Multi-Moment Spatial Analysis of Violence

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Introduction and background

- Count Outcome Examples:
  - Number of Job Changes, Ship Accidents, Strikes, Patents Filed, Crimes Recorded, etc.
- Count Outcome/Model Peculiarities:
  - Low non-negative integers (0,1,2,3,…)
  - Discrete/Skewed
  - May exhibit extra-Poisson variation
  - Preponderance of one or more counts (e.g., 0)
- Correlation Across Space
  - Can be substantive or a nuisance
  - Usually contaminates inferences
  - Particularly difficult to deal with in non-linear models

Overview of this presentation

- What are moments of a distribution?
- Derive a Multi-Moment Generalized Poisson Model:
  - Amount of violence
  - Predictability of violence
  - Mere occurrence of a violent event
  - Substantive spatial structure
- Simulated evidence
- Homicides in Chicago neighborhoods
- Model implications
- Discussion

Distributional Moments

- If x_n are a sample of some random variable, then its
  - r[th] *raw* moments is defined as: $\kappa_r = \frac{1}{N}\sum_n x_n^r$
  - r[th] *central* moments is defined as: $\mu_r = \frac{1}{N}\sum_n (x_n - \mu_1)^r$
  - r[th] *geometric* moments is defined as: $\gamma_r = \frac{1}{N}\sum_n (\log x_n)^r$
- Generically, we can define moments as the mean of various transformations of the original variable x_n

$$\eta_r = \frac{1}{N}\sum_n \psi_r(x_n)$$

Why Study Multiple Moments?

- Typically researchers study only the 1^{st} moment: the expected value. But what about …
  - the expected variance, skewness, etc.
- Example from Financial Econometrics (returns on investment). Investors prefer …
  - Higher returns (i.e., higher mean return)
  - More certain return (i.e., more predictable returns)
  - Left Skewed returns (i.e., surprise gain better than surprise loss)
- Different moments capture different aspects of the phenomenon under study

Setting up the generic problem - I

- *N* Observed Outcomes: $\mathbf{y} = (y_1, \ldots, y_N)$
- *N* Emitted Signals: $\mathbf{s} = (s_1, \ldots, s_N)$
- Approximate relationship at the individual level

$$y_n \approx s_n \qquad \forall n$$

- Partial guidance from theory: *the presence or magnitude of some attributes induce variation in the signals.* I.e., suggests relevance of attributes.

Qualitative Relevance

$$0 \neq \sum_n x_{kn} s_n$$

Quantitative Relevance

$$\sum_n x_{kn} y_n = \sum_n x_{kn} s_n$$

Setting up the generic problem - II

- *Step 1*: Re-parameterize unknown signals into proper probabilities (as weighted sum of propositions)

$$s_n = \mathbf{z}'\mathbf{p}_n = \sum_m z_m p_{mn} \qquad \sum_m p_{mn} = 1, \forall n \qquad (1)$$

- *Step 2*: Quantitative relevance of attributes yield constraints on the probabilities.

$$\sum_n x_{kn} y_n = \sum_n x_{kn} \mathbf{z}'\mathbf{p}_n \qquad \forall k \qquad (2)$$

- *Result*: Ill-posed inversion problem (infinite solutions)

Setting up the generic problem - III

- *IT Solution*: Choose most uncertain / least informative solution using Maximum Entropy (Ed Jaynes)

$$\max_{\mathbf{p}} H = -\sum_n \mathbf{p}_n' \log \mathbf{p}_n \quad \text{s.t. (1) \& (2)}$$

- If corresponding priors ($\mathbf{p}^0$) exist, choose solution that minimizes Cross Entropy

$$\min_{\mathbf{p}} CE = \sum_n \mathbf{p}_n' \log(\mathbf{p}_n / \mathbf{p}_n^0) \quad \text{s.t. (1) \& (2)}$$

- If priors are uniform, both problems are identical

Setting up the generic problem - IV

- Primal *Constrained* Optimization Problem

$$L = \sum_n \mathbf{p}_n' \log(\mathbf{p}_n / \mathbf{p}_n^0) + \sum_n \eta_n (1 - \mathbf{1}'\mathbf{p}_n) + \sum_k \lambda_k \left(\sum_n x_{kn} y_n - \sum_n x_{kn} \mathbf{z}'\mathbf{p}_n \right)$$

- Optimal Solution

$$p_{mn} = \frac{p_m^0 \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda})}{\sum_m p_m^0 \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda})} = \frac{p_m^0 \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda})}{\Omega_n}$$

- Resulting Dual *Unconstrained* Optimization Problem

$$L_* = \sum_{kn} x_{kn} y_n \lambda_k - \sum_n \log \boldsymbol{\Omega}_n$$

The Poisson model derived

- Define Support Space as: $\mathbf{z} = (0,1,2,\ldots,z_M)$
- Assume Priors are: $p_m^0 = {}^{1}\!/_{z_m!}$
- The solution is …

$$p_{mn} = \frac{(1/z_m!)\exp(z_m \mathbf{x}_n' \boldsymbol{\lambda})}{\sum_m (1/z_m!)\exp(z_m \mathbf{x}_n' \boldsymbol{\lambda})} = \frac{(1/z_m!)\exp(z_m \mathbf{x}_n' \boldsymbol{\lambda})}{\exp(\exp(\mathbf{x}_n' \boldsymbol{\lambda}))}$$

$$= \frac{\exp(-\exp(\mathbf{x}_n' \boldsymbol{\lambda}))\exp(\mathbf{x}_n' \boldsymbol{\lambda})^{z_m}}{z_m!} = \frac{\exp(-\alpha_n)\alpha_n^{\ z_m}}{z_m!}$$

- … the Poisson model with a log-link function
- But, the Poisson model could be mis-specified …

Generalized Poisson Model - I

- … potentially mis-specified Poisson model means potentially incorrect priors $1/z_m!$.
- Solution: Parameterize dependence of p_{mn} on priors.
- Re-write solution as:

$$\begin{aligned} p_{mn} &= \boldsymbol{\Omega}^{-1} z_m!^{-1} \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda}) \\ &= \Omega^{-1} z_m!^{-1+\delta} \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda}) \\ &= \Omega^{-1} z_m!^{-1} \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda} + \log z_m! \delta) \end{aligned}$$

