SAMPLING ERROR MODELLING OF POVERTY AND INCOME STATISTICS FOR STATES

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1 Introduction

This paper develops models for sampling error covariance matrices of estimates of age group poverty rates, median income, and per capita income from the Current Population Survey's (CPS) March Supplement. (Not part of the normal CPS estimation procedures, the covariances for 1989 to 1993 were estimated by Bob Fay using `vplex`, a variance estimation program (Fay 1989).) The ultimate objective is to use the sampling error models developed here, in combination with models for the time series of poverty rates and income measures, to improve estimates of the "true" (unobserved) state poverty rates and income measures.

Our models account for three features of the CPS sampling error covariance structure: (1) differences in the variances by state (through random state effects); (2) dependence of variances on sample size and on the level of the estimates (through a generalized variance function, GVF); and (3) sampling error correlations over time (through an autoregressive-moving-average (ARMA) time series model). Section 2 describes our general modelling approach. Then, in Section 3, we discuss details of the model development for the CPS application.

2 General Approach

A general model used in both time series and small area applications starts by writing $y_i = Y_i + e_i$ where the y_i's are direct survey estimators, the Y_i's are the population characteristics ("truth") being estimated, and the e_i's are the sampling errors in the y_i's. In our application the single index i would index both states and years. In matrix-vector notation and assuming normality, the general model for the observed data, $\mathbf{y} = (y_1, \ldots, y_n)'$, is

$$\begin{array}{lll} \mathbf{y} & = \mathbf{Y} + \mathbf{e} & \mathbf{Y} = (Y_1, \ldots, Y_n)', \text{ etc.} \\ \mathbf{Y} & = \mathbf{X}\beta + \mathbf{u} & \mathbf{u} \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}(\psi)) \\ \mathbf{e} & \sim \mathrm{N}(\mathbf{0}, \mathbf{V}(\eta)) & \text{independent of } \mathbf{u}. \end{array} \quad (2.1)$$

The data assumed available are both $\mathbf{y}$ and $\mathbf{C}$, the latter being a direct estimate of the variance-covariance matrix of the sampling errors $\mathbf{e}$. The parameters of the model (2.1) are the $p \times 1$ vector of regression parameters β, and the $r \times 1$ and $m \times 1$ vectors of parameters ψ and η that determine the $n \times n$ covariance matrices $\boldsymbol{\Sigma}(\psi)$ and $\mathbf{V}(\eta)$. Having postulated a model of form (2.1), the task is to use the data $\mathbf{y}$ and $\mathbf{C}$ to make inferences about the parameters (β, ψ, η) and, ultimately, about the true population quantities $\mathbf{Y}$.

In this paper we focus on the use of $\mathbf{C}$ to make inferences about η, which we shall call sampling error modelling. It has been fairly standard in small area estimation to simply assume $\mathbf{V} = \mathbf{C}$ is known (so η corresponds to all distinct elements of $\mathbf{V}$). This is not really a "model" and has the disadvantage that it fails to acknowledge any uncertainty about $\mathbf{V}$. We shall use an approach suggested by Bell and Otto (1993), which is based on assuming a Wishart distribution (DeGroot 1970, Section 5.5) for $\mathbf{C}$ as a working model:

$$\nu\mathbf{C} \sim \text{Wishart}\big(\nu, \mathbf{V}(\eta)\big). \quad (2.2)$$

This model allows us to recognize uncertainty about $\mathbf{V}$ by recognizing uncertainty about the parameters η that determine $\mathbf{V} = \mathbf{V}(\eta)$. Generally, the degrees of freedom parameter ν will also be unknown. In some cases ν can be set to an estimated value (see Bell and Otto 1993). Alternatively, we could let ν depend on model parameters in η (expanding the definition of η, and writing $\nu(\eta)$). In either case, we shall assume η defines the unknown parameters of the model (2.2)

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Though estimates of sampling error variances and covariances are rarely of the simple form that would lead exactly to a Wishart distribution, the model (2.2) may still prove useful since it provides an objective means of using the data $\mathbf{C}$ in making inferences about η, and hence about $\mathbf{V}$. Under a classical (non-Bayesian) approach, the model (2.2) can be estimated by maximizing the Wishart likelihood,

$$\mathcal{L}(\eta|\mathbf{C}) = g(\nu)|\mathbf{V}(\eta)|^{-\nu/2} \times \exp\left(-1/2\,\mathrm{tr}\big(\mathbf{V}(\eta)^{-1}\mathbf{C}\big)\right), \quad (2.3)$$

where $g(\nu)$ includes those terms in the Wishart density not explicitly present in (2.3)—these involve ν but not $\mathbf{V}(\eta)$. Under a Bayesian approach, (2.3) can be multiplied by a (possibly noninformative) prior distribution, $p(\eta)$, which yields something proportional to the posterior $p(\eta|\mathbf{C})$.

