Methodology for Change Variance Estimates:
Testing for a Rise in Child Poverty Rate Greater than Five Percent Between 2000 and 2001

David S. Powers, William R. Bell and Joanna M. Turner
March 7, 2005 (Rev. March 15, 2006[1])

1 Background

The U.S. Census Bureau's Small Area Estimates Branch annually provides the Administration for Children and Families (ACF) in the Department of Health and Human Services (HHS) with model-based estimates of child poverty (ages 0-17). These estimates are used to determine if any states had greater than a 5 percent increase in child poverty rate between two consecutive years. This document addresses change between 2000 and 2001.

The data presented help identify states for which the following equivalent statements are true:

$$\frac{(2001\text{ Poverty Rate}) - (2000\text{ Poverty Rate})}{2000\text{ Poverty Rate}} > 0.05$$

$$(2001\text{ Poverty Rate}) - (2000\text{ Poverty Rate}) > 0.05 \cdot (2000\text{ Poverty Rate})$$

$$(2001\text{ Poverty Rate}) - 1.05 \cdot (2000\text{ Poverty Rate}) > 0$$

This document discusses the derivation of the following estimates and test statistics provided to ACF:

- Variance of (2001 Poverty Rate Estimate – 2000 Poverty Rate Estimate) for children ages 0-17
- Variance of (2001 Poverty Rate Estimate – $1.05\times$(2000 Poverty Rate Estimate)) for children ages 0-17
- *z*-statistics for the test of the null hypothesis that the poverty rate for children ages 0-17 has not increased by more than 5 percent.

The poverty estimates used in this analysis are from the Small Area Income and Poverty Estimates (SAIPE) program. The SAIPE program produces model-based estimates of official poverty as measured by the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). Complete documentation of methods used to produce the 2000 and 2001 state poverty estimates is available under "Documentation" on the SAIPE program's web site, www.census.gov/hhes/www/saipe.html.

[1] Formula (2) has been clarified. See page 10 for details.

Within this document, "change estimate" refers to the 2001 poverty rate for children ages 0-17 minus the 2000 poverty rate for children ages 0-17. Accordingly, "change variance estimate" refers to the variance of this quantity, and "*z*-statistic" refers to the ratio of "change estimate" to the square root of "change variance estimate." Terminology for the five percent change estimates corresponds: "1.05 change estimate" refers to the 2001 poverty rate for children ages 0-17 minus 1.05 times the 2000 poverty rate for children ages 0-17, "1.05 change variance estimate" refers to the variance of this quantity, and "1.05 *z*-statistic" refers to the ratio of "1.05 change estimate" to the square root of "1.05 change variance estimate."

Section 2 below describes the type of hypothesis tests used to assess year-to-year change in the child poverty rates. *Sections 3 and 4* present state and national results for appropriate hypothesis tests. *Section 5* presents mathematical details behind the SAIPE program's poverty estimation, change variance estimation and parameter estimation.

Test results are shown in *Table 1* and *Table 2* located at the end of the document.

2 Hypothesis Tests

The change variance estimate and the change estimate can be used to test whether there is statistically significant evidence that the child poverty rate has increased. Likewise, the 1.05 change variance estimate and the 1.05 change estimate can be used to test whether there is statistically significant evidence that the child poverty rate has increased by more than 5 percent. 1.05 *z*-statistics are created for the one-tailed hypothesis test as follows:

Null Hypothesis: Poverty rate has *not* increased by more than 5 percent

$$(2001\,\text{Poverty Rate}) - 1.05 \cdot (2000\ \text{Poverty Rate}) \le 0$$

Alternative Hypothesis: Poverty rate has increased by more than 5 percent

$$(2001\,\text{Poverty Rate}) - 1.05 \cdot (2000\ \text{Poverty Rate}) > 0$$

Test Statistic (the 1.05 *z*-statistic):

$$z = \frac{((2001\,\text{Poverty Rate Estimate}) - 1.05 \cdot (2000\ \text{Poverty Rate Estimate}))}{\sqrt{\text{Var}((2001\,\text{Poverty Rate Estimate}) - 1.05 \cdot (2000\ \text{Poverty Rate Estimate}))}} \quad (1)$$

Under the SAIPE program's models, *z* has an approximately standard normal distribution when poverty has increased by exactly 5 percent.