- Generalized Poisson: replace δ by $\mathbf{x}_n' \boldsymbol{\beta}$.

$$p_{mn} = \Omega^{-1} z_m!^{-1} \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda} + \log z_m! \mathbf{x}_n' \boldsymbol{\beta})$$

Generalized Poisson Model - II

- How to obtain this generalized solution?
- Simultaneously impose constraints

$$\sum_n x_{kn} y_n = \sum_{nm} x_{kn} z_m p_{mn} \qquad \forall k$$

$$\sum_n x_{kn} \log(y_n !) = \sum_{nm} x_{kn} \log(z_m !) p_{mn} \qquad \forall k$$

- Which yields the desired solution

$$p_{mn} = \Omega^{-1} z_m !^{-1} \exp(z_m \mathbf{x}_n' \boldsymbol{\lambda} + \log z_m ! \mathbf{x}_n' \boldsymbol{\beta})$$

- More generally, impose multi-moment constraints

$$\sum_n x_{kn} \psi_j (y_n) = \sum_{nm} x_{kn} \psi_j (z_m) p_{mn} \qquad \forall k, j$$

Spatial Structure in the Outcomes

- Spatial Autocorrelation: Standard Poisson process

$$\left.\begin{aligned} \log \boldsymbol{\alpha} &= \mathbf{X}\boldsymbol{\lambda} + \rho \mathbf{W} \log \boldsymbol{\alpha} \\ \log \boldsymbol{\alpha} &= (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X}\boldsymbol{\lambda} \\ \log \boldsymbol{\alpha} &= \tilde{\mathbf{X}}\boldsymbol{\lambda} \end{aligned}\right\} \quad \mathbf{y} \sim \text{Poisson}(\boldsymbol{\alpha})$$

- Modified constraints needed

$$\sum_{n} \tilde{x}_{kn} y_n = \sum_{nm} \tilde{x}_{kn} z_m p_{nm}$$

- Generalized Poisson process with spatial autocorrelation among multiple moments

$$\sum_{n} \tilde{x}_{jkn} \psi_j(y_n) = \sum_{nm} \tilde{x}_{jkn} \psi_j(z_m) p_{nm}$$

Testing Hypothesis and Specifications

- Asymptotic Covariance of parameters: approximated by the negative inverted Hessian of the dual objective function:

$$\mathrm{Cov}(\boldsymbol{\theta}) = \left\{ -\frac{\partial^2 L_*}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \right\}^{-1} \quad \text{where} \quad \boldsymbol{\theta} = (\boldsymbol{\lambda}, \boldsymbol{\beta}, \boldsymbol{\rho})'$$

- Nested Specifications: Entropy Ratio Statistic

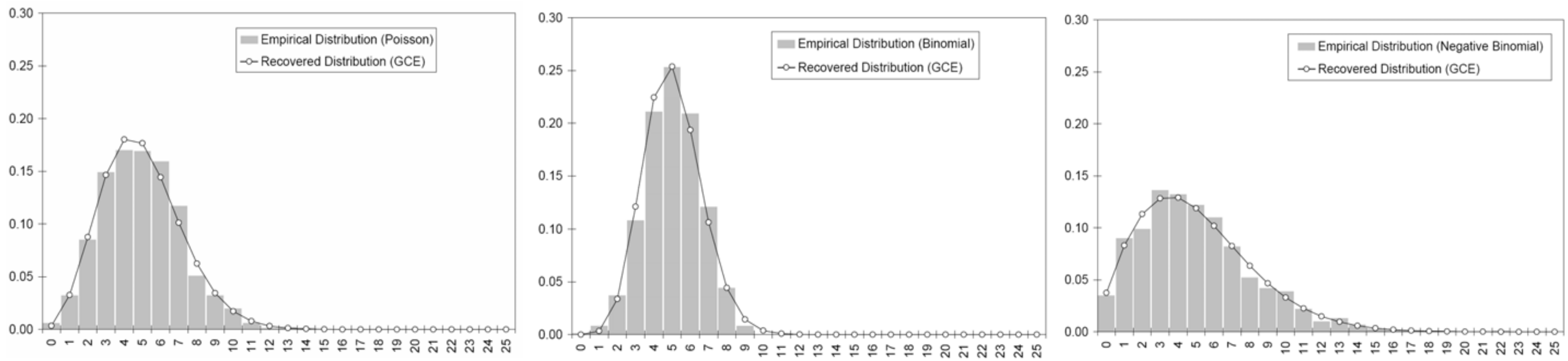
$$ER = 2 \times \left(L_{*(U)} - L_{*(R)} \right) \rightarrow \chi^2_{(R)}$$

- Can always use Huber-White Robust Standard Errors (is we suspect remaining structure in errors)

Yeah! But does it really work …

A Single Randomly Generated Sample

- Does the Generalized Poisson Model recover over-dispersion or under-dispersion in the data?



Some Simulation Results

1. How does the *ER* statistic used for testing unobserved heterogeneity perform?
2. Consequences of ignoring unobserved heterogeneity (with and without nuisance spatial autocorrelation in this heterogeneity)
   - Over rejection under the null, i.e., $\lambda_k = 0$ or $\rho = 0$
   - Does the Generalized Poisson model do better?

