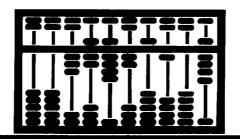# Technical notes

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## Experimental cost-of-living indexes: a summary of current research

Mary F. Kokoski

The Consumer Price Index (CPI) is "a measure of the average change in prices paid by urban consumers for a fixed market basket of goods and services."[1] As a policy tool, it is used to assess changes in households' cost of living and to adjust income or other compensatory payments to households, including wages, salaries, and pensions (private, military, civil service, and Social Security). Since 1985, the schedule of Federal income tax brackets, exemptions, and deductions has been indexed to the CPI.

Over time, the Bureau of Labor Statistics has implemented changes that improve the quality and relevance of the CPI as a measure of changes in the cost of consumption for the average household. In 1978, the sample of consumers surveyed to develop the CPI was broadened to permit the computation of an index, the CPI-U, that would represent the entire urban population (about 80 percent of the noninstitutional U.S. population), rather than only wage earners and clerical workers as did the earlier index, still published as the CPI-W. At that time, the statistical methodology was also improved, employing a scientific probability basis to provide a more representative sample of goods and services consumed. Beginning in 1983, a rental equivalence approach was adopted in the CPI-U so that the index would measure the consumption of housing services and not the investment component of homeownership. Beginning in 1987, the CPI was rebased to reflect the expenditure patterns of households as determined by the Bureau's 1982–84 Consumer Expenditure Survey,[2] which replaced those derived from the 1972–73 expenditure survey, and new commodities (such as personal computers) have been added to the market basket. Refinement of sampling and pricing methodologies is, and will continue to be, an ongoing process.

Even if it were technically flawless, the

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CPI would not be the only possible measure of price change and is not intended to address all issues concerning the welfare effects of price changes. No single index measure could do so. Because it is a fixed-weight Laspeyres index,[3] the CPI cannot incorporate in timely fashion the effects of changes in the consumption patterns of households due to relative price changes—that is, it does not capture the substitution behavior of consumer households between rebasing periods. Also, it represents the *average* price changes affecting consumer units, not the price changes affecting any one household or group of consumers. As a consumption-based index, it excludes the effects of price changes on income tax liabilities, as well as the welfare effects of changes in nonconsumption items (such as environmental amenities or investment values). To meet these concerns, a plethora of measures of price change could be defined: indexes with several sets of expenditure weights, different indexes for various types of households, and indexes that incorporate other variables affecting household welfare. For a specific analytical or policy purpose, it is possible that one of these types of indexes would provide a better measure than the CPI.

The advantages of the CPI are that it represents a clearly defined measurement concept and is constructed on the basis of a well-defined sample of expenditures and prices. As a Laspeyres price index of consumption, its properties with respect to other theoretical indexes of welfare change are well understood.[4] Thus, it is not likely to be replaced by another type of measure in the near future. Other index measures, however, may serve as complements to the CPI in addressing various issues. These alternatives also may provide insights into the character of, and suggest improvements to, the CPI itself.

Three types of new indexes—multiweighted indexes, demographic group-specific indexes, and a price index including income taxes—are currently being analyzed at the research level, using the resources of the 1972–73 and continuing (1980 forward) Consumer Expenditure Surveys. Although preliminary in nature, the results for these alternative index measures supplement past research at the Bureau and suggest the potential usefulness of

these indexes for several of the applications that require a price index measure. These results are summarized below.[5]

### A multiweighted index

A frequently voiced criticism of the fixed-weight CPI is that it inherits a substitution bias; that is, it ignores changes in expenditures made by consumers in response to changes in relative prices. If, for example, the price of beef rises more rapidly than that of poultry, then consumers may substitute poultry for the relatively more expensive beef. Because of this change in consumption patterns, the actual average price change for households would be less than that indicated by an index with expenditure shares fixed at the base-period pattern. The difference between the fixed-weight index and a "true" index, which incorporates substitution behavior, is the substitution bias. Over time, as the cumulative changes in relative prices increase, substitution bias also will increase.

To minimize substitution bias, expenditure weights can be updated periodically or an index with multiple expenditure weights (a "multiweighted index") can be constructed. For the CPI, expenditure weights are updated about once per decade, the most recent revision having occurred in January 1987. Because the Consumer Expenditure Survey has been conducted on a continuing basis since 1980, it is now possible to update expenditure weights more frequently and to produce multiweighted indexes. Several sets of expenditure weights can be used to produce chain-weighted indexes, already in use as official series published by other nations (such as the United Kingdom[6]). In the Laspeyres form, these indexes include expenditure information from a series of past (but not the current) periods. The reference expenditure weights are systematically moved forward over time, thus reducing the potential for substitution bias.[7] "Superlative" indexes,[8] which may be either chained or fixed-base, use expenditure weights from both past and current periods. Under a given set of assumptions, superlative indexes, such as the Fisher and Tornqvist forms,[9] represent "true" cost-of-living indexes, and thus may provide a feasible (and fairly general[10]) measure of the substitution bias inherent in

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the fixed-weight Laspeyres index.

