

SOCIETY OF ACTUARIES
AMERICAN SOCIETY OF PENSION ACTUARIES
JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2006 EA-1 EXAMINATION

Data for Question 1 (3 points)

Smith takes a loan repayable by equal installments at the end of each year for 20 years.

Both the principal and interest portions of the 11th payment are \$100.

Question 1

In what range is the initial amount of the loan?

- (A) Less than \$1,500
- (B) \$1,500 but less than \$1,700
- (C) \$1,700 but less than \$1,900
- (D) \$1,900 but less than \$2,100
- (E) \$2,100 or more

Data for Question 2 (3 points)

Fund information for 2006:

| <u>Date</u> | <u>Fund Value</u> | <u>Contributions</u> | <u>Benefit Payments</u> |
|-------------|-------------------|----------------------|-------------------------|
| 12/31/2005 | \$10,000 | - | - |
| 1/1/2006 | - | \$4,000 | \$1,500 |
| 3/31/2006 | \$12,000 | - | - |
| 4/1/2006 | - | \$2,000 | \$250 |
| 12/31/2006 | X | - | - |

The dollar weighted rate of return was 10% for 2006.

R = the time weighted rate of return for 2006

Question 2

In what range is R?

- (A) Less than 9.2%
- (B) 9.2% but less than 9.5%
- (C) 9.5% but less than 9.8%
- (D) 9.8% but less than 10.1%
- (E) 10.1% or more

Data for Question 3 (4 points)

The following are based on the same assumptions:

- I. The single premium for a life annuity of \$10,000 payable annually to a life aged 50 with first payment at the end of the year is \$123,325.
- II. The 20-year level annual premium for a whole life insurance with a death benefit of \$100,000 payable at the end of the year of death to a life aged 50 is \$3,314.
- III. The 20-year level annual premium for a 20-year endowment insurance of \$100,000 payable at the end of the year of death to a life aged 50 is X.

Interest rate: 5.0%, compounded annually.

Question 3

In what range is X?

- (A) Less than \$3,600
- (B) \$3,600 but less than \$4,000
- (C) \$4,000 but less than \$4,400
- (D) \$4,400 but less than \$4,800
- (E) \$4,800 or more

Data for Question 4 (3 points)

Smith (age 65) has a spouse (age 60) as of 1/1/2006.

Smith is scheduled to commence a life annuity of \$1,000 per month beginning on 1/1/2006.

Instead, Smith elects the actuarially equivalent benefit commencing 1/1/2006 described below:

X per month while both Smith and spouse are alive, plus

X per month to the spouse for life after Smith's death, plus

\$1,000 per month to Smith for life after the spouse's death.

Selected actuarial values:

$$\ddot{a}_{60}^{(12)} = 12.176$$

$$\ddot{a}_{65}^{(12)} = 10.194$$

$$\ddot{a}_{65:60}^{(12)} = 8.023$$

Question 4

In what range is X?

- (A) Less than \$650
- (B) \$650 but less than \$700
- (C) \$700 but less than \$750
- (D) \$750 but less than \$800
- (E) \$800 or more

Data for Question 5 (4 points)

On 1/1/2006, Smith borrows a sum of money repayable in annual payments at the end of each year for 20 years. Each annual payment contains principal and interest.

The principal in the payment at the end of year t equals $\$100t$ (for $t = 1, 2, 3, \dots, 20$). The payment at the end of the year also includes the interest accrued during the year.

Interest rate: 6.0%, compounded annually.

$X =$ the present value at 1/1/2006 of interest payments made during the 20 years of the loan.

Question 5

In what range is X ?

- (A) Less than \$9,000
- (B) \$9,000 but less than \$10,000
- (C) \$10,000 but less than \$11,000
- (D) \$11,000 but less than \$12,000
- (E) \$12,000 or more

Data for Question 6 (4 points)

Terms of a bond:

| | |
|-------------------|------------------------|
| Face amount | \$1,000 |
| Redemption amount | \$1,000 |
| Term | 20 years |
| Coupons | 5.0%, payable annually |

The bond is issued at a discount, which is amortized over the 20-year life of the bond.

