Multifactor productivity slips in the nonrubber footwear industry

While output per employee hour rose modestly from 1958 to 1986, multifactor productivity for this industry declined on average, more so in the period before 1973

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For many years, the Bureau of Labor Statistics has published, a labor productivity measure for the footwear industry termed output per employee hour.[1] Many factors influence movements in labor productivity, for example, technological change, changes in the skills and efforts of the work force, economies of scale, the amount of capital input per worker, and the amount of intermediate purchases input per worker. This article presents a supplementary productivity measure for the footwear industry—multifactor productivity—in which output is related to the combined inputs of labor, capital, and intermediate purchases. This measure differs from the traditional measure in that it accounts for the last two influences in the input measure and therefore does not reflect the impact of these influences in the productivity residual.

From 1958 to 1986, output per employee hour in the footwear industry rose at an average rate of 0.6 percent per year, well below the 2.5-percent rate for manufacturing as a whole. Multifactor productivity actually declined over the period by an average 0.9 percent per year. The rise in output per employee hour reflected changes in the contribution of capital per hour, of intermediate purchases per hour, and of other sources (multifactor productivity). The development of the multifactor productivity measure indicates that the low rate of growth in output per employee hour was caused not by declining amounts of capital or intermediate purchases available to labor over the period, but rather by the influence of other factors. The influence of capital, referred to here as the *capital effect*, is measured as the change in the capital-labor ratio multiplied by the share of capital income in the total output. The influence of intermediate purchases, referred to here as the *intermediate purchases effect*, is measured as the change in the intermediate purchases–labor ratio multiplied by the share of intermediate purchases in the total output. The capital effect showed an increase of 0.6 percent per year over the period 1958–86, while the intermediate purchases effect rose 0.9 percent. The decline in multifactor productivity was more than offset by these increases in the capital effect and intermediate purchases effect. Multifactor productivity suffered at least in part from a slow pace of development and diffusion of new technology in the industry.

Underlying the 0.9-percent annual decrease in multifactor productivity was an output decline of 3.0 percent

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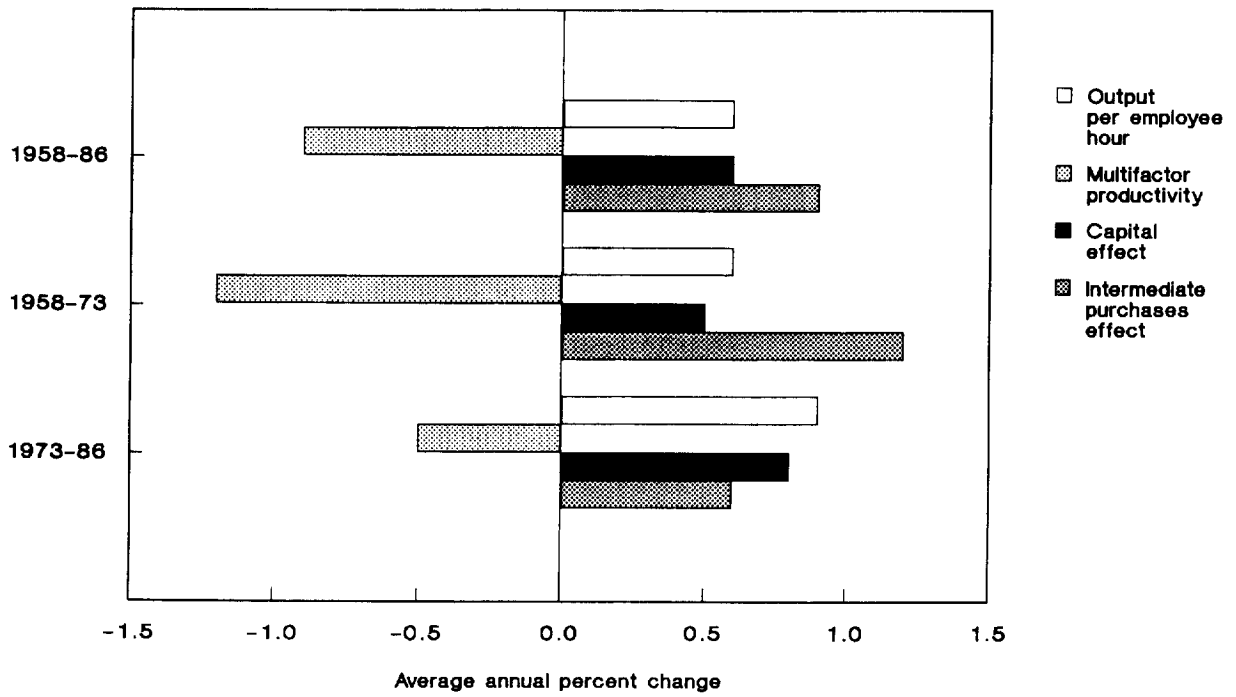
Chart 1. Output per employee hour rose in nonrubber footwear--despite multifactor productivity's fall--as capital and intermediate purchases increased relative to labor