In mathematical terms, the fixed-weight Laspeyres price index is:

$$(1) \quad L_{t,r} = \sum_{i=1}^{N} p_{it}\, q_{ir} \Big/ \sum_{i=1}^{N} p_{ir}\, q_{ir}$$

where $p_{it}$ is the price of good $i$ in period $t$; $q_{it}$ is the quantity of good $i$ consumed in period $t$; $r$ is the reference period for the consumption vector $q$ (= $q_{1r}$, $q_{2r}$, . . . , $q_{Nr}$); and $L_{t,r}$ is the Laspeyres price index for period $t$ compared to period $r$. The expenditures $p_{ir}\, q_{ir}$ ($i = 1, \ldots, N$) that compose the reference-period market basket are the result of household consumption decisions made in period $r$ at the prevailing set of prices $p_r$ (= $p_{1r}, \ldots, p_{Nr}$). The index in equation (1) compares the cost of purchasing the market basket $q_r$ under the new prices $p_t$ and the reference-period prices $p_r$; it ignores the possibility that consumers are likely to change their expenditure patterns by purchasing a new market basket $q_t$ at the new prices $p_t$. If, for example, the price of good $i$ rises rapidly and the price of good $j$ remains the same, then it is likely that $q_{it} < q_{ir}$ and $q_{jt} > q_{jr}$ (unless $i$ and $j$ are complementary goods). By maintaining the reference expenditures $q_{ir}$ and $q_{jr}$, the Laspeyres index provides an upper bound to the effects of the price changes on consumer welfare.[11] This discrepancy is the source of substitution bias and, as previously noted, it is likely to increase over time as differences in relative prices increase.[12]

A chained Laspeyres index is constructed by multiplying, or "chaining," a series of period-to-period Laspeyres indexes, and is given by:

$$(2) \quad L_{t,r}^c = r_{,r+1} \cdot L_{r+1,r+2} \cdot \ldots \cdot L_{t-1,t}$$

$$\left(\frac{\sum_i p_{i,r+1} q_{i,r}}{\sum_i p_{i,r} q_{i,r}}\right) \cdot \left(\frac{\sum_i p_{i,r+2} q_{i,r+1}}{\sum p_{i,r+1} q_{i,r+1}}\right) \cdots$$

$$\cdot \left(\frac{\sum_i p_{i,t}\, q_{i,t-1}}{\sum_i p_{i,t-1}\, q_{i,t-1}}\right)$$

where $L_{t,r}^c$ is the chained Laspeyres index for reference period $r$ and comparison period $t$. As equation (2) shows, the reference period for each component index is its previous period, so that the index includes the expenditure weights from periods $r$, $r+1$, $r+2$, . . . $t-1$, not just the earliest period $r$. Because the divergence in relative prices, and thus expenditure patterns, is likely to be smaller between two adjacent periods than between period $r$ and any comparison period $r+2$, . . . , $t$, the chained index would be expected to inherit less sub-

stitution bias than the fixed-weight index in equation (1).[13]

In both the fixed-base and the chain-weight contexts, the Fisher's ideal index and the Tornqvist index may be constructed. These superlative indexes in effect represent the geometric means of two price indexes, one based on period $r$ as the reference level of consumer welfare and one based on the period $t$ level of welfare as the reference. Because of the way in which they incorporate more than one expenditure pattern, the superlative indexes are themselves true cost-of-living indexes under certain rather flexible assumptions about consumer preferences.

The fixed-base $(T_{t,r})$ and chain-weighted $(T_{t,r}^c)$ Tornqvist indexes are given by (3a) and (3b), respectively:

$$(3a) \quad T_{t,r} \equiv \prod_{i=1}^{N} (p_{it}/p_{ir})^{(s_{ir} + s_{it})/2}, \text{ and}$$

$$(3b) \quad T_{t,r}^c \equiv \prod_{\tau=r+1}^{t} T_{\tau,\tau-1}$$

where $s_{ir} = p_{ir}\, q_{ir} / \Sigma p_{ir}\, q_{ir}$ and $s_{it} = p_{it}\, q_{it} / \Sigma p_{it}\, q_{it}$. The fixed-base $(F_{t,r})$ and chain-weighted $(F_{t,r}^c)$ Fisher's ideal indexes are given by:

$$(4a) \quad F_{t,r} \equiv \left(\frac{\Sigma_i p_{it}\, q_{ir}}{\Sigma_i p_{ir}\, q_{ir}}\right)^{\frac{1}{2}} \cdot \left(\frac{\Sigma_i p_{it}\, q_{it}}{\Sigma p_{ir}\, q_{it}}\right)^{\frac{1}{2}},$$

and

$$(4b) \quad F_{t,r}^c \equiv \prod_{\tau=r+1}^{t} F_{\tau,\tau-1}$$

Unlike their Laspeyres counterparts in equations (1) and (2), the superlative index forms require expenditure data for both past periods and the current period $t$. This limits their usefulness as up-to-date official series because data on consumer expenditures require time for collection and processing. However, these indexes can serve as a benchmark for assessing the magnitude of the substitution bias inherent in the Laspeyres indexes for recent past periods. By comparing (1) to (3a), (3b), (4a), or (4b), the degree of bias in the Laspeyres form can be tracked over time.[14]

In a preliminary investigation of these issues, expenditure data from the 1982–84 Consumer Expenditure Surveys were aggregated by calendar quarter for 160 categories of expenditures.[15] These expenditure categories correspond closely to those used to construct the CPI. From these data, both quarterly and annual average indexes were constructed. The calculated fixed-base ("direct") and chained indexes are presented in table 1. The reference period for the quarterly indexes is first-quarter 1982,

while that for the annual indexes is 1982.