The amount the bond is written up in the 16th year is 50% greater than the amount the bond is written up in the 11th year.

Question 6

In what range is the issue price of the bond?

- (A) Less than \$600
- (B) \$600 but less than \$650
- (C) \$650 but less than \$700
- (D) \$700 but less than \$750
- (E) \$750 or more

Data for Question 7 (4 points)

Data from a select and ultimate mortality table:

| x | $q_{[x]}$ | $q_{[x]+1}$ | $q_{[x]+2}$ | q_{x+3} | $x+3$ |
|-----|-----------|-------------|-------------|-----------|-------|
| 50 | 0.074 | 0.094 | 0.114 | 0.126 | 53 |
| 51 | 0.076 | 0.096 | 0.116 | 0.128 | 54 |
| 52 | 0.078 | 0.098 | 0.118 | 0.130 | 55 |
| 53 | 0.080 | 0.100 | 0.120 | 0.132 | 56 |
| 54 | 0.083 | 0.103 | 0.123 | 0.135 | 57 |

$$\ell_{55} = 13,200$$

$$Z = d_{[53]+2} - d_{[53]+1}$$

Question 7

In what range is Z ?

- (A) Less than 120
- (B) 120 but less than 150
- (C) 150 but less than 180
- (D) 180 but less than 210
- (E) 210 or more

Data for Question 8 (3 points)

Data for a stationary population:

Average age at death for all whose deaths occur between ages 50 and 60 is 55.567.

$$T_{50} = 2,020,000$$

$$T_{60} = 1,200,000$$

$$\ell_{50} = 87,500$$

Question 8

In what range is the number of deaths between ages 50 and 60 in this stationary population?

- (A) Less than 12,000
- (B) 12,000 but less than 12,500
- (C) 12,500 but less than 13,000
- (D) 13,000 but less than 13,500
- (E) 13,500 or more

Data for Question 9 (3 points)

Selected actuarial values:

$$a_{\overline{n}|} = 7.02358$$

$$v^n = 0.50835$$

Question 9

In what range is $(Ia)_{\overline{3n}|}$?

- (A) Less than 119
- (B) 119 but less than 125
- (C) 125 but less than 131
- (D) 131 but less than 137
- (E) 137 or more

Data for Question 10 (4 points)

$$s(x) = 1 - .005x - .00005x^2, \quad 0 \leq x \leq 100$$

Y = the probability that all of three lives exactly ages 40, 45, and 50 on 1/1/2006 will die during the five-year period beginning on 1/1/2011.

Question 10

In what range is Y?

- (A) Less than 0.00042
- (B) 0.00042 but less than 0.00047
- (C) 0.00047 but less than 0.00052
- (D) 0.00052 but less than 0.00057
- (E) 0.00057 or more

Data for Question 11 (2 points)

Selected actuarial values:

$$\ell_x = 1000$$

$$q_x^{(1)} = 0.050$$

$$q_x^{(2)} = 0.030$$

$$q_x^{(3)} = 0.300$$

Decrements (1) and (3) are uniformly distributed throughout the year.

Decrement (2) occurs at the end of the year.

Y = the number of decrements at age x due to cause (2).

Question 11

In what range is Y ?

- (A) Less than 19.50
- (B) 19.50 but less than 20.00
- (C) 20.00 but less than 20.50
- (D) 20.50 but less than 21.00
- (E) 21.00 or more

Data for Question 12 (4 points)

Terms of a bond:

| | |
|-------------------|-------------------------------|
| Face amount | \$1,000 |
| Redemption amount | \$1,000 |
| Term | 10 years |
| Coupons | 6%, payable semi-annually |
| Yield rate | 5%, convertible semi-annually |

Question 12

In what range is the Macaulay duration of the bond?

- (A) Less than 7.57
- (B) 7.57 but less than 7.64
- (C) 7.64 but less than 7.71
- (D) 7.71 but less than 7.78
- (E) 7.78 or more

Data for Question 13 (3 points)

Terms of a bond:

| | |
|-------------------|--------------------------------------|
| Face amount | \$1,000 |
| Redemption amount | \$1,000 |
| Term | 10 years |
| Coupons | 3%, payable semi-annually |
| Yield rate | 4% nominal, compounded semi-annually |

Immediately after the fifth coupon payment is made, the bond is sold to yield a rate of 5% nominal, compounded monthly.