per year and a 2.1-percent average annual drop in combined inputs. The decline in multifactor productivity slackened on average after 1973. (See table 1.) Although there have been year-to-year fluctuations, multifactor productivity fell at only a 0.5-percent rate after that year, compared with the 1.2-percent average rate of decrease prior to 1973. Output per employee hour also improved in the post-1973 period relative to the earlier period, but it was well below the manufacturing average for both periods.

Trends in the individual inputs varied considerably over the 1958–86 period. (See table 2.) While labor input dropped at a rate of 3.5 percent per year, capital input rose a scant 0.1 percent per year, and intermediate purchases declined at a 1.9-percent rate. Combined inputs, the weighted aggregate of these components, declined at a 2.1-percent rate per year. Thus, over the whole period, labor input showed the most rapid decline, followed by the lesser drop in intermediate purchases, while capital input showed a slight gain.

Although the growth in output per employee hour was well below average in the footwear industry over the period 1958–86, there was no post-1973 slowdown, as there was for manufacturing as a whole and most other industries. Output per employee hour in the manufacturing sector fell off from a rate of 2.9 percent in the period 1958–73 to a rate of 2.4 percent between 1973 and 1986. (It had declined to a rate of 1.6 percent in the period 1973–81, but began recovering in the mid-1980's.) For the footwear industry, output per employee hour actually accelerated somewhat from a rate of 0.6 percent in the earlier period to 0.9 percent in the post-1973 period.[2]

The relative contributions of the capital effect, the intermediate purchases effect, and multifactor productivity to changes in output per employee hour were not the same in the pre- and post-1973 periods. During the pre-1973 period, a 0.6-percent gain in output per employee hour was obtained from a 1.2-percent decline in multifactor productivity plus a 0.5-percent annual gain in the capital effect plus a 1.2-percent increase in the intermediate purchases effect. (Rounding produces the one-tenth of a point discrepancy when the addends are summed.) In the period 1973–86, a 0.9-percent average annual gain in output per employee hour was obtained from a 0.5-percent per year average drop in multifactor productivity plus a 0.8-percent rise in the capital effect plus a 0.6-percent increase in the intermediate purchases effect. (See chart 1.) Thus, in both periods output per employee hour recorded an increase, despite a decline in multifactor productivity, as a result of increases in the amount of capital per worker hour and intermediate purchases per worker hour.

For the period 1958–86 as a whole, the increase in the capital effect was 0.6 percent, resulting from a gain of 0.1 percent in capital input, while labor input was declining at a rate of 3.5 percent per year. Although the capital effect did not change much from the pre- to the post-1973 period (0.5 percent and 0.8 percent), the changes in the components differed. In the period 1958–73, capital input grew at a rate of 1.6 percent per year, while labor input declined by 1.7 percent per year. In the period 1973–86, capital fell by 1.5 percent per year, but labor fell even faster at a 5.3-percent rate.

The intermediate purchases effect showed a 0.9-percent gain over the period 1958–86. This increase resulted from a 1.9-percent decline per year in intermediate purchases, more than offset by a 3.5-percent average drop in hours. The intermediate purchases effect did fall off somewhat from a pre-1973 rate of 1.2 percent to a post-1973 rate of 0.6 percent. Underlying the pre-1973 growth was a 0.7-percent increase in intermediate purchases and a 1.7-percent drop in labor hours. The somewhat slower rate during 1973–86 resulted from a decline in intermediate purchases of 4.2 percent coming closer to matching a drop in labor of 5.3 percent than was the case in the earlier period.