The substitution bias may be measured in percentage terms using the equation:

$$(5) \quad Bias\,1 = \left(\frac{L - S}{S - 100}\right) \cdot 100$$

where $S$ represents the "true" or, in this case, superlative index.[16] Alternatively, it may be measured in terms of the cumulative difference in price change from the reference period:

$$(6) \quad Bias\,2 = L - S$$

or in terms of the period-to-period difference in measured price change:

$$(7) \quad Bias\,3 = \left(\frac{L_t}{L_{t-1}} - \frac{S_t}{S_{t-1}}\right) \cdot 100$$

To measure the substitution bias over the 1982–84 period, the difference between the direct Laspeyres index and the corresponding Tornqvist $(T)$ and chained Tornqvist $(T^c)$ indexes are calculated using each of the above measures. The results are provided in table 2. Analogous measures using the Fisher index also may be constructed, but are not presented here because the values of the two superlative indexes are very close.[17]

The quarterly indexes indicate an increase of about 11 percent in the cost of living from first-quarter 1982 to fourth-quarter 1984, while the annual average change from 1982 to 1984 is about 7 percent. Generally, the fixed-base Laspeyres index increased more quickly than the chained Laspeyres index, a result consistent with substitution among goods in response to price changes (although by no means a proof of such behavior). The correspondence between the fixed-base and chained superlative indexes is somewhat closer than for the Laspeyres form, although it is not exact to one decimal place except for the annual indexes. Some of this discrepancy may be attributed to large differences between $p_r$ and $p_t$ or between $p_{\tau-1}$ and $p_{\tau}$,[18] or to differences in preferences across periods.[19] Measurement error may play a significant role, and the variance of quarterly expenditure survey-derived cost weights may be high.

This means that the measures of substitution bias in table 2 differ somewhat when based on $T$ or $T^c$. The measure Bias 1 ranges from 3 to 11 percent across the quarterly series and from 1.4 to 3 percent for the annual series. Bias 2, a cumulative measure, indicates that the absolute divergence between $L$ and the superlative measure generally increases over time. The period-to-period Bias 3 ranges from 0.1 to 0.3 percent. It is generally higher for comparisons

## Table 1. Direct and chained indexes for total consumption expenditures, selected periods, 1982–84

| Period | Laspeyres | Tornqvist | Fisher |
|---|---|---|---|
| | Direct Indexes | | |
| **Quarterly index** | | | |
| **1982:** | | | |
| I | 100.0 | 100.0 | 100.0 |
| II | 101.3 | 101.3 | 101.3 |
| III | 103.1 | 103.0 | 103.0 |
| IV | 103.7 | 103.5 | 103.5 |
| **1983:** | | | |
| I | 103.9 | 103.8 | 103.8 |
| II | 105.3 | 105.1 | 105.1 |
| III | 106.5 | 106.1 | 106.2 |
| IV | 107.2 | 106.8 | 106.8 |
| **1984:** | | | |
| I | 108.3 | 107.6 | 108.0 |
| II | 109.4 | 109.0 | 109.0 |
| III | 110.5 | 110.2 | 110.3 |
| IV | 111.3 | 110.7 | 110.7 |
| **Annual index** | | | |
| 1982 | 100.0 | 100.0 | 100.0 |
| 1983 | 103.4 | 103.3 | 103.3 |
| 1984 | 107.3 | 107.2 | 107.2 |
| | Chained Indexes | | |
| **Quarterly index** | | | |
| **1982:** | | | |
| I | 100.0 | 100.0 | 100.0 |
| II | 101.3 | 101.3 | 101.3 |
| III | 103.0 | 102.9 | 103.0 |
| IV | 103.4 | 103.4 | 103.4 |
| **1983:** | | | |
| I | 103.5 | 103.5 | 103.5 |
| II | 104.9 | 104.9 | 104.9 |
| III | 106.0 | 106.0 | 106.0 |
| IV | 106.7 | 106.7 | 106.7 |
| **1984:** | | | |
| I | 107.6 | 107.7 | 107.7 |
| II | 108.7 | 108.8 | 108.8 |
| III | 109.9 | 110.0 | 110.0 |
| IV | 110.8 | 110.8 | 110.8 |
| **Annual index** | | | |
| 1982 | 100.0 | 100.0 | 100.0 |
| 1983 | 103.4 | 103.3 | 103.3 |
| 1984 | 107.3 | 107.2 | 107.2 |

NOTE: Annual indexes are based on annual average expenditures and price relatives.

to $T$, although the significance of this result is suspect.