The CPS sample design is such that, since 1985, samples for different states are drawn independently. By ordering the observations $\mathbf{y}$ so that all the y_i for each state occur in succession, $\mathbf{V}(\eta)$, and correspondingly $\mathbf{C}$, will be block diagonal. Thus, the diagonal blocks $\mathbf{C}_s$ of $\mathbf{C}$ will be assumed to have independent Wishart distributions analogous to (2.2). The next section develops this model using generalized variance functions (GVFs) and time series models to define $\mathbf{V}(\eta)$.

3 Modelling the CPS Sampling Errors

In this section we develop models for the sampling errors in the state CPS estimates of age group poverty rates, median income, and per capita income. Section 3.1 analyzes variation of the sampling error variances over states and years. Section 3.2 examines the relation of the variances to the estimates using generalized variance functions (GVFs). Section 3.3 analyzes the correlations of the sampling errors over time. These are all preliminary analyses to Sections 3.4 and 3.5, which put the results together into a full model for the state sampling error covariance matrices $\mathbf{C}_s$.

3.1 Preliminary Analyses of the Variances

To check whether the sampling error variances vary by state and by year, we did an analysis of variance (ANOVA) with the weighted log variances for each statistic. The variances, v_{st}, were weighted by the state sample sizes, n_{st}, defined as the number of households in sample for state s and year t. The log transformation was used to make the data more normal.

$$\log(n_{st}v_{st})) = \log(Y_{st}^2) + \mathrm{State}_s + \mathrm{Year}_t + z_{st}, \quad s = 1, \ldots, 51 \text{ and } t = 1989, \ldots, 1993. \quad (3.1)$$

Since we do not have the true values, Y_{st}, we substitute the corresponding direct estimates y_{st} in (3.1).

The usual ANOVA F-statistics are suspect for (3.1) because the z_{st}'s may not be independent over time, so we concentrated on simply examining the mean squares (MS). The MSs for Years are an order of magnitude smaller than those for States for the age 0 to 4 poverty rate and both income statistics, but only marginally smaller for the other poverty rates. We suspect that some of the variation over years and states can be captured by a generalized variance function (GVF) that permits a more general dependence of the variances on the level of the data than is accounted for by $\log(Y_{st}^2)$ only. This may lessen the need for a State effect and even eliminate the need for a Year effect.

3.2 Generalized Variance Functions

We modelled the relation of the variances to their estimates using GVF models (Wolter, 1985, p. 203) that extend (3.1) as follows:

$$\begin{aligned} \log(n_s v_{st})) &= \log(\mathrm{GVF}(Y_{st})) + \mathrm{State}_s + \mathrm{Year}_t \\ &+ z_{st}, \quad (3.2) \\ s &= 1, \ldots, 51 \text{ and} \\ t &= 1989, \ldots, 1993. \end{aligned}$$

The GVFs take on one of the forms in Table 1. (We show the GVFs in terms of the variance, whereas Wolter gives them in terms of the relative variance.) Note that for the poverty rates, which are proportions, the usual binomial distribution theory suggests a model of the form $\alpha + \beta Y + \gamma Y^2$ with $\alpha = 0$ and $\gamma = -\beta$. We use a bias corrected version of Akaike's AIC (Hurvich and Tsai, 1991) to discriminate between our models (the model with minimum AIC being favored). The AIC results in Table 1 show that for median income, the AICs are not very different (within 2 of each other) except that for (v), which is significantly worse. For per capita income, however, models (iii) and (v) are preferred over the others.

For Poverty Rates, the AIC results in Table 1 show that all the GVFs tried are a major improvement over the constant relative variance model, (i), with models (iv) and (vi) consistently being the two best models. AICs for models (ii), (iii), and (v) are not that much higher, however, so that our general conclusion is that while use of a GVF more general than the constant relative variance model is important, the particular choice among (*ii*) to (*vi*) may not be essential.