Table 1. Multifactor and related productivity indexes, 1958–86

[1977=100]

Year	Multifactor productivity	Output per employee hour	Output per unit of capital	Output per unit of intermediate purchases
1958	113.0	86.2	150.0	123.1
1959	119.0	90.7	165.6	128.3
1960	116.0	89.4	156.3	125.2
1961	117.7	90.0	159.4	127.5
1962	118.0	90.7	159.7	127.4
1963	118.8	94.2	153.3	127.2
1964	117.7	94.0	153.7	124.7
1965	115.1	93.0	149.9	121.3
1966	115.9	94.6	150.4	121.3
1967	105.8	92.6	134.8	106.3
1968	108.3	95.9	139.8	107.7
1969	101.6	89.8	120.5	104.1
1970	103.7	96.6	118.7	103.8
1971	102.8	98.3	114.7	101.9
1972	101.5	95.9	115.6	100.8
1973	101.0	94.9	106.1	103.1
1974	97.8	93.8	98.2	100.0
1975	97.1	97.6	94.3	97.6
1976	99.4	98.2	101.8	99.3
1977	100.0	100.0	100.0	100.0
1978	101.6	101.8	100.0	101.9
1979	106.5	99.7	98.2	113.0
1980	99.2	98.0	94.6	101.0
1981	95.6	95.0	91.1	97.2
1982	100.3	106.0	88.1	101.6
1983	99.2	104.1	85.2	101.8
1984	97.6	105.0	77.9	101.6
1985	91.4	105.4	69.4	93.5
1986	91.2	107.4	64.7	94.3
	Average annual rates of change (percent)			
1958–86	-0.9	0.6	-3.0	-1.1
1958–73	-1.2	.6	-2.7	-1.8
1973–86	-.5	.9	-3.1	-.3

Technological change

Technological change has come slowly to the footwear industry. Automation of the industry on a mass scale has been hampered by a number of factors, including the high cost of the necessary equipment and the small size of most of the firms in the industry. In addition, for years the industry lacked a uniform last-grading system needed to facilitate the making of shoes in a wide variety of shoe lengths and widths. With the advent of affordable computers and computer-aided design, it is now possible to design a shoe pattern and grade it for production in different sizes and widths within hours. However, frequent style changes are a fact of life for the footwear industry, and difficulty in adapting the production equipment to these changes is still a problem. Moreover, frequently changing styles do not allow for the long production runs required to make the purchase of the equipment feasible. Accordingly, most of the improvements in technology have been of an incremental nature involving improvements in existing machine designs, and even these improvements have not spread rapidly throughout the industry. As a result, the footwear industry has remained very labor intensive.

Much of the technological change that occurred in the footwear industry during the late 1950's and 1960's was directed at reducing labor costs. This emphasis was strengthened as competition from low-cost imports rose. These imports benefit from low labor costs. For example, the introduction of injection molding made it possible to use liquid plastic to mold the upper material of the shoe onto the sole using no stitching and very little labor. Similarly, the process of affixing preformed soles and heels to uppers using cement also saved time and labor costs. This process was accompanied by an increase in the use of premolded "unit bottoms" purchased from outside the industry, thus saving further on labor costs to the industry.

Many of the technologies introduced during the 1970's and 1980's, along with the increased use of synthetic materials that coincided with the introduction of these new technologies, resulted in savings in both labor and intermediate purchases. Synthetic materials for shoe uppers, for example, were more uniform in weight and quality and could be cut in layers with automatic machinery. This saved time and labor costs and also reduced the amount of materials wastage. Similarly, the flow molding process, whereby designs are embossed onto a thermoplastic upper from a mold, reduced both labor and materials costs. The advent of computerized equipment has allowed even more savings: computer-controlled cutting and computer-controlled stitching, though not widespread in the industry, have tended to reduce the amount of labor time involved and the amount of damage done to materials. More recently, computer-aided design lets manufacturers respond rapidly to style changes by reducing the time

Table 2. Output and input indexes, 1958–86
[1977=100]

