
Units:	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}	10^6 cm^{-1}
$1S_{1/2}$	−1237.5(6)	−1268.5(7)	−1300.2(7)	−1332.5(7)	−1365.5(8)	−1399.3(8)	−1433.8(8)	−1469.2(9)
$2S_{1/2}$	−322.25(8)	−330.67(8)	−339.30(9)	−348.12(9)	−357.2(1)	−366.4(1)	−375.9(1)	−385.7(1)
$2P_{1/2}$	−322.79(5)	−331.23(5)	−339.88(6)	−348.73(6)	−357.79(6)	−367.08(7)	−376.61(7)	−386.37(7)
$2P_{3/2}$	−272.44(3)	−278.22(3)	−284.08(3)	−290.01(3)	−296.00(3)	−302.06(4)	−308.19(4)	−314.39(4)
$3S_{1/2}$	−137.31(2)	−140.73(2)	−144.22(3)	−147.78(3)	−151.42(3)	−155.14(3)	−158.94(3)	−162.82(3)
$3P_{1/2}$	−137.47(2)	−140.90(2)	−144.39(2)	−147.96(2)	−151.61(2)	−155.33(2)	−159.14(2)	−163.03(3)
$3P_{3/2}$	−122.450(9)	−125.08(1)	−127.75(1)	−130.44(1)	−133.17(1)	−135.93(1)	−138.73(1)	−141.56(1)
$3D_{3/2}$	−122.4696(5)	−125.1028(5)	−127.7681(5)	−130.4657(5)	−133.1957(6)	−135.9583(6)	−138.7539(6)	−141.5824(7)
$3D_{5/2}$	−118.8110(5)	−121.2849(5)	−123.7856(5)	−126.3129(5)	−128.8670(6)	−131.4480(6)	−134.0558(6)	−136.6905(7)