A single one-tailed test would be appropriate to test for an increase greater than 5 percent in a particular state. However, since we are testing for an increase greater than 5 percent in all 50 states and Washington, D.C., applying one-tailed tests separately for each state would be inappropriate. In particular, if no state had an increase greater than 5 percent and we performed this test separately for each state, then the probability we would conclude one or more states had an increase greater than 5 percent may be larger than the stated significance level. This is referred to as the problem of "multiple comparisons."

In order to test whether there has been a child poverty rate increase greater than 5 percent in any of the 51 states, we follow the *Bonferroni* approach. The Bonferroni approach addresses the problem of multiple comparisons by using a critical value such that, if all the null hypotheses for a set of tests were true, the probability that one or more of these tests would yield a statistically significant result will be no larger than the specified significance level.

3 State Results

We use a 10 percent significance level for our hypothesis tests. For a set of 51 tests (for the 50 states and Washington, D.C.) with the standard normal *z*-statistic, the Bonferroni 10 percent one-tailed critical value is 2.88. If any state has a 1.05 *z*-statistic greater than 2.88, then there is evidence that the true child poverty rate for that state increased by more than 5 percent. We find that no states have a 1.05 *z*-statistic greater than 2.88 when comparing 2000 and 2001 child poverty rates. Thus, using the Bonferroni test, *we do not find statistical evidence that any state has a child poverty rate increase greater than 5 percent between 2000 and 2001.*

As described, the Bonferroni approach is appropriate for answering the question, "Is there evidence that *any* state had a child poverty rate increase exceeding 5 percent?" A different critical value would be appropriate to test for evidence of a child poverty rate increase greater than 5 percent in a *particular* state that was selected in advance, i.e., not selected based on looking at the results for all the states. The critical value when an individual state is selected in advance is 1.28, the cutoff for the one-tailed test with 10 percent significance level.

No state has a 1.05 *z*-statistic greater than 1.28. Therefore, even ignoring multiple comparison issues and running separate 10 percent tests for each state individually, no states show a statistically significant increase in child poverty rate greater than 5 percent between 2000 and 2001.

The results for each state are presented in Table 1 and Table 2. Table 1 contains the point estimates, and Table 2 contains the standard errors and *z*-statistics. The critical value of 2.88 should be used when checking for statistically significant evidence (at the 10 percent level) that any state had a child poverty rate increase, or an increase greater than 5 percent, and the critical value of 1.28 should be used by individual states examining their own results separately.

4 National Results

In addition to state-level child poverty rates, we also consider the child poverty rate at the national level. The official national poverty estimates are direct estimates from the ASEC. Standard errors for these estimates are computed using formula (1) and Table 3 of U.S. Census Bureau's "Source and Accuracy of Estimates for Poverty in the United States," available at www.census.gov/hhes/poverty/poverty00/pov00src.pdf for 2000 and at www.census.gov/hhes/poverty/poverty01/p60-219sa.pdf for 2001.[2] These estimates and standard errors are the latest available as of the posted release date and are not subsequently updated.

The first line of Table 1 and Table 2 contains point estimates, standard errors and *z*-statistics for the ASEC estimate of the poverty rate for the United States as a whole. We see from Table 1 that, while the estimated U.S. poverty rate increased between 2000 and 2001, the percent increase was only 0.6 percent, which is considerably less than five percent. Hence, the *z*-statistic does not reject the null hypothesis (*z*-statistic = -1.77 < 1.28), and we do not find evidence (at the 10 percent level of significance) of greater than a five percent increase in the United States' child poverty rate between 2000 and 2001.

To compute the 1.05 *z*-statistic at the national level, we use equation (1) for *z* given in Section 2. The variance in the denominator is computed as:[3]

$$\mathrm{Var}((\text{2001 Poverty Rate Estimate}) - 1.05\cdot(\text{2000 Poverty Rate Estimate})) = s_x^{\,2} + (1.05\cdot s_y)^2 - 2rs_x(1.05\cdot s_y) \quad ,$$

where

s_x = standard error of 2001 poverty rate for children ages 0-17 in poverty

$1.05\cdot s_y$ = standard error of 1.05 × 2000 poverty rate for children ages 0-17 in poverty

r = correlation coefficient for year-to-year comparisons of ASEC poverty estimates of proportions.