Year	Output	Combined inputs	Employee hours	Capital	Intermediate purchases
1958	134.9	119.4	156.5	89.9	109.6
1959	148.4	124.7	163.7	89.6	115.6
1960	141.0	121.6	157.8	90.2	112.6
1961	142.0	120.7	157.8	89.1	111.4
1962	144.2	122.1	159.0	90.3	113.1
1963	140.3	118.1	149.0	91.5	110.3
1964	143.4	121.9	152.6	93.3	114.9
1965	143.9	125.0	154.8	96.0	118.6
1966	148.3	128.0	156.8	98.6	122.3
1967	138.6	131.0	149.7	102.8	130.3
1968	147.4	136.0	153.7	105.4	136.9
1969	132.1	129.9	147.1	109.6	126.8
1970	130.1	125.5	134.7	109.6	125.4
1971	122.6	119.2	124.7	106.9	120.3
1972	121.5	119.7	126.6	105.1	120.5
1973	114.0	112.9	120.2	107.4	110.6
1974	105.8	108.2	112.8	107.8	105.8
1975	98.2	101.2	100.7	104.2	100.6
1976	101.7	102.3	103.5	99.9	102.4
1977	100.0	100.0	100.0	100.0	100.0
1978	99.5	97.9	97.7	99.5	97.6
1979	94.0	88.3	94.3	95.7	83.2
1980	90.3	91.1	92.2	95.5	89.4
1981	87.9	91.9	92.5	96.5	90.4
1982	86.3	86.0	81.4	97.9	84.9
1983	80.8	81.4	77.6	94.8	79.4
1984	71.8	73.5	68.3	92.2	70.7
1985	62.1	67.9	59.0	89.6	66.4
1986	55.3	60.6	51.5	85.4	58.6
	Average annual rates of change (percent)				
1958–86	-3.0	-2.1	-3.5	0.1	-1.9
1958–73	-1.2	.0	-1.7	1.6	.7
1973–86	-4.5	-4.0	-5.3	-1.5	-4.2

involved in designing and grading patterns. Also, the process allows the operator to adjust the pattern to maximize the amount of usable material.[3]

Output

Between 1958 and 1986, output of nonrubber footwear declined at an average annual rate of 3.0 percent. Although the industry attained slightly higher levels of output in 1959 and 1966 relative to 1968, there was a general, though slight, upward trend in output between 1958 and 1968. After 1968, output declined in every year (except for a small gain in 1976), falling to less than one-half the 1968 level in 1986.

Since the late 1950's, output of the U.S. footwear industry has been adversely affected by a variety of factors. In particular, competition from foreign manufacturers has eroded the U.S. industry's share of the total domestic consumption of footwear. Moreover, despite large increases in disposable income in the United States, per capita consumption of shoes has not increased substantially over the period. Imports of nonrubber footwear went from less than 27 million pairs in 1960 to more than 940 million pairs in 1986, a 35-fold increase. The ratio of imports to U.S. consumption of nonrubber shoes rose from 4 percent in 1960 to 80 percent in 1986, in quantity terms.[4] In value terms, however, imports account for a smaller proportion, almost 67 percent of U.S. consumption in 1986. These numbers reflect the continuing concentration of domestic production in a higher priced segment of the market.

The product mix of U.S. production of footwear has also changed. While U.S. production of all types of footwear has declined since the late 1950's, the contraction in output has been especially severe in women's and in misses' and children's shoes. Output of women's shoes fell about 70 percent between 1958 and 1986. Women's shoes made up more than 46 percent of all nonrubber footwear produced domestically in 1958; by 1986, the proportion had fallen to 34 percent. Production of misses' and children's shoes also declined dramatically, by about 87 percent between 1958 and 1986. The proportion of misses' and children's shoes to total nonrubber footwear fell from about 12 percent in 1958 to less than 4 percent in 1986.

Although output of men's shoes declined by almost 43 percent between 1958 and 1986, the rate at which production of men's shoes fell was slower than that for women's or for misses' and children's shoes. As a result, the proportion of men's shoes produced rose from 17 percent of all nonrubber footwear in 1958 to 24 percent in 1986.

Capital

Over the period 1958–86 as a whole, the flow of services from the capital stock in the industry rose slightly, by 0.1 percent per year on average. From 1958 to 1968, when output trended slightly upward, capital input increased at a 1.6-percent rate per year. From 1968 to 1986, when output was declining substantially, capital input fell, but at a much slower rate (–1.2 percent) than the drop in output.

Capital rose almost steadily, though rather slowly, reaching a peak in 1970, 22 percent above the 1958 level. Capital input then declined almost every year thereafter. This pattern was similar to that of output, but capital rose more than output in the earlier period and fell more slowly than output in the latter period.