X = the absolute value of the difference between the book value and the sale price.

Question 13

In what range is X?

- (A) Less than \$58.00
- (B) \$58.00 but less than \$60.00
- (C) \$60.00 but less than \$62.00
- (D) \$62.00 but less than \$64.00
- (E) \$64.00 or more

Data for Question 14 (5 points)

Smith (age 45) purchases a single premium annuity on 1/1/2006 that has the following characteristics:

| | |
|---------------|---|
| Payments | \$1,000 annually at the beginning of each year starting on 1/1/2026 (age 65) and payable for life. |
| Death benefit | If death occurs before age 65, five annual payments of \$1,000 will be made with the first payment due at the end of the year of Smith's death. |

Selected actuarial values:

$$\ddot{a}_{45:\overline{20}|} = 10.9961$$

$$\ddot{a}_{45} = 13.1949$$

$${}_{20}P_{45} = 0.8771$$

Interest rate: 7.0%, compounded annually

Question 14

In what range is the single premium for this annuity?

- (A) Less than \$2,425
- (B) \$2,425 but less than \$2,440
- (C) \$2,440 but less than \$2,455
- (D) \$2,455 but less than \$2,470
- (E) \$2,470 or more

Data for Question 15 (4 points)

Smith (age 65) purchases a single premium annuity on 1/1/2006 that has the following provisions:

- Payments \$1,000 annually for Smith's life with the first payment on 12/31/2006.
- Death benefit A survivor annuity of \$1,000 per year will be shared by Smith's twin children (age 35 on 1/1/2006) beginning on the 12/31 following Smith's death. Payments are made only if both children are alive.

Selected actuarial values:

$$\begin{aligned}a_{35} &= 13.5119 \\a_{65} &= 8.7004 \\a_{35:35} &= 13.1360 \\a_{35:65} &= 8.6451 \\a_{\overline{35:35:65}} &= 13.8888\end{aligned}$$

Question 15

In what range is the single premium for this annuity?

- (A) Less than \$13,200
- (B) \$13,200 but less than \$13,400
- (C) \$13,400 but less than \$13,600
- (D) \$13,600 but less than \$13,800
- (E) \$13,800 or more

Data for Question 16 (3 points)

The following annuities are actuarially equivalent:

- I. A life annuity of \$1,000 per month payable at the beginning of each month starting at age 55.
- II. A life annuity that provides for the payment of X per month payable at the beginning of each month from age 55 to age 62 and (X - \$800) per month thereafter.

Selected actuarial values:

$$\ddot{a}_{55}^{(12)} = 11.3300$$

$$\ddot{a}_{55:\overline{7}|}^{(12)} = 5.5000$$

Question 16

In what range is X?

- (A) Less than \$1,100
- (B) \$1,100 but less than \$1,200
- (C) \$1,200 but less than \$1,300
- (D) \$1,300 but less than \$1,400
- (E) \$1,400 or more

Data for Question 17 (3 points)

| | <u>Account A</u> | <u>Account B</u> |
|------------------------|--|---------------------------------|
| Deposits | \$100 on 1/1/2006 \$100 on 4/1/2006 \$100 on 7/1/2006 \$100 on 10/1/2006 | \$400 on 5/1/2006 |
| Credited interest rate | $\delta = 6.0\%$ at 1/1/2006 δ increases to 6.35% on 3/1/2006 and remains at this level for the remainder of the year | $d^{(6)}$ for the entire period |

At 12/31/2006, Account A and Account B have the same value.

Question 17

In what range is $d^{(6)}$?

- (A) Less than 0.0597
- (B) 0.0597 but less than 0.0602
- (C) 0.0602 but less than 0.0607
- (D) 0.0607 but less than 0.0612
- (E) 0.0612 or more

Data for Question 18 (3 points)

A 10-year bond is purchased at par value with annual coupons.