According to the economic index number literature, multiweighted indexes, which are now feasible because of the continuous collection of Consumer Expenditure Survey data, may inherit less substitution bias than the fixed-weight Laspeyres index. In this analysis, multiweighted indexes have been used to assess the magnitude of substitution bias in fixed-weight indexes such as the CPI. Further research is under way to explore alternative methods of producing multiweighted indexes, particularly to better exploit the panel structure

of the expenditure survey data and to examine the effects of seasonal (or other systematic) variations in relative prices and expenditures in the quarterly context.[20]

### Indexes for demographic groups

Although used for a variety of policy purposes, the CPI–U represents the effects of price changes on an average or "representative" household. To the extent that a given household's expenditure patterns differ from the population average, the effects of relative price changes on this household will not be reflected by the CPI value. For example, families with a relatively large share of their consumer expenditure devoted to food will experience a greater increase in their cost of living than will households with a small expenditure share for food when food price increases are relatively large. Similarly, an elderly household may be more greatly affected by changes in prices of household fuels and medical care than by changes in prices of furniture and entertainment commodities.

If it is assumed that the expenditure patterns of households are related to the households' demographic attributes, then a price index for a demographic subgroup might, on average, be more representative of the price changes experienced by a household in that subgroup than would the CPI for all households. Production of a reliable set of subgroup price indexes requires information on the relationship between demographic attributes and expenditures by households, as well as the variance of con-

sumption patterns within and between population subgroups.

Analysis of the variation in price indexes across households has been done by Robert Michael and Robert Hagemann.[21] In Hagemann's study, data from the interview portion of the 1972–73 Consumer Expenditure Survey were used to perform regressions of demographic attributes on household-specific price indexes. His results indicate that, while some demographic attributes had a statistically significant influence on expenditure patterns, variability in expenditure patterns within subgroups was still at least as large as that across groups. To assess the importance of this result for producing subgroup indexes for more-aggregate demographic groups, experimental subgroup indexes for 1972 expenditure survey households have now been constructed.[22] Subgroup mean expenditure shares (rather than household-specific indexes) were used to construct the subgroup indexes, which were limited to households that rented their dwellings. Age (64 and under, over 64) of reference person,[23] household composition (singles, couples, families, single parents), and retirement status (of reference person) provided the demographic group disaggregation for the 1972 sample of renter households.[24] The indexes were based on 54 categories of goods and services.

The demographic group indexes and 1972 sample sizes are presented in table 3. The results show that, over the 8-year period, the Laspeyres indexes may diverge by

## Table 2. Substitution bias estimates, selected periods, 1982–84

| Period | Bias 1 | | Bias 2 | | Bias 3 | |
|---|---|---|---|---|---|---|
| | Laspeyres (L) versus Tornqvist (T) | Laspeyres (L) versus chained Tornqvist (Tᶜ) | Laspeyres (L) versus Tornqvist (T) | Laspeyres (L) versus chained Tornqvist (Tᶜ) | Laspeyres (L) versus Tornqvist (T) | Laspeyres (L) versus chained Tornqvist (Tᶜ) |
| **Quarterly estimate** | | | | | | |
| **1982:** | | | | | | |
| I | — | — | — | — | — | — |
| II | 0 | 0 | 0 | 0 | 0 | 0 |
| III | 3.3 | 6.9 | .1 | .2 | .1 | .5 |
| IV | 5.7 | 8.8 | .2 | .3 | .1 | . |
| **1983:** | | | | | | |
| I | 2.6 | 11.4 | .1 | .4 | -.1 | 3 |
| II | 3.9 | 8.2 | .2 | .4 | .1 | 3 |
| III | 6.6 | 8.3 | .4 | .5 | .2 | 2 |
| IV | 5.9 | 7.5 | .4 | .5 | 0 | 4 |
| **1984:** | | | | | | |
| I | 9.2 | 7.8 | .7 | .6 | .3 | .1 |
| II | 4.4 | 6.8 | .4 | .6 | .3 | 0 |
| III | 2.9 | 5.0 | .3 | .5 | .1 | -.1 |
| IV | 5.6 | 4.6 | .6 | .5 | .3 | 0 |
| **Annual estimate** | | | | | | |
| 1982 | — | — | — | — | — | — |
| 1983 | 3.0 | 3.0 | .1 | .1 | .1 | .1 |
| 1984 | 1.4 | 1.4 | .1 | .1 | 0 | 0 |

as much as 4 index points across demographic subgroups. In addition, the indexes are higher for larger households (for example, families and couples versus singles).

In table 4, each of the demographic-group Laspeyres index values is compared to the index value for all renters, and also to the index for the general age and household composition classifications to which the group belongs. For example, the index for families with householders under age 64 is compared to the indexes for: all renters (column 1); all households with reference persons under age 64 (column 2); and all families (column 3). The results show that, in most cases, age subgroup indexes do not come closer to the demographic group indexes (and may even differ more from them) than does the all-renters index. Even for the over-64 group, the sample from which expenditure shares for a CPI for retired persons might be obtained, an over-64 subgroup index provides a poorer approximation of the index for over-64 singles than does the all-renters index. Likewise, an index for all retirees does not necessarily well represent the retired singles. For the under-64 households subgroup, however, indexes based upon household composition perform relatively well.