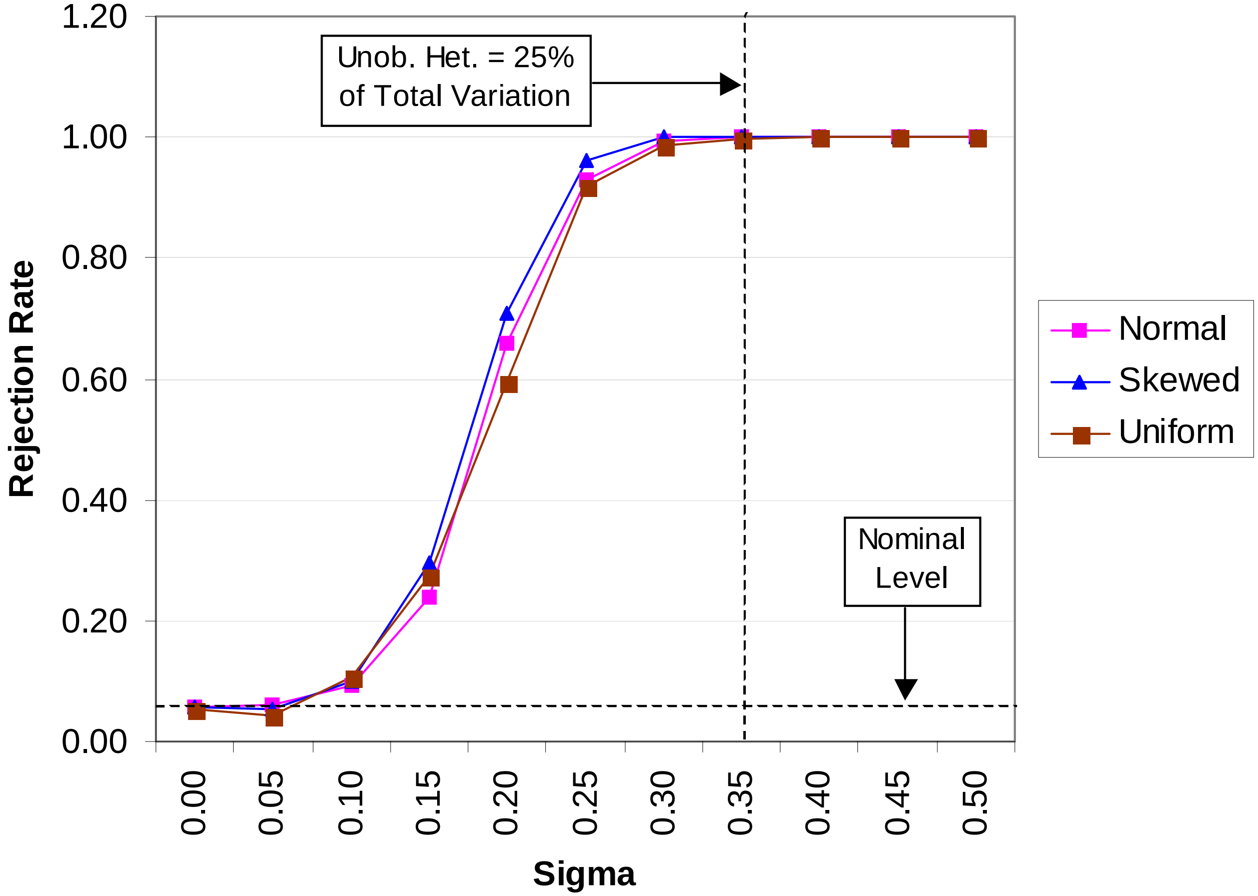
- Simulation design: $y_n \sim \text{Po}(\alpha_n)$ *N*=343

where $$\log \alpha_n = \underbrace{\beta_0 + \beta_1 \times N(0,1)}_{\text{fixed across repeated samples}} + e_n \qquad \beta_0 = \beta_1 = 1$$

with varying form and structure in e_n across 500 reps.

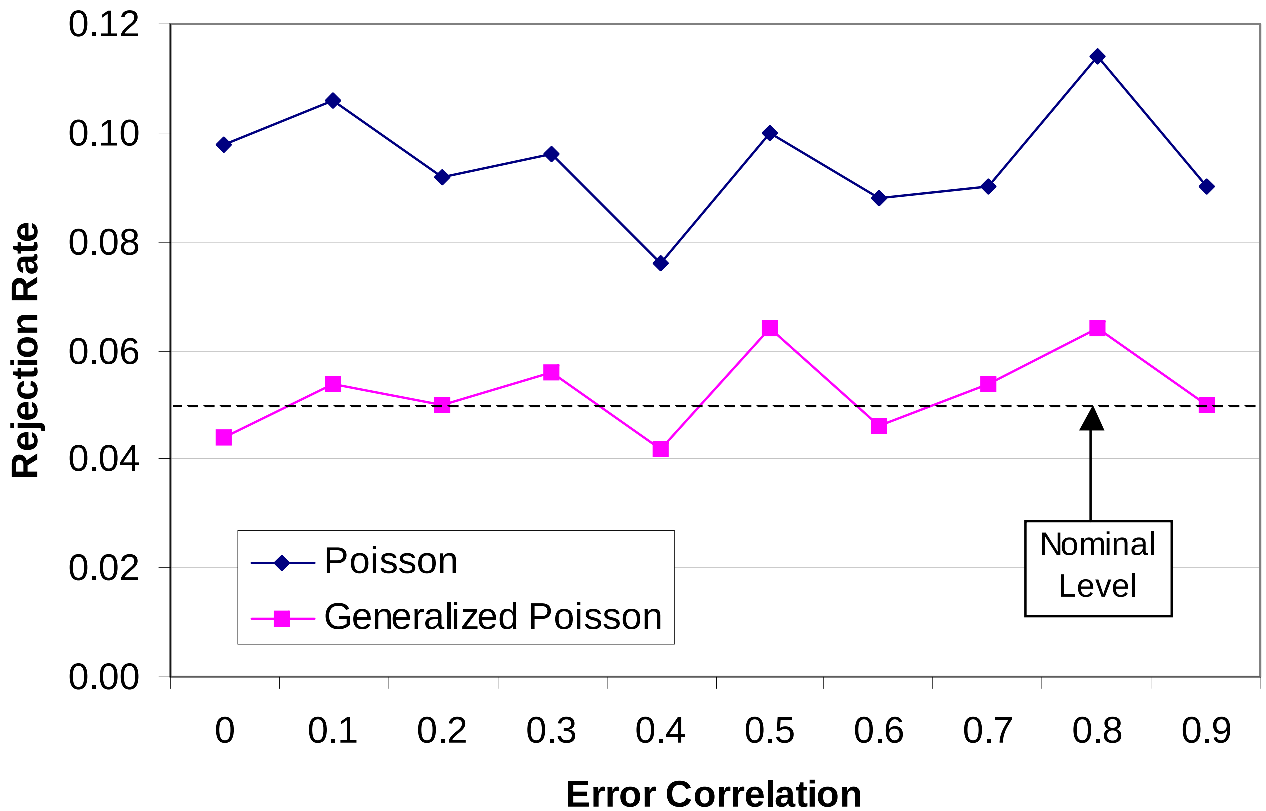
Simulation Results – I
ER Test for Unobserved Heterogeneity

- DGP $e_n = \sigma \times u_n$



Simulation Results – II
Wald Test for Lagrange Multiplier

- DGP $\beta_1 = 0$ with $e_n = 0.35 \times \underbrace{(\mathbf{I} - \rho_e \mathbf{W})^{-1} N(0,1)}_{\text{standardized}}$



OK! So, maybe it works with simulated data …

What about the real world … ?