Capital input includes the services in the production process yielded by the structures (mostly buildings) in which production takes place, the land on which the structures stand, the equipment used in producing output (both in direct production activities and in support activities), and the inventories of finished goods, work in process, and materials and supplies that the firm holds. These categories of capital input—structures, land, equipment, and inventories—did not always move together. In the period 1958–68, when total capital grew at a 1.6-percent average annual rate, equipment grew at almost the same rate (1.4 percent), but structures and land rose more slowly (both at 0.3 percent), while inventories increased 2.2 percent per year. In the period 1968–86, when total capital fell by 1.2 percent per year, structures and land continued to increase slightly (by 0.2 percent per year), while inventories dropped by a substantial 2.1

percent per year and equipment fell 0.9 percent per year.

Labor

Employment in the nonrubber footwear industry declined from 227,000 workers in 1958 to 75,900 workers in 1986. Footwear employment has responded closely to changes in output throughout the period, declining only slightly during the early to mid-1960's. Between 1958 and 1968, employment declined by 5.3 percent. From 1968 to 1986, however, employment fell 65 percent, or an average of 4.7 percent per year.

In many industries, there is a lag between the time that demand rises or falls off and the time that employee hours are increased or reduced. This lag occurs because it is difficult for managers to predict how long changes in demand will last, and in many cases employers are reluctant to lay off skilled personnel because it can be costly to rehire them or train new personnel when demand rises again. As can be seen in table 3, in most years the declines (gains) in output after 1968 were matched quickly by reductions (increases) in employee hours. The reductions in employee hours occurred because of both layoffs at existing plants and plant closures over the period. From 1967 to 1982, the number of footwear establishments declined from 1,083 to 751, a loss of more than 20 plants per year on average. Since 1982, plant closures have continued.

Intermediate purchases

Intermediate purchases consist of the raw materials, energy (purchased fuels and electricity), and purchased services used in the production of the industry's output. Materials constitute more than 80 percent of the value of intermediate purchases for the nonrubber footwear industry, and by far the largest component of materials for this industry is leather. Intermediate purchases declined at an average rate of 1.9 percent per year between 1958 and 1986. However, in the earlier part of the period, from 1958 to 1968, intermediate purchases rose at a relatively rapid 1.8-percent rate per year on average. In comparison, output increased at an average annual rate of only 0.3 percent during that period. From 1968 to 1973, intermediate purchases fell at an average annual rate of 3.5 percent per year, more closely in line with the rate at which output fell (−4.4 percent). As a result, the rate of decline in the productivity of intermediate purchases, that is, the ratio of output to intermediate purchases, which fell by 1.5 percent per year from 1958 to 1968, eased to a decrease of 0.9 percent per year between 1968 and 1973, and to 0.3 percent after 1973.

Despite some year-to-year volatility in leather prices, the period 1958–68 was one of moderate price increases in intermediate purchases. Between 1958 and 1968, prices of intermediate purchases increased by about 0.9 percent per year on average. In contrast, the later period was characterized by much larger price increases in both leather and petrochemical-based inputs (affecting many synthetic materials). Between 1968 and 1986, intermediate purchases prices were increasing at an average annual rate of 7.7 percent.

Table 3. Annual percent changes in output and employee hours in the footwear industry, 1968–86

Year	Output	Employee hours
1968–69	−10.4	−4.3
1969–70	−1.5	−8.4
1970–71	−5.8	−7.4
1971–72	−.9	1.5
1972–73	−6.2	−5.1
1973–74	−7.2	−6.2
1974–75	−7.2	−10.7
1975–76	3.6	2.8
1976–77	−1.7	−3.4
1977–78	−.5	−2.3
1978–79	−5.5	−3.5
1979–80	−3.9	−2.2
1980–81	−2.7	.3
1981–82	−1.8	−12.0
1982–83	−6.4	−4.7
1983–84	−11.1	−12.0
1984–85	−13.5	−13.6
1985–86	−11.0	−12.7

The more rapidly rising intermediate purchases prices after 1968 provided an incentive for manufacturers to find ways of conserving on intermediate purchases consumption. For example, during the 1960's footwear manufacturers shifted to using more synthetic materials. These synthetic materials, such as plastic, vinyl, and other poromeric materials, are more uniform in weight and quality and therefore allow less wastage. Tanners responded by supplying leathers that were more uniform than before, with the less desirable parts removed. Improvements in cutting, such as the use of laser technology, water-jet cutting, and piecework systems, were introduced to reduce wastage also. Other technological changes, such as the numerically controlled upper roughing machines used for roughing the leather, also reduced damage to materials.