Here, *r* equals 0.45 since we are using the expanded ASEC sample and comparing estimates for all people.[4]

[2] Standard errors for national poverty *rates* are computed as the ratio of the standard error of the national estimated number in poverty to the Population Estimates Program national population estimate (adjusted to represent the population covered by the ASEC). The national population estimate has non-sampling error but no sampling error.
[3] This is the standard ASEC generalized variance formula for the *standard error of a difference* (squared). See formula (3) and the accompanying text on page 7 of U.S. Census Bureau's "Source and Accuracy of Estimates for Poverty in the United States," available at www.census.gov/hhes/poverty/poverty00/pov00src.pdf for 2000.
[4] See Table 7 on page 9 of U.S. Census Bureau's "Source and Accuracy of Estimates for Poverty in the United States," available at www.census.gov/hhes/poverty/poverty00/pov00src.pdf for 2000.

5 Mathematical Details

5.1 State Poverty Model

The SAIPE program's poverty models employ both direct survey-based estimates of poverty from the ASEC and regression predictions of poverty based on administrative records and Census 2000 data. The SAIPE program's state poverty model is defined as follows:

$$
\begin{aligned}
y_i &= Y_i + e_i \qquad e_i \sim N(0, V_{ei}) \\
Y_i &= X_i\beta_i + u_i \quad\; u_i \sim N(0, \sigma_{ui}^2 I)
\end{aligned}
\quad ,
$$

where

y_i = vector of 51 state ASEC estimates of poverty ratios for a given age group and a given year,

Y_i = vector of "true" poverty ratios for a given age group and a given year,

X_i = matrix of predictor variables for a given age group and a given year; β_i contains the corresponding regression coefficients,

u_i = vector of model errors for a given age group and a given year, assumed independent across states; σ_{ui}^2 is their common variance,

e_i = vector of sampling errors for a given age group and a given year, assumed independent across states; V_{ei} is the diagonal matrix giving the sampling error variances for each state for a given age group and a given year.

Poverty ratios for children ages 0-4 and children ages 5-17 are modeled separately. The subscript i = 1, 2, 3, 4 indexes the four ASEC equations for the two years (2000 and 2001) and two age groups (0-4 and 5-17) according to the following scheme:

i = 1: y_1 = 2000 ASEC estimated poverty ratio for children ages 0-4
i = 2: y_2 = 2000 ASEC estimated poverty ratio for children ages 5-17
i = 3: y_3 = 2001 ASEC estimated poverty ratio for children ages 0-4
i = 4: y_4 = 2001 ASEC estimated poverty ratio for children ages 5-17

The coefficient vector, β_i, and the model error variance, σ_{ui}^2, are estimated by Bayesian techniques, treating the estimated sampling error variances, V_{ei}, as known. (Estimation of V_{ei} and σ_{ui}^2 is discussed in *Section 5.4*.) The Bayesian techniques combine the regression predictions with the direct ASEC estimates, weighting the contribution of these two components on the basis of their relative precision, in order to obtain model-based estimates of child poverty rates by state.

Starting in 2001, the year in which 2000 income data are collected, the ASEC estimates are obtained from a significant expansion of the sample. The expanded sample is referred to as the SCHIP sample expansion because it was designed to improve the statistical reliability of certain estimates used in the funding formula for the State Children's Health Insurance Program (SCHIP). The estimated sampling error variances, V_{ei}, and sampling error correlations, σ^2_{ui}, discussed in this document are based upon SCHIP-expanded samples, which are now the ASEC standard.

For full documentation of the SAIPE program's state poverty estimation, please see www.census.gov/hhes/www/saipe/documentation.html.

5.2 Poverty Rates, Ratios and Universes

Poverty ratios for children ages 0-4 are defined as the ASEC estimated number of children ages 0-4 in poverty divided by the ASEC estimated population ages 0-4. Likewise, poverty ratios for children ages 5-17 are defined as the ASEC estimated number of children ages 5-17 in poverty divided by the ASEC estimated population ages 5-17. Poverty *rates* differ from poverty *ratios* in that they have different denominators. Poverty rates have as their denominator the ASEC demographic *poverty universe* (described below), whereas poverty ratios have as their denominator the ASEC *population estimate*.

Both the ASEC demographic poverty universe and ASEC population estimate exclude people in military barracks and institutional group quarters since the ASEC does not sample from these groups. However, the poverty universe also excludes children ages 0-14 not related to householder by birth, marriage or adoption since there is no elder relative to answer the income portion of the ASEC questionnaire for these children, and income questions are not asked of children under age 15. (For further discussion of poverty measurement, see www.census.gov/hhes/poverty/povdef.html. For further discussion of ASEC concepts and definitions, see www.census.gov/population/www/cps/cpsdef.html.)