Overall, it appears that variations in expenditure patterns within broadly defined demographic groups may cause demographic subgroup indexes to be less representative of the households within the subgroup than would an aggregate price index. In this analysis, indexes based upon household composition may hold some measure of promise, but further research

## Table 4. Differences in Laspeyres indexes within subgroups[1]
[In index points]

| Demographic group | All renters | Subgroup by— | |
| --- | --- | --- | --- |
| | | Age | Household composition |
| All singles | -3.1 | — | — |
| All couples | -1.3 | — | — |
| All families | 2.7 | — | — |
| All single parents | -1.3 | — | — |
| Under age 64 | -.1 | — | — |
| Singles | -3.1 | -3.5 | -.4 |
| Couples | -1.9 | -1.8 | -.6 |
| Families | 2.7 | 2.8 | -.0 |
| Single parents | -1.3 | -1.2 | -.0 |

| | | | Retirement status |
| --- | --- | --- | --- |
| Over age 64: | | | |
| All retirees | .7 | .1 | — |
| Singles | -1.7 | -2.2 | — |
| Retired | -1.6 | -2.2 | -2.3 |
| Couples | 1.5 | .9 | — |
| Retired | 2.7 | .1 | 1.9 |

[1] Using data from table 3, the difference is calculated as the value of the detailed index less the value of the more-aggregate subgroup index.

## Table 3. Price indexes across demographic groups for 1980
[1972=100]

| Demographic group | Laspeyres index | 1972 sample size |
| --- | --- | --- |
| All renters[1] | 184.2 | 3,443 |
| All singles | 181.1 | 1,144 |
| All couples[2] | 182.9 | 663 |
| All families | 187.0 | 940 |
| All single parents | 183.0 | 301 |
| Under age 64 | 184.1 | 2,779 |
| Singles | 180.7 | 747 |
| Couples[2] | 182.4 | 531 |
| Families | 186.9 | 892 |
| Single parents | 182.9 | 298 |
| Over age 64 | 184.8 | 664 |
| All retirees | 185.0 | 371 |
| Singles | 182.6 | 397 |
| Retired | 182.7 | 249 |
| Couples | 185.7 | 132 |
| Retired | 186.9 | 76 |

[1] The analysis was limited to renter households because of lack of information on rental equivalence of owned residence in the 1980 survey.

[2] Husband-wife households without children present.

will be needed to explore this area more fully. An even more important need is to assess the variance of these indexes and comparative measures. With the availability of expenditure survey data for 1980–85, a more detailed and rigorous analysis of the relationship between household attributes and expenditure is possible. Larger sample sizes may permit greater disaggregation of the population into more specific demographic groups, as well as a greater disaggregation of the commodity and service categories. In addition, the importance of the period(s) chosen for analysis should be investigated.

## The tax and price index

The CPI measures the change in total expenditure needed to enable the consumer to purchase a fixed market basket of commodities and services under a new set of prices. In many cases, however, it is used to adjust components of income (such as wages and pensions), and not household expenditures, as the prices of expenditure goods change. Thus, it can be argued that an "income-based" cost-of-living measure[25] would be more appropriate than an "expenditure-based" measure such as the CPI. An income-based cost-of-living (ICOL) index would assess the changes in pretax income needed by consumers to enable them to achieve the same level of consumption as before the price changes. Under a progressive tax system, it is expected that index computation in terms of pretax income will differ from that in terms of consumption expenditures because a larger

share of the consumer's income must be devoted to tax payments as income increases. Such an income-based index would also reflect changes in tax rates and the structure of the tax system as these variables affect the difference between pre-tax and disposable income.[26]

Robert Gillingham and John Greenlees have constructed a fixed-weight pretax income index, the tax and price index (TPI), to capture the impact of Federal and State income taxes on the welfare effects of price changes.[27] It includes a market basket of consumer goods and services and (unlike the CPI) also incorporates real savings,[28] charitable contributions, and nonconsumption goods (such as professional memberships and consumer financial services) in the set of expenditures for households.[29] This set of expenditures was determined for each household in the 1972–73 expenditure survey sample as its reference expenditure pattern. Future consumption, public goods, and environmental variables were not explicitly included, nor was income from sources other than wages and salaries and self-employment.