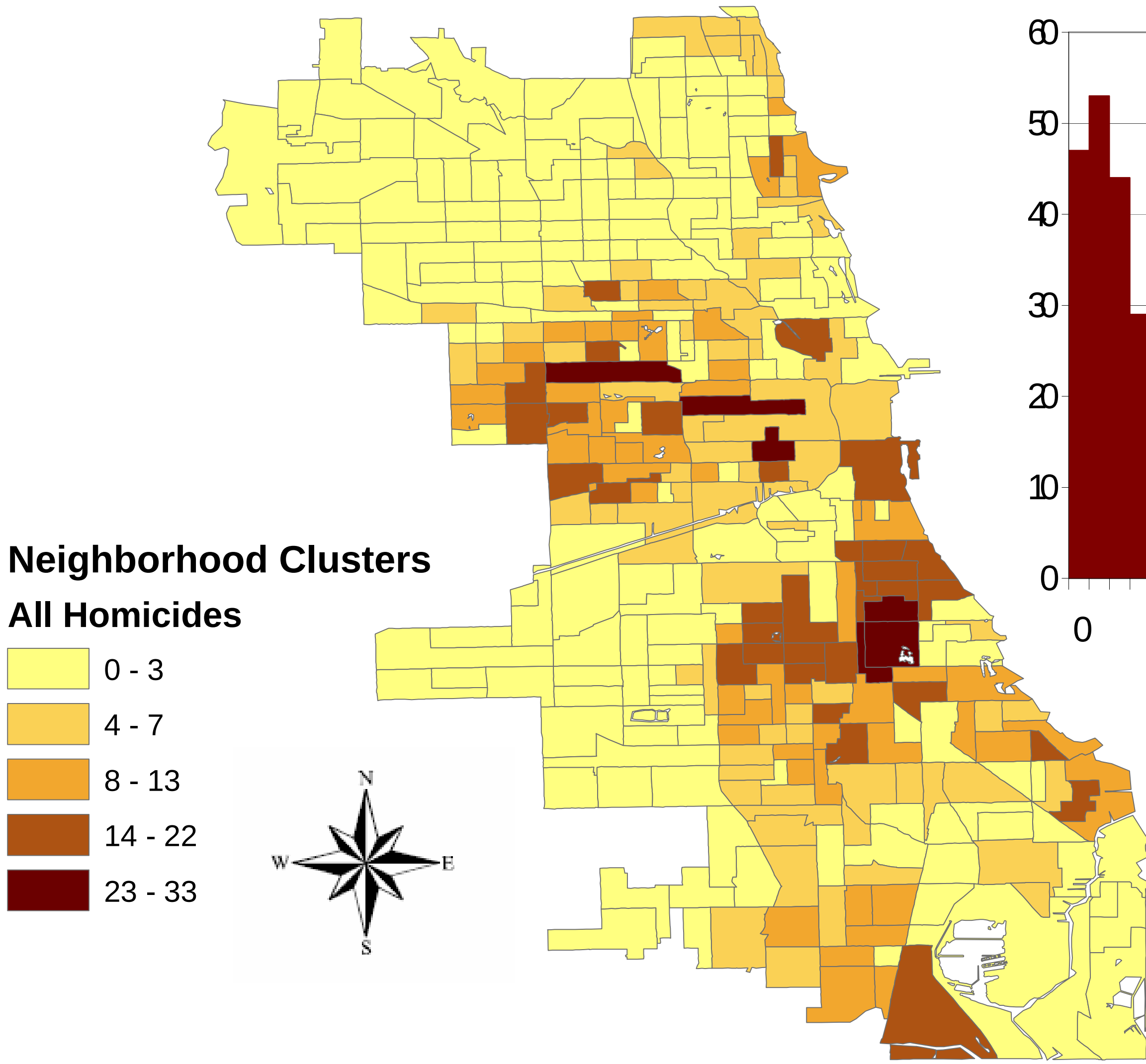
Application: Homicide Counts in Chicago Neighborhoods (1989-1990)

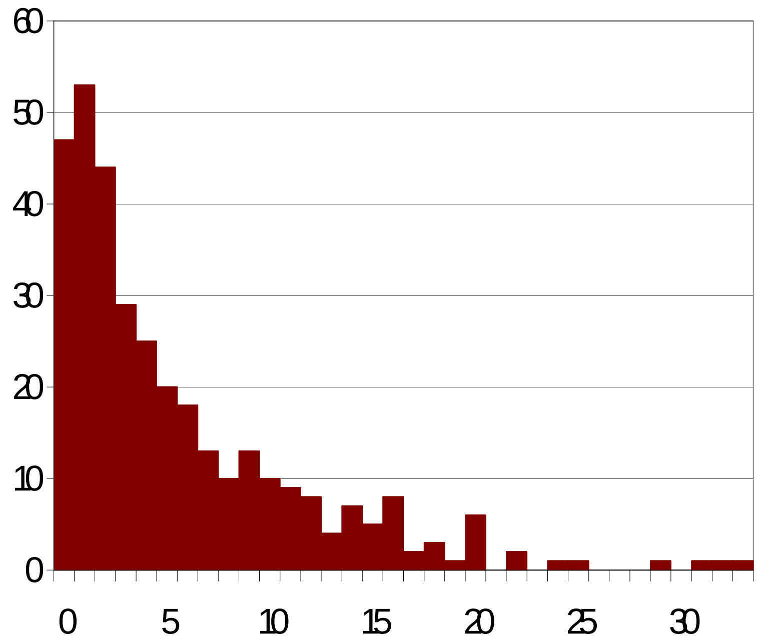
- Counts of homicides recorded in each of 343 Chicago neighborhood (PHDCN) between 1989-1991.