D = Macaulay duration if the yield to maturity is 5.0%

Question 18

In what range is D?

- (A) Less than 7.75
- (B) 7.75 but less than 7.85
- (C) 7.85 but less than 7.95
- (D) 7.95 but less than 8.05
- (E) 8.05 or more

Data for Question 19 (4 points)

Mortality Table A

$$\ell_x = 1000\sqrt{240 - 2x}, \quad 0 \leq x \leq 120$$

Mortality Table B

Constant force of mortality = $2\mu_{43}$ from
Mortality Table A

$Z = {}_{2|}q_{41}$ under Mortality Table B

Question 19

In what range is Z ?

- (A) Less than 0.014
- (B) 0.014 but less than 0.017
- (C) 0.017 but less than 0.020
- (D) 0.020 but less than 0.023
- (E) 0.023 or more

Data for Question 20 (4 points)

A fire department admits 40 rookies each year at the age of 27, each of whom has a complete expectation of life of 55 years.

The firefighters are retired upon reaching age 52.

The fire department has reached a stationary population with 850 members having an average age of 38 years.

X = the average age at death of the current firefighters.

Question 20

In what range is X ?

- (A) Less than 79
- (B) 79 but less than 82
- (C) 82 but less than 85
- (D) 85 but less than 88
- (E) 88 or more

Data for Question 21 (2 points)

Given the following spot rates from a yield curve:

| <u>Duration</u> | <u>Annual yield</u> |
|-----------------|---------------------|
| 1 year | 2.00% |
| 2 years | 4.00% |
| 3 years | 5.00% |

A three-year bond with annual coupons of X is purchased at par value for \$5,000.

Question 21

In what range is X?

- (A) Less than \$243
- (B) \$243 but less than \$245
- (C) \$245 but less than \$247
- (D) \$247 but less than \$249
- (E) \$249 or more

Data for Question 22 (2 points)

Given the following spot rates from a yield curve:

| <u>Duration</u> | <u>Annual yield</u> |
|-----------------|-------------------------|
| 1 year | 2.00% |
| 2 years | 4.00% |
| 3 years | 5.00% |

Future forward rates are based on current spot rates.

X = the one-year forward rate implied by the yield curve at the end of the second year.

Question 22

In what range is X?

- (A) Less than 3.50%
- (B) 3.50% but less than 5.00%
- (C) 5.00% but less than 6.50%
- (D) 6.50% but less than 8.00%
- (E) 8.00% or more

Data for Question 23 (3 points)

Data from a double decrement table:

$${}_tq_x^{(1)} = 0.1t, \quad 0 \leq t \leq 1$$

$${}_tq_x^{(2)} = ct, \quad 0 \leq t \leq 1; \quad c \text{ is a constant, } 0 \leq c \leq 1$$

$$q_x^{(T)} = 0.28$$

Question 23

In what range is $q_x^{(2)}$?

- (A) Less than 0.175
- (B) 0.175 but less than 0.185
- (C) 0.185 but less than 0.195
- (D) 0.195 but less than 0.205
- (E) 0.205 or more

Data for Question 24 (3 points)

Selected values from a 1-year select and ultimate mortality table:

$$p_x = 0.955$$

$$q_{[x]} = 0.5q_x$$

Other data:

Interest rate: 6.0%, compounded annually

$$A_x = 0.5842$$

Question 24

In what range is $\ddot{a}_{[x]}$?

- (A) Less than 7.000
- (B) 7.000 but less than 7.200
- (C) 7.200 but less than 7.400
- (D) 7.400 but less than 7.600
- (E) 7.600 or more

Data for Question 25 (4 points)

Perpetuity: Annual end-of-year payments starting at \$1 and increasing by \$2 each year

Selected actuarial values:

$$\ddot{s}_{\overline{2n}|} = 72$$

$$\ddot{a}_{\overline{n}|} = 6$$

$$i > 0\%$$

Question 25

In what range is the present value of this perpetuity?