Table 1: AICs OF GVF MODELS

INCOME STATISTICS

		AIC	
	GVF	**Med**	**P.C.**
(*i*)	γY^2 ($\gamma = e^\mu$ in (3.1))	197.5	224.4
(*ii*)	$\alpha + \beta Y$	196.8	227.4
(*iii*)	$\alpha + \beta Y + \gamma Y^2$	195.5	211.2
(*iv*)	$(\alpha + \beta / Y)^{-1}$	196.9	223.3
(*v*)	$(\alpha + \beta / Y + \gamma / Y^2)^{-1}$	204.1	214.6
(*vi*)	αY^β	197.1	227.6

POVERTY RATES

	AIC for Age			
GVF	**0–4**	**5–17**	**18–64**	**65+**
(*i*)	315.9	314.8	231.1	406.2
(*ii*)	217.4[3]	254.2	217.3	361.2
(*iii*)	220.6	255.6	218.6	363.0
(*iv*)	217.4[2]	250.4[2]	215.1[1]	359.8[1]
(*v*)	220.6	253.4	218.4	363.2
(*vi*)	217.3[1]	250.2[1]	215.8[2]	360.4[2]

The superscripts [1], [2], and [3] show the first, second, and third best fitting models. The third model is shown only when its AIC is close to those of the first two.

3.3 Analysis of the Correlations

As noted earlier, in recent years CPS samples for different states have been drawn independently, so that sampling error correlations between states are approximately zero. Correlations over time between sampling errors in CPS estimates for any given state are generated by correlations in individual responses over time and by the nature of the CPS sample design. The 4–8–4 CPS rotation pattern leads to autocorrelation in monthly sampling errors that has been investigated for labor force characteristics by Train, Cahoon, and Makens (1978), Dempster and Hwang (1993), and Adam and Fuller (1992). Time series models for sampling errors in monthly CPS estimates have been developed by Tiller (1992) and Bell and Hillmer (1994). (Sampling error autocorrelation in monthly CPS estimates is also affected by composite estimation, which is not done for the annual March supplement estimates.)

Sampling error autocorrelation in the annual CPS estimates that we analyze here should follow a simpler pattern, since the 4–8–4 monthly rotation scheme produces a 50% sample overlap one year apart, and no overlap two or more years apart. If samples comprising different rotation groups were selected independently, this would mean that sampling errors more than one year apart would be approximately uncorrelated, which would correspond to a moving average model of order one (MA(1)). However, two aspects of the CPS design can lead to sampling error autocorrelation extending beyond the sample overlap. The first is the practice of replacing households that rotate out of the sample by neighboring households, since neighbors probably exhibit correlation in economic characteristics such as income. The second aspect is the fact that primary sampling units (PSUs) are redrawn only for CPS redesigns that occur about every 10 years, so that the between PSU component of sampling error probably contributes autocorrelation for many years due to PSU overlap. Train, Cahoon, and Makens (1978) estimated nonzero sampling error correlations between monthly CPS estimates at time points with no sample overlap. We thus want to examine estimated sampling error autocorrelations for evidence that they are nonzero at lags beyond one year. We postulated an autoregressive-moving average model of order 1,1 (ARMA(1,1)) as a more general structure to allow for this. If the autoregressive parameter is zero, this model reduces to the MA(1) model that corresponds to sampling error autocorrelation only at the one year lag where sample overlap occurs.

Before determining an appropriate time series model to account for sampling error autocorrelation, two other questions should be addressed. First, does the sampling error autocorrelation appear stationary, that is, does $\mathrm{Corr}(e_{st}, e_{s,t-l})$ depend only on the lag l separating the two estimates, and not on the year t? Second, do the autocorrelations show essentially the same pattern for all the states? Notice that building a separate time series model for each state is impractical, given that we have estimated sampling error autocorrelations only for five years for each state.

ANOVA models for each measure were fit to investigate the contributions of state, lag, and year within lag effects to the variation in the (transformed) correlation estimates. The lag effects were estimated to be the most important, followed by the state effects. The mean squares of the year effects nested within the lags were at most 8 percent of the lag effect mean squares. This suggested stationarity may be a reasonable assumption, and that any true variation over states in autocorrelation is secondary to the lag effect.