- Explanatory variables include (all census)
  - **LPOP**: Natural log of residential population (scale)
  - **RESDEP**: Resource depravation index (measuring concentrated disadvantage)
  - **RESST**: Percent of neighborhood households where the head of household has lived for more than 5 years (measuring residential stability)
  - **YMEN**: Young men as a % of population
  - **PNFH**: % of Non-family households

- Spatial Link Matrix: First Order Queen Criterion

Observed Spatial Distribution of the Homicide Count





Results:
All Homicides, Various Models

	Poisson	Negative Binomial	Zero-Inflated Generalized Poisson Model			Spatial Zero-Inflated Generalized Poisson Model		
			Amount of Violence	Predictability of Violence	Some Violence	Amount of Violence	Predictability of Violence	Some Violence
Intercept	-7.800 *	-8.380 *	-4.887 *	0.718	-7.373	-5.786 *	0.643	-1.424
LPOP	0.953 *	0.973 *	0.465 *	0.034	1.040	0.603 *	-0.012	0.342
RESDEP	0.714 *	0.816 *	0.841 *	-0.201 *	1.577 *	0.950 *	-0.244 *	0.725
RESST	0.151 *	0.198 *	0.127	-0.017	0.268	0.061	0.015	-0.044
YMEN	0.554 *	0.868 *	0.960 *	-0.341 *	-1.017	0.869 *	-0.319 *	-0.170
PNFH	0.124	0.238	0.384	-0.152	1.259	-0.156	0.093	2.389
Scale	…	0.171 *	…	…	…	…	…	…
ρ	…	…	…	…	…	0.211 *	0.467 *	-0.710
R^2	61%	59%			72%			74%

* = $p < 0.05$; + = $p < 0.1$

Results:
Expressive Homicides, Various Models

	Poisson	Negative Binomial	Zero-Inflated Generalized Poisson Model			Spatial Zero-Inflated Generalized Poisson Model		
			Amount of Violence	Predictability of Violence	Some Violence	Amount of Violence	Predictability of Violence	Some Violence
Intercept	-9.085 *	-9.298 *	-6.674 *	0.740	-1.390	-7.059 *	0.747	0.326
LPOP	1.076 *	1.057 *	0.609 *	0.055	0.467	0.661 *	0.035	0.278
RESDEP	0.825 *	0.938 *	0.928	-0.214 *	1.290 *	0.952 *	-0.227 *	1.283 *
RESST	0.028	0.059	-0.153	0.081	0.949	-0.147	0.081	0.956
YMEN	0.307 *	0.594 *	1.747 *	-0.816 *	-3.902 *	1.606 *	-0.760 *	-3.461 *
PNFH	0.192	0.355	0.838	-0.369	-0.732	0.867	-0.369	-1.042
Scale	…	0.178 *	…	…	…	…	…	…
ρ	…	…	…	…	…	0.084	0.218	-0.251
R^2	60%	58%			72%			73%

* = $p < 0.05$; + = $p < 0.1$

Results: Instrumental Homicides, Various Models

	Poisson	Negative Binomial	Zero-Inflated Generalized Poisson Model			Spatial Zero-Inflated Generalized Poisson Model		
			Amount of Violence	Predictability of Violence	Some Violence	Amount of Violence	Predictability of Violence	Some Violence
Intercept	-8.691 *	-8.506 *	-4.122	-2.271	-0.885	-3.372	-3.779 *	-1.077
LPOP	0.902 *	0.862 *	0.429	0.294	-0.058	0.417 +	0.406 *	-0.031
RESDEP	0.787 *	0.855 *	0.749 *	-0.179	0.237	0.260 *	0.263 *	0.794 *
RESST	-0.133	-0.147	-0.084	0.048	-0.307	0.239	-0.187	-0.687 +
YMEN	0.697 *	0.865 *	0.356	-0.120	1.271	0.100 +	0.215	1.510 +
PNFH	0.349	0.468	-0.573	0.418	1.941	-0.642 *	0.790 *	1.677
Scale	…	0.243 *	…	…	…	…	…	…
ρ	…	…	…	…	…	-0.782 *	0.955 *	0.320
R^2	42%	42%			45%			51%

* = p < 0.05; + = p < 0.1

Interesting Model Implications

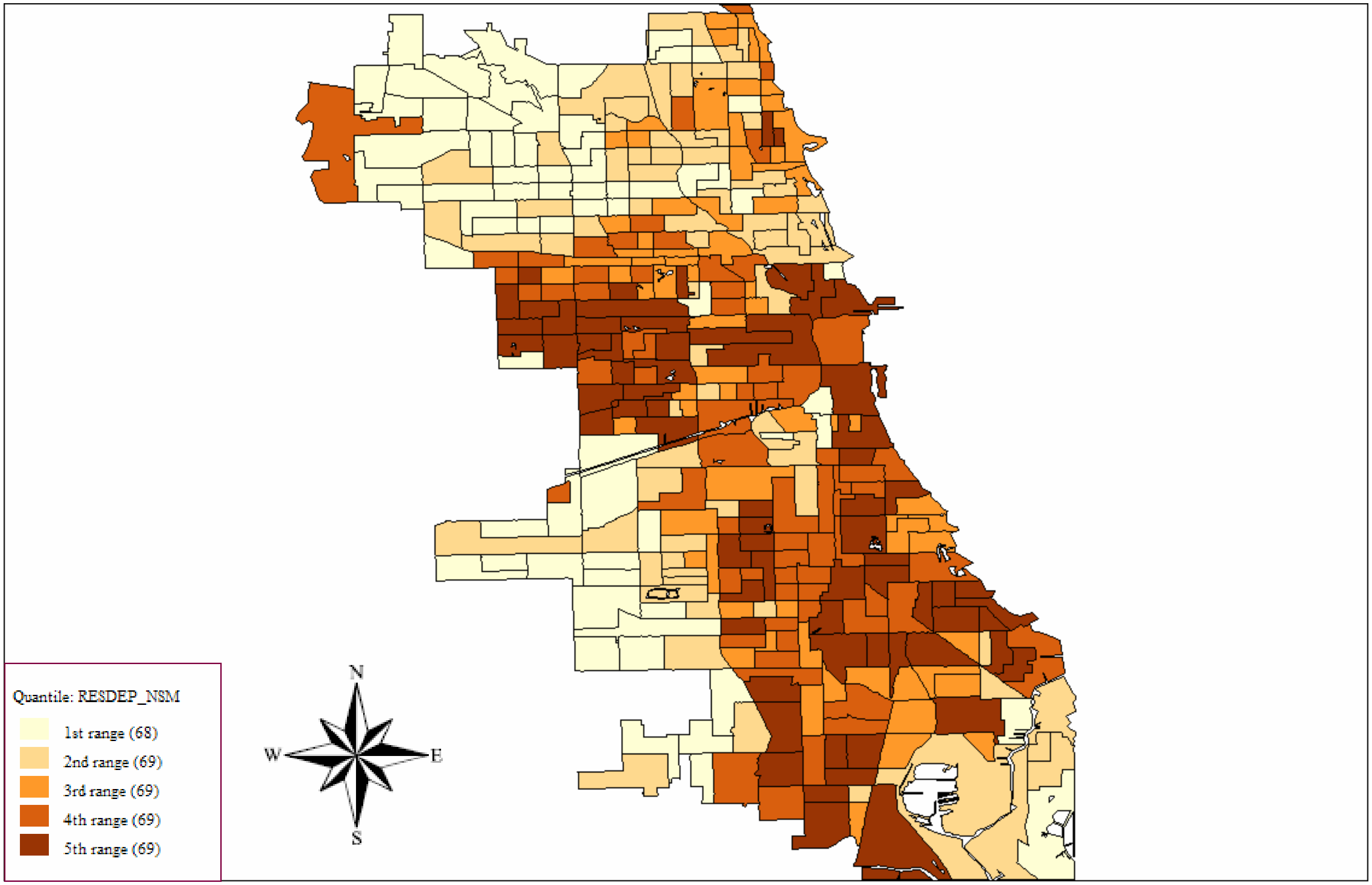
- Tease out decomposition of marginal effects