The SAIPE program's state models are run using poverty ratios instead of poverty rates since construction of the poverty rates is more straightforward and more reliable. In computing poverty ratios we use ASEC weighted estimates in both the numerators and denominators (as opposed to demographic population estimates in the denominators) because the positive correlation among these ASEC estimates reduces the variance of the resulting poverty ratios. (For further discussion of denominators for poverty rates, see www.census.gov/hhes/www/saipe/techdoc/inputs/denom.html.)

We convert model-based estimates of poverty ratios for children ages 0-4 and children ages 5-17 into estimates of poverty rates for children ages 0-17 by the following steps:

- Multiply the SAIPE program's model-based estimates of poverty *ratios* for each combination (i) of age group and year by corresponding demographic population estimates in order to obtain estimates of the number of children ages 0-4 and 5-17 in poverty in each state. The demographic population estimates are available from the U.S. Census Bureau's population estimates program, and we adjust them to represent the population covered by the ASEC.
- Multiply the estimated number in poverty in each state by a raking factor (defined in Section 5.3) for each combination (i) of age group and year so that the resulting state estimated numbers in poverty sum to the ASEC national estimate for that combination of age group and year.
- For each state add the raked estimate of the number of children ages 0-4 in poverty to the raked estimate of the number of children ages 5-17 in poverty to get the raked estimate of the number of children ages 0-17 in poverty for a given year.
- Form the estimated poverty *rates* for children ages 0-17 by dividing the estimated number of children ages 0-17 in poverty by the demographic poverty universe estimate for children ages 0-17 (poverty universe for children ages 0-4 plus poverty universe for children ages 5-17).

Note that in the first step we multiply the estimated poverty ratios by the demographic estimates of population rather than by the ASEC estimates of population. The demographic estimates of population have no sampling error and, though they contain other (nonsampling) errors, are considered to be more accurate than population estimates constructed from ASEC sample data. The demographic population estimates are thus more appropriate for multiplying the estimated poverty ratios, though the ASEC population estimates are more suitable as denominators for the poverty ratios (due to their correlation with the poverty ratio numerators, as noted above).

The ASEC estimates we model use data from interviews conducted in February, March, and April of a given year (the survey year, SY) regarding income from the previous year (the income year, IY). The relevant population estimates and poverty universes to use as denominators in the poverty rates and poverty ratios are those for the survey year. Therefore, the estimated poverty ratios and poverty rates for 2000 use population estimates and poverty universes for 2001, and the estimated poverty ratios and poverty rates for 2001 use population estimates and poverty universes for 2002.

5.3 Change Variance Estimates

This section describes mathematical details behind the computation of change variance estimates and 1.05 change variance estimates. The square roots of these variance estimates form the denominators of the z-statistics and 1.05 z-statistics used to assess change in child poverty rates.

We represent the demographic population estimates in mathematical notation as:

N_{1k} = 2001 demographic population estimate for children ages 0-4 in state k,

N_{2k} = 2001 demographic population estimate for children ages 5-17 in state k,

$N_{3k} =$ 2002 demographic population estimate for children ages 0-4 in state k,

$N_{4k} =$ 2002 demographic population estimate for children ages 5-17 in state k,

and we represent the poverty universes as:

U_{1k} = 2001 demographic poverty universe estimate for children ages 0-4 in state k,

U_{2k} = 2001 demographic poverty universe estimate for children ages 5-17 in state k,

U_{3k} = 2002 demographic poverty universe estimate for children ages 0-4 in state k,

U_{4k} = 2002 demographic poverty universe estimate for children ages 5 –17 in state k.

We define scaling factors for the two age groups in each year as:

$$r_{1k} = \frac{N_{1k}}{U_{1k} + U_{2k}}, \quad r_{2k} = \frac{N_{2k}}{U_{1k} + U_{2k}}$$
$$r_{3k} = \frac{N_{3k}}{U_{3k} + U_{4k}}, \quad r_{4k} = \frac{N_{4k}}{U_{3k} + U_{4k}},$$

and we define the raking factor for each combination (i) of age group and year as:

$$RF_i = \frac{\text{CPS direct national estimate of number in poverty for age group - year combination } i}{\sum_k (\text{model - based estimate of number in poverty for state } k \text{ for age group - year combination } i)}.$$

The scaling factors are used to turn the estimated poverty ratios into estimated poverty rates by weighting the ratios in proportion to the number of people in each age group for the given year. The raking factors are used to scale the state poverty ratio estimates such that, when they are multiplied by the state demographic population estimates, the products sum to the national ASEC estimate of the number of children in poverty.