Lastly, we estimated the means by lag over states and years of the transformed autocorrelations for each statistic and backtransformed them. These are shown in the following table:

Table 2: CPS CORRELATIONS BY LAG

	Lag			
Statistic	**1**	**2**	**3**	**4**
Per Capita Income	0.53	0.31	0.27	0.23
Median Income	0.52	0.30	0.27	0.20
Poor 0–4	0.37	0.14	0.12	0.09
Poor 5–17	0.36	0.19	0.17	0.12
Poor 18–64	0.40	0.22	0.19	0.15
Poor 65 and over	0.30	0.09	0.06	0.07

These results show, as expected, the highest correlation at lag 1, but with evidence of additional correlation at lags 2 through 4. The patterns are reasonably consistent with the postulated ARMA(1,1) model, although the fit would be better if the correlations at lags 3 and 4 showed faster decay towards zero. An ARMA(1,2) model could be used to capture this pattern.

3.4 A Model for the State Sampling Error Covariance Matrices $\mathbf{C}_s$

The analyses of sections 3.1 to 3.3 established that the sampling errors in the CPS estimates of annual state income and poverty characteristics (i) are autocorrelated, with autocorrelation extending beyond lag 1, and (ii) are heteroscedastic, with variances depending on sample size, level (of estimates), and state. The model we shall use for the $\mathbf{C}_s$ accounts for autocorrelation with a time series model (e.g., ARMA(1,1)), and for heteroscedasticity through scaling by sample size, use of a GVF, and use of state effects on variances. It is also possible that variances depend on time, apart from their dependence on level, and that autocorrelation varies by state. But these effects appear secondary to those we shall account for in our model, and they also would be difficult to include in the model. Thus, we shall ignore these possible effects in the remainder of the analysis.

As noted in Section 2, our model for the $\mathbf{C}_s$ will be based on the Wishart distribution, with a parametric representation of $E(\mathbf{C}_s) = \mathbf{V}_s(\eta)$. To account for the effects noted above, $\mathbf{V}_s(\eta)$ can take the following form:

$$\mathbf{V}_s(\eta) = \omega_s \mathbf{D}_s(\alpha,\beta,\gamma)\mathbf{R}(\phi,\theta)\mathbf{D}_s(\alpha,\beta,\gamma) \qquad (3.3)$$

where ω_s is the effect of state s on the covariances, $\mathbf{D}_s(\alpha,\beta,\gamma)$ is a diagonal matrix with entries corresponding to square roots of GVFs divided by sample sizes n_{st} (e.g, $\sqrt{(\alpha+\beta y_{st}+\gamma y_{st}^2)/n_{st}}$, with estimates y_{st} replacing the true values Y_{st}), and $\mathbf{R}(\phi,\theta)$ is a 5×5 correlation matrix corresponding to a time series model such as ARMA(1,1) ($e_{st} = \phi e_{s,t-1} + \varepsilon_{st} - \theta\varepsilon_{s,t-1}$). The full model can then be defined by assuming $\nu\mathbf{C}_s$ has a Wishart$(\nu, \mathbf{V}_s(\eta))$ distribution, with $\mathbf{V}_s(\eta)$ given by (3.3) and ν the degrees of freedom (assumed the same for each state).

A problem with the model (3.2) is the number of state effect parameters ω_s (51) for the amount of data available (effectively 255 estimated variances, since the 510 estimated autocorrelations don't contribute information about the ω_s). In fact, some convergence problems were experienced when fitting the models (3.2) (requiring tinkering with initial values to resolve these), and it was suspected that these problems were due to the high ratio of parameters to data. As an alternative to reduce the number of model parameters while still allowing for differing state effects ω_s, the ω_s are assumed to be random effects, with $\tau_s = 1/\omega_s$ coming from a Gamma distribution constrained so that $\mathrm{E}(\omega_s) = 1$. This implies a Gamma$(a+1, a^{-1})$ distribution for τ_s defined by the one parameter a. Thus, the model is

$$\begin{aligned} \nu\mathbf{C}_s &= \mathbf{W}_s/\tau_s \qquad (3.4) \\ \mathbf{W}_s &\sim \text{independent Wishart}(\nu, \widetilde{\mathbf{V}}_s(\eta)) \\ \tau_s &\sim i.i.d.\ \text{Gamma}(a+1, a^{-1}) \end{aligned}$$