Summary

Output per employee hour in the footwear industry over the period 1958–86 grew only 0.6 percent per year. This low rate of growth reflected increases in the amount of intermediate purchases and capital relative to labor, offsetting a decline of 0.9 percent per year in multifactor productivity. The decline in multifactor productivity was concentrated in the pre-1973 period; multifactor productivity declined at a slower rate on average during the post-1973 period. Output per employee hour did not slow down after 1973, but it was still well below the manufacturing average in both the pre- and post-1973 periods.

Domestic production of footwear has fallen by more than half since the late 1960's, as imports have risen rapidly since that time. Productivity growth in the industry has been hampered partly by a slow pace of technological change and a slow rate of adoption of whatever new technology has been introduced.

FOOTNOTES

[1]This labor productivity measure was introduced by the Bureau in July 1965 in *Indexes of Output per Man-hour 1949–63*.

[2]The conclusion that labor productivity in this industry experienced no slowdown in the 1970's and early 1980's holds regardless of the choice of initial and terminal years. With 1973 as the breakpoint, none of the growth rates ending in 1978 or later is significantly below any of the rates beginning in 1965 or earlier.

[3]For further examination of the changes in technology in the footwear industry, see *Technology and Its Impact on Labor in Four Industries*, Bulletin 2263 (Bureau of Labor Statistics, November 1986).

[4]*Current Industrial Reports*, Series MA31A, U.S. Department of Commerce, various issues.

APPENDIX: Multifactor productivity measurement

Methodology and data definitions

The following is a brief summary of the methods and data underlying the multifactor productivity measure for the footwear industry. A technical note, describing the procedures and data in more detail, is available from the authors at the Office of Productivity and Technology, Bureau of Labor Statistics, Washington, DC 20212.

Output. The output measure for the footwear industry is based on the weighted change in the quantity of production of eight types of shoes and slippers as reported in the Bureau of the Census' Current Industrial Reports, series MA31A. The weights are computed as the share obtained by each type of shoe in the total value of production of all nonrubber footwear.

For multifactor measures for individual industries, output is defined as total production, rather than the alternative of value added. For a value-added measure, intermediate inputs are subtracted from total production. Consequently, an important difference between the multifactor productivity measures that BLS publishes for individual industries and those for aggregate sectors of the economy is that the latter measures are constructed within a value-added framework. For the major sectors of the economy, intermediate transactions tend to cancel out; intermediate inputs are much more important in production at the industry level.

Further, output in industry measures is defined as total production which "leaves" an industry in a given year in the form of shipments plus net changes in inventories of finished goods and work in process. Shipments to other establishments within the same industry are excluded because they represent double counting, which distorts the productivity measures.

Labor. Employee-hour indexes, which represent the labor input, measure the aggregate number of employee hours. These hours are the sum of production worker hours, from *Censuses of Manufactures* and *Annual Surveys of Manufactures* (U.S. Department of Commerce), and nonproduction worker hours, derived by multiplying the number of nonproduction workers from the Census publication by an estimate of nonproduction worker average annual hours. The labor input data are the same as those used in the published BLS output per hour series.

Capital. A broad definition of capital input, including equipment, structures, land, and inventories, is used to measure the flow of services derived from the stock of physical assets. Financial assets are not included.

For productivity measurement, the appropriate concept of capital is "productive" capital stock, which represents the stock used to produce the capital services employed in current production. To measure the productive stock, it is necessary, for each type of asset, to take account of the loss of efficiency of the asset as it ages. That is, assets of different vintages have to be aggregated. For the measures in this article, a concave form of the age/efficiency pattern (slower declining efficiency during earlier years) is chosen.

In combining the various types of capital stock, the weights applied are implicit rental prices of each type of asset. They reflect the implicit rate of return to capital, the rate of depreciation, capital gains, and taxes. (For an extensive discussion of capital measurement, see *Trends in Multifactor Productivity, 1948–81*, Bulletin 2178 (Bureau of Labor Statistics, 1983).)

Intermediate purchases. Intermediate purchases primarily include materials, fuels, electricity, and purchased business services. Materials measured in real terms refer to items consumed or put into production during the year. Freight charges and other direct charges incurred by the establishment in acquiring these materials are also included. The data from which the intermediate inputs are derived include all purchased materials and fuels regardless of whether they were purchased by the individual establishment from other companies, transferred to it from other establishments of the same company, or withdrawn from inventory during the year. An estimate of intraindustry transactions is removed from materials and fuels data.