$$\frac{\partial \ln(\hat{s}_n)}{\partial x_{kn}} = \frac{\hat{\phi}_n(1,1)}{\hat{s}_n}\lambda_{k1} + \frac{\hat{\phi}_n(1,2)}{\hat{s}_n}\lambda_{k2} + \frac{\hat{\phi}_n(1,3)}{\hat{s}_n}\lambda_{k3}$$

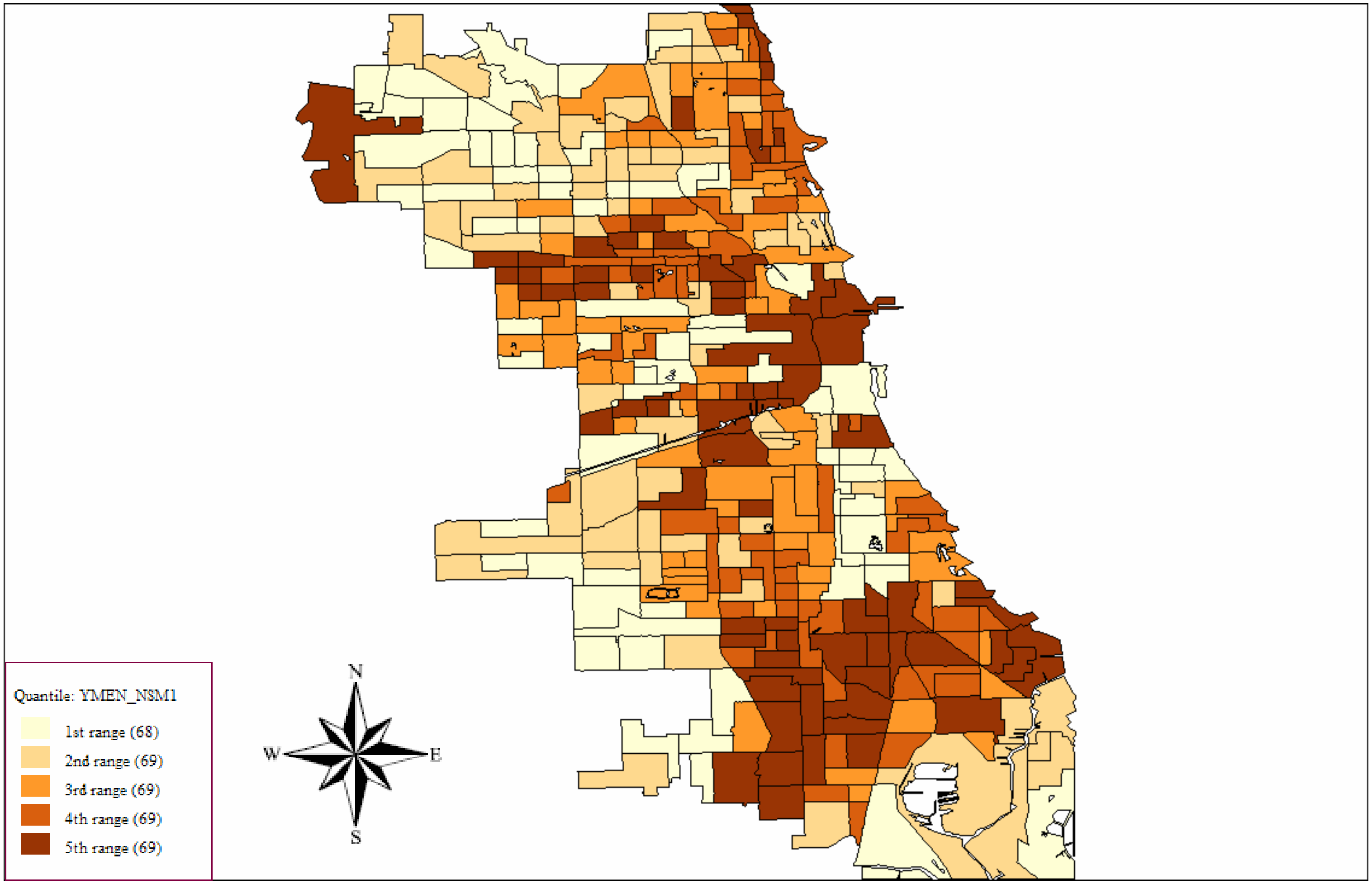
  where $\hat{\phi}(i,j)$ is the covariance between the i^{th} and j^{th} moments.