with $\widetilde{\mathbf{V}}_s(\eta)$ given by dropping ω_s from (3.3). The degrees of freedom parameter, ν, is assumed common across states s. The density of $\mathbf{C}_s$ can be shown to be

$$\begin{aligned} \mathrm{p}(\mathbf{C}_s) &= g(\nu,a)\left[a + \frac{\nu}{2}\mathrm{tr}(\widetilde{\mathbf{V}}_s(\eta)^{-1}\mathbf{C}_s)\right]^{-(a+1+\frac{\nu k}{2})} \\ &\times\ |\widetilde{\mathbf{V}}_s(\eta)|^{-\nu/2}|\mathbf{C}_s|^{(\nu-k-1)/2} \end{aligned}$$

where

$$\begin{aligned} g(\nu,a) &= \left[\pi^{k(k-1)/4}\prod_{j=1}^{k}\Gamma(\frac{\nu-j+1}{2})\right]^{-1} \\ &\times\ \frac{a^a}{\Gamma(a)}\Gamma(a+1+\nu k/2)(\frac{\nu}{2})^{\nu k/2}. \end{aligned}$$

The density $\mathrm{p}(\mathbf{C}_s)$ is the likelihood function for η that can be maximized in classical inference or used to develop the posterior of η for Bayesian inference.

Note that as $a \to 0$ the Gamma$(a+1, a^{-1})$ distribution becomes diffuse, essentially letting the ω_s be fixed (unrelated) state effects. As $a \to \infty$ the Gamma$(a+1, a^{-1})$ distribution becomes degenerate at 1, implying no state effects ($\omega_s = 1$ for all s).

As another way to understand the model (3.4), note that the Gamma$(a+1, a^{-1})$ distribution for τ_s in (3.4) is the same as $\frac{a+1}{a}$ Gamma$(a+1, (a+1)^{-1})$ (note DeGroot 1970, p. 39), and Gamma$(a+1, (a+1)^{-1})$ is the same as the $\chi^2_{2(a+1)}/2(a+1)$ distribution. Also, in the univariate ($k=1$) case, the Wishart distribution for W_s in (3.4) becomes that of $\tilde{\mathbf{V}}_s(\eta)$ (now a scalar) times a χ^2_ν random variable. It is thus easy to see that the distribution for $\mathbf{C}_s$ in the univariate case is that of $\frac{a}{a+1}\tilde{\mathbf{V}}_s(\eta)$ times an $F(\nu, 2(a+1))$ random variable. For $k > 1$ then, apart from the $\frac{a}{a+1}$ factor, the distribution of $\mathbf{C}_s$ implied by (3.4) is something like a multivariate generalization of the F-distribution. (Though the label multivariate F has been used for distributions related to the joint distribution of only the diagonal elements of $\mathbf{C}_s$; see Johnson and Kotz (1972, pp. 240–243).) Unless a is "large," the model (3.4) implies a longer tail in the distribution of $\mathbf{C}_s$ than the Wishart (or χ^2). This is needed to accomodate the variation across states.

A related random effect variance model was proposed by Kleffe and Rao (1992), and studied further by Arora and Lahiri (1995). The model used was simpler than (3.4), being for the univariate case and assuming independent sampling errors for different small area estimators. It also differed from (3.4) in that a distribution was assumed directly for the small area variances, rather than allowing random small area effects on the variances (as was done in assuming ω_s random to get from (3.3) to (3.4)). The random variance distribution was left unspecified by Kleffe and Rao, while Arora and Lahiri assumed a gamma distribution for the precisions (reciprocals of the variances).

3.5 Sampling Error Model Estimation

We estimated by maximum likelihood 24 different variants of the model (3.4) for the sampling error covariance matrices, $\mathbf{C}_s$, for each statistic. The variants corresponded to all combinations of 8 GVFs and 3 ARMA models. The GVFs included those in Table 1, plus a constant variance GVF, and the CPS GVF, $\beta Y + \gamma Y^2$, which is (*iii*) with $\alpha = 0$. The ARMA models tried were the AR(1), ARMA(1,1), and ARMA(1,2). Table 3 summarizes some of the results.