Annual estimates of the cost of services purchased from other business firms are also required for multifactor productivity measurement in a total output framework. Some examples of such services are legal services, communica-

tions services, and repair of machinery. An estimate of the constant-dollar cost of these services is included in the intermediate purchases input.

Capital, labor, and intermediate purchases income shares. Weights are needed to combine the indexes of the major inputs into a combined input measure. The weights for the footwear industry are derived in two steps: first, an estimate of income in current dollars for each input is derived, and then the income of each input is divided by the total income of all inputs.

Conceptual framework

The multifactor productivity measure presented here is computed by dividing an index of output by an index of combined inputs of capital, labor, and intermediate purchases. The framework for measurement is a production function describing the relation of output and inputs and an index formula that is consistent with this production function.

The general form of the production function underlying the multifactor productivity measures is postulated to be

$$(1) \qquad Q(t) = Q(K(t), L(t), M(t), t),$$

where $Q(t)$ is total output, $K(t)$ is input of capital services, $L(t)$ is input of labor services, $M(t)$ is input of intermediate purchases, and t is time.

Differentiating equation *(1)* with respect to time, and with some algebraic manipulations, the sources-of-growth equation is,

$$(2) \qquad \frac{\dot{Q}}{Q} = A + w_k \frac{\dot{K}}{K} + w_l \frac{\dot{L}}{L} + w_m \frac{\dot{M}}{M},$$

where A is the rate of change of multifactor productivity, w_k is output elasticity (percentage change in output due to a 1-percent change in input) with respect to the capital input, w_l is output elasticity with respect to the labor input, and w_m is output elasticity with respect to the intermediate purchases input. (The dot over a variable indicates the derivative of the variable with respect to time.)

Equation *(2)* shows the rate of change of output as the sum of the rate of change of multifactor productivity and a weighted average of rates of change of capital, labor, and intermediate purchases inputs. Now, if competitive input markets are assumed, then each input is paid the value of its marginal product. The output elasticities in equation *(2)* can then be replaced by the factor income shares,

$$w_k = \frac{P_k K}{P_q Q},$$

$$w_l = \frac{P_l L}{P_q Q}, \text{ and}$$

$$w_m = \frac{P_m M}{P_q Q},$$

where P_q is the price of output and P_k, P_l, and P_m are the prices paid for the capital, labor, and intermediate purchases inputs, respectively. Furthermore, if constant returns to scale are assumed, then $w_k + w_l + w_m = 1$.

Equation *(2)* can be rewritten as

$$(3) \qquad A = \frac{\dot{Q}}{Q} - w_k \frac{\dot{K}}{K} - w_l \frac{\dot{L}}{L} - w_m \frac{\dot{M}}{M}$$

In this expression, the growth of multifactor productivity can be seen as a measure of economic progress: it measures the increase in output over and above the gain due to increases in inputs.

Equation *(2)* can also be transformed into a contribution equation which allows for an analysis of the change in output per employee hour. First, subtract $\dot{L}/L$ from both sides of equation *(2)*. Because the weights sum to unity, apply the term $(w_k + w_l + w_m)$ to the $\dot{L}/L$ term inserted on the right-hand side. Next, gather terms with the same weight and derive the following equation:

$$(4) \qquad \frac{\dot{Q}}{Q} - \frac{\dot{L}}{L} = w_k\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right) + w_m\left(\frac{\dot{M}}{M} - \frac{\dot{L}}{L}\right) + A$$

The left side of equation *(4)* is the growth rate of output per employee hour. The terms in parentheses on the right side are, in order, the rates of change in the capital-labor ratio and the intermediate purchases-labor ratio. Thus, the rate of growth in output per employee hour can be decomposed into the weighted sum of changes in these ratios plus the change in multifactor productivity.

Equations *(2)*, *(3)*, and *(4)* are Divisia indexes which require continuous data for computation. The BLS multifactor indexes are actually constructed according to a Tornqvist formula which represents a discrete approximation to the Divisia index. The rate of change in output or an input is calculated as the difference from one period to the next in the natural logarithms of the variables. For example, $\dot{Q}/Q$ is calculated as $ln\ Q(t) - ln\ Q(t-1)$. Indexes are then constructed from the antilogarithms of this differential. The weights w_k, w_l, and w_m are calculated as the arithmetic averages of the respective shares in time periods t and $t-1$.