For each household in the sample, information was gathered to determine its Federal and State and local income tax burden for any given level of household income. This information includes tenure status (homeowners versus renters), filing status (it was assumed, for lack of data, that all married couples filed jointly), State and county of residence, family composition (for example, number of dependents), deductible expenditures (such as charitable contributions and sales taxes paid), and so forth. The *State Tax Handbook* and *State Tax Guide*,[30] *Your Federal Income Tax*,[31] and copies of the 1040 and 1040A forms[32] provided the specific tax rules, brackets, and rates to be applied. The computation of the TPI then involved solution of the following problem: "given a [reference] value of consumption expenditures $M_x$, what is the value of gross [pretax] income $Y$ such that after-tax income equals $M_x$?"[33]

In general terms, the TPI is given by:

$$(8) \quad TPI_{t,r} \equiv \frac{\left( \sum_{i=1}^{N} p_{it} q_{ir} + T_t \right)}{\left( \sum_{i=1}^{N} p_{ir} q_{ir} + T_r \right)}$$

$$\equiv \frac{\left( \sum_{i=1}^{N} p_{it} q_{ir} + \tau(p_t, t_t; q_r, z_r, S_r) \right)}{\left( \sum_{i=1}^{N} p_{ir} q_{ir} + \tau(p_r, t_r; q_r, z_r, S_r) \right)}$$

### Table 5. Tax and price indexes, 1967–85

[1967=100]

| Period | Consumer Price Index[1] | Federal tax | State tax | FICA tax | Tax and price index |
|---|---|---|---|---|---|
| 1967 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| 1968 | 103.6 | 120.4 | 120.0 | 114.6 | 106.4 |
| 1969 | 108.0 | 136.2 | 144.2 | 126.9 | 112.8 |
| 1970 | 113.1 | 129.5 | 150.9 | 128.2 | 116.2 |
| 1971 | 118.0 | 126.3 | 172.7 | 140.0 | 120.5 |
| 1972 | 121.6 | 124.1 | 195.4 | 157.1 | 124.0 |
| 1973 | 129.3 | 141.9 | 221.1 | 205.4 | 134.4 |
| 1974 | 142.8 | 173.5 | 266.2 | 244.0 | 151.4 |
| 1975 | 154.4 | 187.6 | 308.5 | 261.1 | 164.0 |
| 1976 | 163.4 | 202.3 | 349.3 | 280.4 | 174.5 |
| 1977 | 173.9 | 223.1 | 390.5 | 300.5 | 187.1 |
| 1978 | 186.1 | 256.8 | 446.9 | 334.7 | 203.4 |
| 1979 | 204.9 | 287.9 | 498.7 | 412.0 | 225.9 |
| 1980 | 228.4 | 362.0 | 549.9 | 468.0 | 258.1 |
| 1981 | 250.2 | 434.7 | 695.7 | 579.3 | 290.2 |
| 1982 | 264.8 | 424.7 | 738.7 | 625.6 | 302.8 |
| 1983 | 275.4 | 401.4 | 815.4 | 667.7 | 310.7 |
| 1984 | 286.9 | 401.7 | 840.7 | 711.5 | 321.8 |
| 1985 | 296.6 | 414.1 | 855.0 | 779.3 | 333.3 |

| Period | **Percent change** | | | | |
|---|---|---|---|---|---|
| 1967–68 | 3.6 | 20.4 | 20.0 | 14.6 | 6.4 |
| 1968–69 | 4.2 | 13.1 | 20.0 | 10.7 | 6.0 |
| 1969–70 | 4.7 | -4.9 | 4.6 | 1.0 | 3.0 |
| 1970–71 | 4.3 | -2.5 | 14.5 | 9.2 | 3.7 |
| 1971–72 | 3.0 | -1.7 | 13.1 | 12.2 | 2.9 |
| 1972–73 | 6.4 | 14.4 | 13.1 | 30.8 | 8.4 |
| 1973–74 | 10.5 | 22.3 | 20.4 | 18.8 | 12.7 |
| 1974–75 | 8.1 | 8.1 | 15.9 | 7.0 | 8.3 |
| 1975–76 | 5.8 | 7.8 | 13.2 | 7.4 | 6.4 |
| 1976–77 | 6.4 | 10.3 | 11.8 | 7.2 | 7.2 |
| 1977–78 | 7.0 | 15.1 | 14.5 | 11.4 | 8.7 |
| 1978–79 | 10.1 | 12.1 | 11.6 | 23.1 | 11.1 |
| 1979–80 | 11.5 | 25.7 | 19.3 | 13.6 | 14.2 |
| 1980–81 | 9.5 | 20.1 | 16.9 | 23.8 | 12.4 |
| 1981–82 | 5.9 | -2.3 | 6.2 | 8.0 | 4.3 |
| 1982–83 | 4.0 | -5.5 | 10.4 | 6.7 | 2.6 |
| 1983–84 | 4.2 | .1 | 3.1 | 6.6 | 3.6 |
| 1984–85 | 3.4 | 3.1 | 1.7 | 9.5 | 3.6 |
| Average annual rate of change | 6.3 | 8.6 | 12.8 | 12.3 | 7.0 |

[1] As computed for the 7,242 households in the 1972–73 Consumer Expenditure Survey sample.

where, for reference period $r$ and comparison period $t$: $p_i$ is the price of consumption good $i$; $q_i$ is the quantity of good $i$ consumed; and $T$ is the tax burden, which is a function, $\tau$, of prices ($p$), tax parameters ($t$), consumption ($q$), expenditure on non-consumption goods ($z$), and the demographic attributes that determine tax filing status and liabilities ($S$). Because tax liabilities vary by household, the TPI was constructed at the household level for each of the 7,242 expenditure survey households in the sample and averaged to derive an aggregate index. An expenditure-based price index, analogous to the CPI, was derived for this sample as well for comparison with the TPI.[34]

The resulting TPI, "CPI," and indexes of changes in the various income tax components are given in table 5, where the reference expenditures $p_r q_r$ are for 1972–73.