- Examine variation in marginal effects across space
  - Map out variations in effects
  - Study what correlates with high/low effects

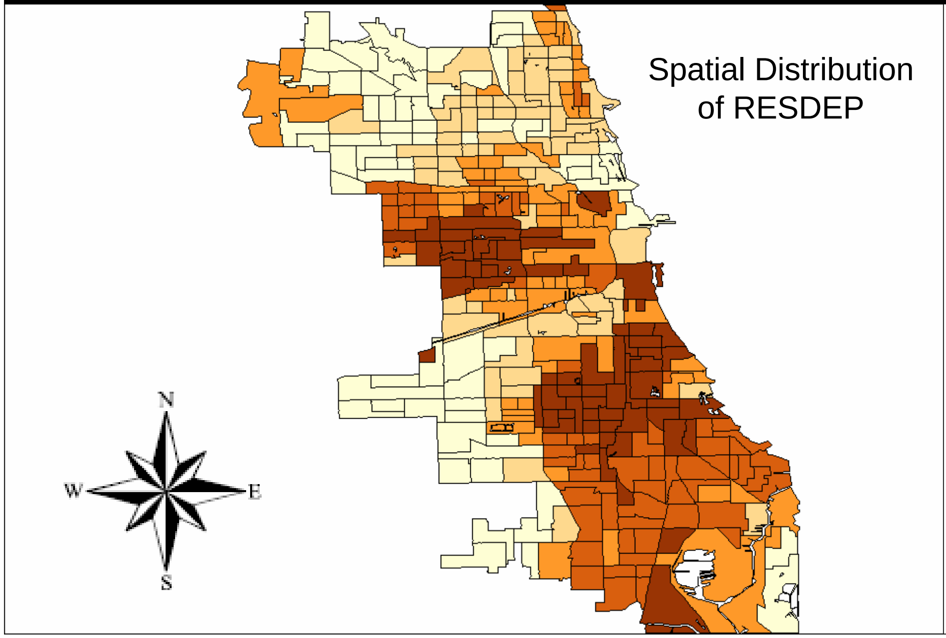
Spatial Variation in the Effect of RESDEP