Consistent with the preliminary results of section 3.2, for the poverty rates the constant relative variance model fit poorly. Its AICs were higher than those of the best fitting GVF by about 90 to 180, depending on the age group. The constant variance model also fit poorly, its AICs being about 140 or 150 higher than the best. Among the other GVF models, the AIC differences were at most 5. The results for three candidate GVFs are shown in Table 3. The $(\alpha + \beta/Y + \gamma/Y^2)^{-1}$ GVF had the lowest AIC for each age group except 0–4, for which the CPS variance formula, $\beta Y + \gamma Y^2$, was best. However, as the AIC differences are not great, for the poverty rates any GVF other than constant variance or constant relative variance might be used. In particular, the CPS variance formula might be picked for its relatively good fit, familiarity, and theoretical appeal.

Table 3: AIC DIFFERENCES AND a AND ν PARAMETER ESTIMATES

Poverty Rates

Age	GVF	∇AIC	a	ν
0–4	$\alpha + \beta Y + \gamma Y^2$	-0.4	47.2	20.2
0–4	$\beta Y + \gamma Y^2$	-2.0	46.8	20.2
0–4	$(\alpha + \beta/Y + \gamma/Y^2)^{-1}$	0.0	46.6	20.2
5–17	$\alpha + \beta Y + \gamma Y^2$	0.8	28.5	18.7
5–17	$\beta Y + \gamma Y^2$	2.6	27.9	18.7
5–17	$(\alpha + \beta/Y + \gamma/Y^2)^{-1}$	0.0	27.8	18.8
18–64	$\alpha + \beta Y + \gamma Y^2$	1.9	130.8	20.1
18–64	$\beta Y + \gamma Y^2$	4.6	113.3	20.0
18–64	$(\alpha + \beta/Y + \gamma/Y^2)^{-1}$	0.0	146.1	20.1
65$^+$	$\alpha + \beta Y + \gamma Y^2$	0.2	16.6	14.7
65$^+$	$\beta Y + \gamma Y^2$	0.3	16.3	14.7
65$^+$	$(\alpha + \beta/Y + \gamma/Y^2)^{-1}$	0.0	16.6	14.7

Income

Statistic	GVF	∇AIC	a	ν
P.C.	γY^2	22.3	17.4	22.3
P.C.	$\alpha + \beta Y + \gamma Y^2$	1.7	16.2	23.0
P.C.	$(\alpha + \beta/Y + \gamma/Y^2)^{-1}$	0.0	17.6	23.0
Med	γY^2	8.7	22.3	24.0
Med	$\alpha + \beta Y + \gamma Y^2$	1.5	24.8	24.2
Med	$(\alpha + \beta/Y + \gamma/Y^2)^{-1}$	0.0	25.5	24.2

∇AIC is the difference between the AIC of the given model and the $(\alpha + \beta/Y + \gamma/Y^2)^{-1}$ model for that statistic. a is the random effect parameter, and ν is the degrees of freedom.

For the income statistics, the $(\alpha + \beta/Y + \gamma/Y^2)^{-1}$ GVF again provided the best fit, though the AICs for the $\alpha + \beta Y + \gamma Y^2$ and $(\alpha + \beta/Y)^{-1}$ GVFs were very close. Any of these three GVFs might be used. (Results for the first two of these GVFs are given in Table 3.) The other GVFs tried did not fit very well. However, the fit of the constant relative variance model, (γY^2, see Table 3) was not terrible, so if strong weight were given to simplicity of the model, this GVF might be used.

Of the time series models, our preliminary analysis in Section 3.3 was borne out. The slow decay of the autocorrelations (Table 2.) was fit best by the ARMA(1,2) model. This model had the lowest AIC for all the statistics. (For the 65^+ poverty rates the AR(1) model achieved approximately the same AIC.) On average over the other models, the AICs were 15 higher for the ARMA(1,1) and 27 higher for the AR(1). This makes the ARMA(1,2) the clear choice.

Estimates of the random effects parameter, a, and the degrees of freedom parameter, ν, showed significant variation between statistics (a more so than ν), but little variation between alternative (reasonably fitting) models for a given statistic. This result is encouraging; we would not like to see dependence of a or ν on the GVF or ARMA model chosen. Future research will look more closely at the nature of the random effects, and will explore methods of allowing for uncertainty about them when using the model (2.1) to make inferences about the true population quantitities, Y_{st}.

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