The percentage changes in these indexes are also shown.

The results show that all tax components and the TPI have risen more quickly than the prices of market commodities. In 1985, the TPI exceeded the "CPI" by 36.7 index points. Thus, even if pretax incomes were indexed by the "CPI," the households would, on average, still have experienced a loss in after-tax purchasing power. The percentage changes in the indexes show that the tax components and the TPI were generally increasing more quickly than prices up through 1981. The "bracket creep" effect appears to have caused much of the divergence between the annual average rates of increase in the TPI and the "CPI." Changes in tax system parameters also influenced the TPI series: the tax surcharge in 1968–69; the Personal Exemption Credit and Earned Income Credit in 1975; the Credit

for the Elderly and the Taxable Income Credit in 1976, which reduced the rise in taxes; the lower marginal rate schedules in 1979; and the Economic Recovery Tax Act of 1981, which reduced the rate of increase in the TPI in 1982–85. Despite the increasing rates for the State tax and FICA components, the TPI increased more slowly than the "CPI" during 1981–84. In 1985, the first year in which Federal income tax brackets were indexed by the Consumer Price Index, the increase in the TPI was only 0.2 index points greater than the increase in the "CPI."

The TPI provides a complement to the published CPI which may be used to address such issues as the effect of indexing Federal tax brackets by the CPI; the effect of differential tax treatment of population subgroups (homeowners, retirees, and so forth); and the net effect of substituting indirect (such as sales) for direct (income) taxes. If, for example, a value-added or sales tax were implemented to replace some or all of the revenues generated by the income tax, the TPI would reflect both the increase in the prices of goods and the reduction in income tax burdens; the CPI (which incorporates sales taxes) would capture only the sudden increase in retail prices. However, neither index, as now constructed, would capture changes in other taxes, such as those on unearned (asset) income, or the effects of tax changes on the consumption of leisure. Improvements to the tax and price index may be made, such as the inclusion of direct taxes on asset income and the incorporation of greater detail in the expenditure categories and demographic attributes. Consideration of the labor-leisure choice in this framework is also an interesting possibility.

RESEARCH IS UNDER WAY to expand upon the results of this report and to possibly produce similar experimental indexes on a production basis. The availability of data from the continuing Consumer Expenditure Survey provides a basis for improving upon this preliminary analysis. Production versions of these indexes would thus be comparable to the revised CPI, which has expenditure weights based on 1982–84 household consumption data. □

### Footnotes

[1] *BLS Handbook of Methods, Volume II: The Consumer Price Index*, Bulletin 2134–2 (Bureau of Labor Statistics, April 1984)

[2] The Consumer Expenditure Survey, conducted by the Bureau of the Census for the Bureau of Labor Statistics, provides detailed information on the expenditures and demographic attributes of a representative sample of U.S. households. It consists of two distinct components—the Interview Survey, which collects information from a rotating panel of households for five consecutive calendar quarters each; and the Diary Survey, which consists of two weekly diaries of specific expenditures recorded by each participating household. The Interview component is designed to capture large and/or infrequent household expenditures (such as major appliances) and the Diary component is designed to collect information on smaller, more frequent types of expenditures (as for food items).

[3] It is actually a modified Laspeyres index, although the expenditure weights are fixed for a reference period.

[4] Robert Pollak, "The Treatment of Taxes in the Consumer Price Index, " in W. Erwin Diewert and Claude Montmarquette, eds., Price Level Measurement (Ottawa, Statistics Canada, 1983); and Robert Gillingham, "A Conceptual Framework for the Consumer Price Index," in Proceedings, Business and Economic Statistics Section (American Statistical Association, 1974), pp. 254–65.

[5] A detailed and more technical discussion is available in Mary F. Kokoski, "A Report on Experimental Indices of the Cost of Living," Report no. 751 (Bureau of Labor Statistics, 1987); and Robert Gillingham and John Greenlees, "The Impact of Direct Taxes in the Cost of Living," Journal of Political Economy, Vol. 95, 1987, pp. 775–96.

[6] R. F. Fowler, "Some Problems of Index Number Construction," Studies in Official Statistics Research Series No. 3 (London, Department of Employment and Productivity, 1970).

[7] Household expenditure patterns may change over time independently of changes in relative prices. By including continuous updates of expenditure information, the multiweighted index also accommodates the changes in consumer preferences due to shifting tastes, lifestyles, and so forth.

[8] W. Erwin Diewert, "Exact and Superlative Index Numbers," Journal of Econometrics, Vol. 4, 1976, pp. 115–45.

[9] W. Erwin Diewert, "The Theory of the Cost-of-Living Index and the Measurement of Welfare Change," in W. Erwin Diewert and Claude Montmarquette, eds., Price Level Measurement (Ottawa, Statistics Canada, 1983), pp. 163–223.