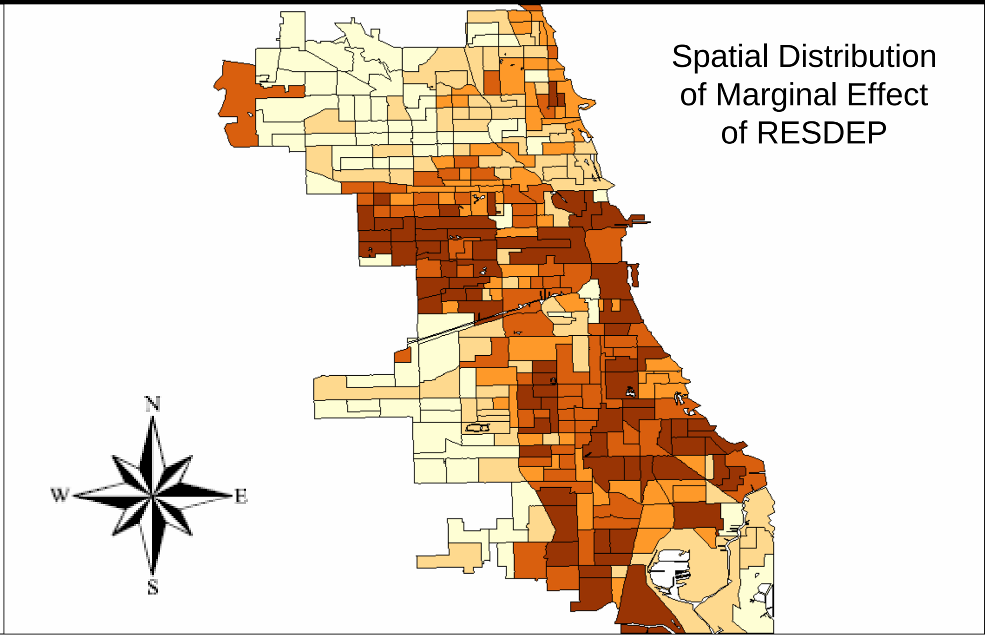


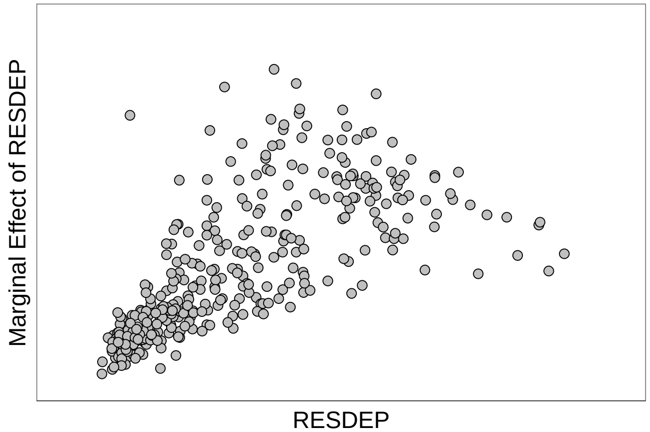
Spatial Variation in the Effect of YMEN



Marginal Effects of RESDEP versus RESDEP







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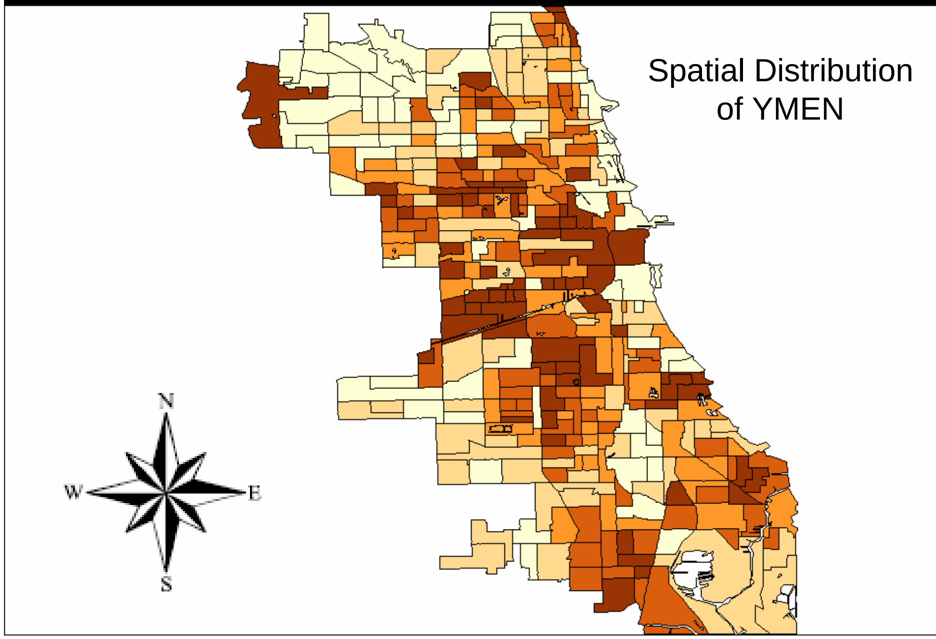
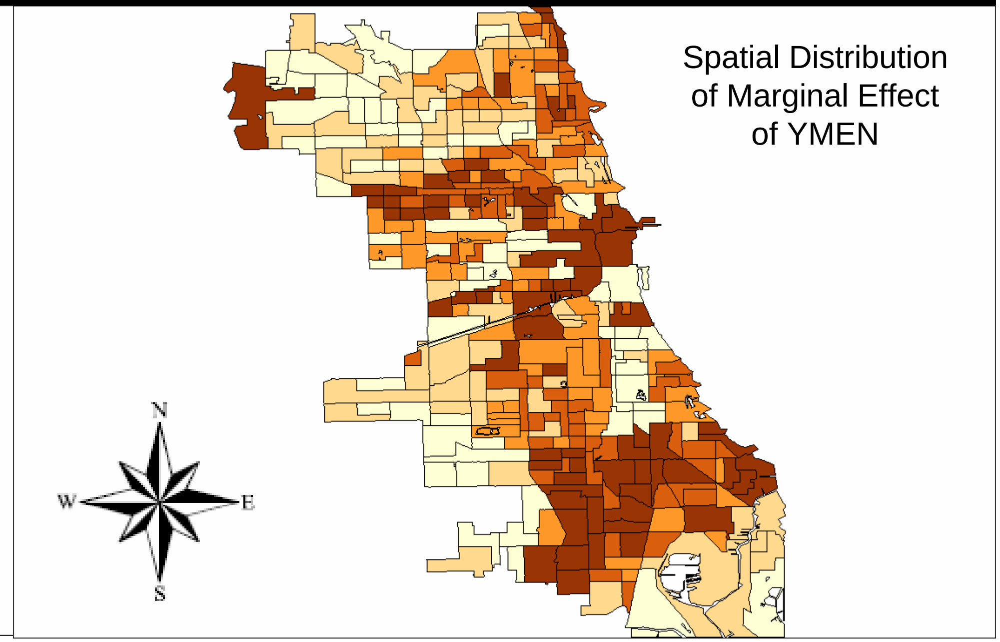
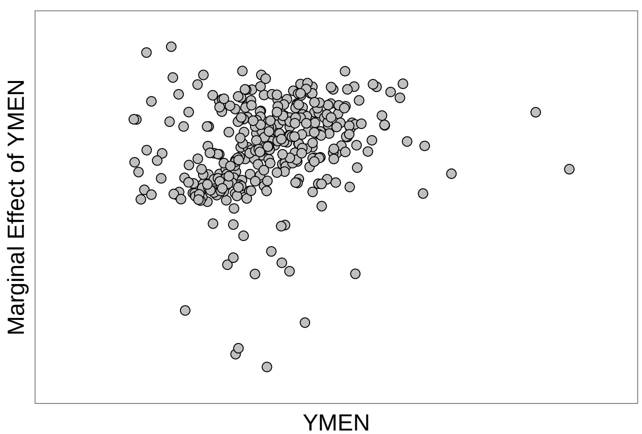
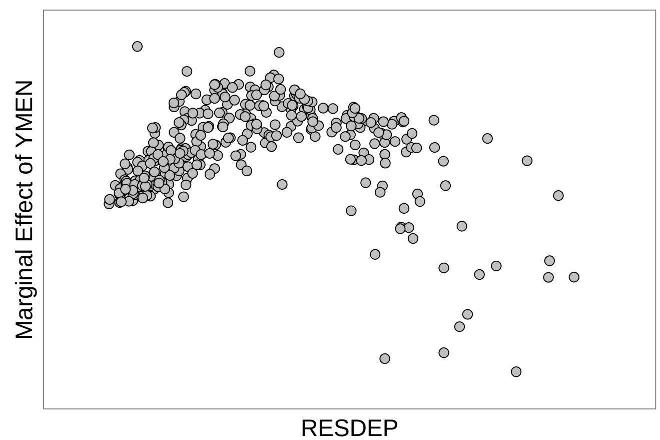
Marginal Effects of YMEN versus YMEN and RESDEP



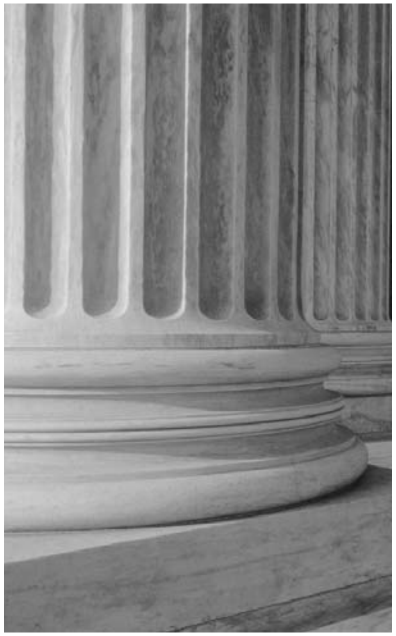






Concluding Thoughts

- Approach presented here has several good features
  - Minimal distributional assumptions invoked a-priori
  - Extends easily to incorporate several real-word data/sample features
  - Other non-sample knowledge can be included
  - Yields varying effects across space
  - Can yield very precise policy recommendations
- Further Extensions
  - Endogenous predictors (e.g., resource depravation ?)
  - Simultaneous Equation Count Models (e.g., different types of homicides)
  - Space-time models
- To do …
  - More Simulations
  - More cross method comparisons (e.g., Bayesian)



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