- (A) Less than \$130
- (B) \$130 but less than \$142
- (C) \$142 but less than \$154
- (D) \$154 but less than \$166
- (E) \$166 or more

Data for Question 26 (3 points)

Interest rate: 5.0%, compounded annually

$$p_x = 0.9 \text{ for all } x$$

Question 26

In what range is $A_{\overline{x}|x}$?

- (A) Less than 0.52
- (B) 0.52 but less than 0.58
- (C) 0.58 but less than 0.64
- (D) 0.64 but less than 0.70
- (E) 0.70 or more

Data for Question 27 (4 points)

The following are actuarially equivalent:

- I. \$100,000
- II. X payable annually at the end of each year during the joint life of two lives both age 40, payable for 10 years certain and up to 20 years thereafter while at least one is alive.

Selected annuity and mortality values:

$${}_{10}P_{40} = 0.8848$$

$${}_{20}P_{50} = 0.5217$$

$$a_{50} = 12.522$$

$$a_{70} = 6.293$$

$$a_{50:50} = 9.695$$

$$a_{70:70} = 4.054$$

Interest rate: 4.0%, compounded annually

Question 27

In what range is X?

- (A) Less than \$6,000
- (B) \$6,000 but less than \$6,100
- (C) \$6,100 but less than \$6,200
- (D) \$6,200 but less than \$6,300
- (E) \$6,300 or more

Data for Question 28 (3 points)

A multiple decrement table is constructed based on the following three single decrement tables:

| <u>Decrement #1</u> | | <u>Decrement #2</u> | | <u>Decrement #3</u> | |
|-----------------------|-------------------------|-----------------------|-------------------------|-----------------------|-------------------------|
| <u>x</u> | <u>l_x</u> | <u>x</u> | <u>l_x</u> | <u>x</u> | <u>l_x</u> |
| 25 | 100 | 25 | 100 | 25 | 100 |
| 26 | 90 | 26 | 80 | 26 | 70 |

Uniform distribution of decrement is assumed in each of the above single decrement tables.

Question 28

In what range is $q_{25}^{(2)}$?

- (A) Less than 0.154
- (B) 0.154 but less than 0.161
- (C) 0.161 but less than 0.168
- (D) 0.168 but less than 0.175
- (E) 0.175 or more

Data for Question 29 (3 points)

Selected actuarial values:

$$\ell_0 = 1140$$

$$\ell_{53} = 270$$

$$p_x = 0.75, x \geq 50$$

Deaths are uniformly distributed over $[0,50]$.

Question 29

In what range is the number of deaths occurring between ages 45.50 and 55.75?

- (A) Less than 558.0
- (B) 558.0 but less than 560.0
- (C) 560.0 but less than 562.0
- (D) 562.0 but less than 564.0
- (E) 564.0 or more

Data for Question 30 (3 points)

Data for a single premium life insurance policy:

Age at issue 40

Benefit \$10,000 payable at end of year of death

If death occurs before age 50, an additional benefit equal to 50% of the single premium accumulated with interest at 5.0%, compounded annually, is payable at the end of the year of death.

Selected actuarial values:

$$A_{40} = 0.31549$$

$${}_5E_{40} = 0.71823$$

$${}_5E_{45} = 0.71230$$

Interest rate: 5.0%, compounded annually

Question 30

In what range is the net single premium?

- (A) Less than \$3,300
- (B) \$3,300 but less than \$3,700
- (C) \$3,700 but less than \$4,100
- (D) \$4,100 but less than \$4,500
- (E) \$4,500 or more

2006 EA-1 EXAMINATION
ANSWER KEY

| Question | Answer |
|----------|--------|
| 1 | D |
| 2 | A |
| 3 | C |
| 4 | B |
| 5 | D |
| 6 | C |
| 7 | A |
| 8 | B |
| 9 | D |
| 10 | B |
| 11 | B |
| 12 | D |
| 13 | D |
| 14 | B |
| 15 | B |
| 16 | E |
| 17 | A |
| 18 | E |
| 19 | E |
| 20 | E |
| 21 | C |
| 22 | D |
| 23 | C |
| 24 | D |
| 25 | B |
| 26 | B |
| 27 | B |
| 28 | C |
| 29 | D |
| 30 | B |