[10] The alternative, parametric approach to constructing true cost-of-living indexes requires econometric estimation of consumer demand relationships. The resulting "parametric" true cost-of-living indexes are then dependent upon the assumptions and functional form chosen for the consumer demand model.

[11] A theoretical derivation and proof of this assertion are available in Diewert, "Exact and Superlative Index Numbers."

[12] Steven Braithwait, "The Substitution Bias of the Laspeyres Price Index: An Analysis Using Estimated Cost-of-Living Indexes," American Economic Review, Vol 70, 1980, pp. 64–77.

[13] Some concern has been expressed in the context of chained indexes about the problem of "chain drift." In the most general sense, drift may occur if approximation or measurement errors in the period-to-period indexes cumulate as these indexes are multiplied together to construct the chain. Specific characterizations of "drift" have not been fully resolved in the literature as yet. See Bohdan J. Szulc, "Linking Index Number Series," in W. Erwin Diewert and Claude Montmarquette, eds., Price Level Measurement (Ottawa, Statistics Canada, 1983); and F. G. Forsyth and R. F. Fowler, "The Theory and Practice of Chain Price Index Numbers," Journal of the Royal Statistical Society, Vol. 144, 1981, Part 2, pp. 224–46.

[14] Marilyn Manser and Richard McDonald, "An Analysis of Substitution Bias in Inflation Measurement, 1959–82," Econometrica, Vol. 56, 1988, pp. 909–30.

[15] The data base included reports from both the Diary and Interview portions of the survey.

[16] P. J. Lloyd, "Substitution Effects and Biases in Nontrue Indices," American Economic Review, Vol. 65, 1975, pp. 301–13.

[17] See Manser and McDonald, "An Analysis."

[18] Recall that the superlative indexes represent the geometric means of two price indexes with different reference periods. The values of these implicit component indexes may differ because a different set of reference expenditures serves as the basis for the comparison of indexes.

[19] Consumers may require more than 3 months to accommodate price changes by adjusting their expenditures, and may also exhibit seasonal patterns in their consumption behavior.

[20] Seasonal or other cyclical patterns in prices and expenditures can, in principle, cause the chained index and the direct index to diverge, or "drift."

[21] Robert Michael, "Variation Across Households in the Rate of Inflation," Journal of Money, Credit, and Banking, Vol. 11, 1979, pp. 32–46; and Robert Hagemann, "The Variability of Inflation Rates Across Household Types," Journal of Money, Credit, and Banking, Vol. 14, 1982, pp. 494–510.

[22] These research indexes were created by using the average expenditure patterns of the given demographic groups as cost weights for the national CPI price series of each individual commodity category. It is possible that these population subgroups may have different geographic distributions, purchase different varieties of goods, shop in different outlets, or pay different prices (an example being senior citizen discounts). Such effects will not be reflected in the indexes presented here. (See Commissioner of Labor Statistics Janet L. Norwood, Statement before the Special Committee on Aging, U.S. Senate, June 29, 1987.)

[23] In the Consumer Expenditure Survey, the reference person is the first member of the consumer unit that the survey respondent mentions as an owner or renter of the household premises during the initial interview.

[24] Data are from the Interview portion of the expenditure survey only.
Preliminary comparisons of these results to those for a 1980 sample (comparable for renters only) has been done, but sample sizes in the latter survey are too small to permit discussion here.

[25] Robert Pollack, "The Treatment of Taxes in the Consumer Price Index," unpublished paper (Bureau of Labor Statistics, 1972).

[26] It is implicitly assumed that income tax payments and government benefits supplied from these revenues are not correlated. Government services (such as defense) are not included in the consumers' market basket of goods and services. (For a discussion of indexes that incorporate such public goods, see Steven A. Cobb, "Interarea Cost of Living Measurement with Nonmarket Goods: A Demand Systems Approach," Working Paper No. 140 (Bureau of Labor Statistics, 1983).) FICA taxes are treated likewise, because the link between the payment of these taxes and future Social Security benefits is somewhat tenuous.

[27] See Gillingham and Greenlees, "The Impact."

[28] This is monetary savings divided by a price index.

[29] These expenditures have investment components, that is, they yield benefits in future periods but are purchased in the current period. Such expenditures are inappropriate in an expenditure-based index such as the CPI (see Gillingham, "A Conceptual Framework"), but are affected by the tax code.

[30] Commerce Clearing House, Inc. State Tax Handbook, various annual issues; and Commerce Clearing House, Inc., State Tax Guide, various issues.

[31] U.S. Internal Revenue Service, Your Federal Income Tax, Publication 17, various annual issues.

[32] U.S. Internal Revenue Service, Individual Income Tax Returns, Forms 1040 and 1040A, various annual issues.

[33] Gillingham and Greenlees, "The Impact."

[34] The indexes here were based on a disaggregation of consumption expenditures into 37 categories of goods and services.
Both the TPI and "CPI" in this analysis employ the equivalent rent measure for homeownership. This rental equivalence approach has been adopted by the official CPI under the 1987 revision.