Letting R_i be a 51x51 diagonal matrix with the r_{ik} terms (scaling factors) on the diagonal, the vector of contributions from the ages 0-4 group and the ages 5-17 group to the year 2001 poverty rate can be written as $R_3 \cdot Y_3$ and $R_4 \cdot Y_4$, respectively. The raked estimators of these are then $R_3 \cdot RF_3 \hat{Y}_3$ and $R_4 \cdot RF_4 \hat{Y}_4$. Likewise, the vector of contributions from the ages 0-4 group and the ages 5-17 group to the year 2000 poverty rate are $R_1 \cdot Y_1$ and $R_2 \cdot Y_2$, respectively, and the raked estimators of these are $R_1 \cdot RF_1 \hat{Y}_1$ and $R_2 \cdot RF_2 \hat{Y}_2$.

The error in the change estimate can then be written as:

$$[R_3(Y_3 - RF_3\hat{Y}_3) + R_4(Y_4 - RF_4\hat{Y}_4)] - [R_1(Y_1 - RF_1\hat{Y}_1) + R_2(Y_2 - RF_2\hat{Y}_2)],$$

where $Y_i - RF_i\hat{Y}_i$ is the error in the raked poverty ratio estimates for combination (*i*) of age group and year. The diagonal of the variance matrix of the above expression will be the change variance estimates. Similarly, the error in the 1.05 change estimate can be written as:

$$[R_3(Y_3 - RF_3\hat{Y}_3) + R_4(Y_4 - RF_4\hat{Y}_4)] - 1.05[R_1(Y_1 - RF_1\hat{Y}_1) + R_2(Y_2 - RF_2\hat{Y}_2)],$$

and the diagonal of the variance matrix of this expression will be the 1.05 change variance estimates.

Bell (1999) determined that the vector of prediction errors, $Y_i - RF_i\hat{Y}_i$, for combination (*i*) of age group and year can be expressed as:

$$Y_i - RF_i\hat{Y}_i = A_i \cdot u_i + (A_i - I)\cdot e_i + A_i X_i \beta_i ,$$

where

$$A_i = (1 - RF_i)I + RF_i(I - H_i)(I - M_i) ,$$

$$H_i = \sigma^2_{ui}\Sigma_i^{-1}, \quad \Sigma_i = \sigma^2_{ui}I + V_{ei}, \quad \text{and } M_i = X_i(X_i'\Sigma_i^{-1}X_i)^{-1}X_i'\Sigma_i^{-1} .$$

The term $A_i X_i \beta_i$ can be rewritten as $(1 - RF_i) \times X_i\beta_i$. This is, fundamentally, a bias term that arises from raking to national totals under the model assumption that the regression function $X_i\beta_i$ produces unbiased estimates. (The raking factor, RF_i, also includes some random estimation error.) The model is, of course, an approximation, and the raking is done because it is believed to reduce possible bias arising from failure of the model assumptions. We therefore ignore this bias term in computing measures of error for the raked estimates and compute the covariance matrix based on just the contribution of the first two terms, $A_i \cdot u_i + (A_i - I)\cdot e_i$, to the error.

Proceeding with the assumption that the term $A_i X_i \beta_i$ can be ignored, the errors in the change estimate and the 1.05 change estimate can both be expressed as:

$$\begin{aligned} &R_3[A_3 \cdot u_3 + (A_3 - I)\cdot e_3] + R_4[A_4 \cdot u_4 + (A_4 - I)\cdot e_4] \\ &\quad + \tilde{R}_1[A_1 \cdot u_1 + (A_1 - I)\cdot e_1] + \tilde{R}_2[A_2 \cdot u_2 + (A_2 - I)\cdot e_2] \end{aligned} ,$$

where $\tilde{R}_1$ and $\tilde{R}_2$ are $-1.05R_1$ and $-1.05R_2$, respectively, for the error in the 1.05 change estimate and are $-R_1$ and $-R_2$, respectively, for the error in the change estimate.

The covariance matrix of the above expression can be written as: [5]

$$\sum_i \sum_j [\overline{R}_i \cdot (A_i - I)]Cov(e_i, e_j)[\overline{R}_j \cdot (A_j - I)]' + \sum_i \sum_j [\overline{R}_i \cdot A_i]Cov(u_i, u_j)[\overline{R}_j \cdot A_j]', \quad (2)$$

where, for the 1.05 change variance estimates:

$\overline{R}_i = -1.05R_i$ when i = 1 or 2, and $\overline{R}_i = R_i$ when i = 3 or 4,

and, for the change variance estimates:

$\overline{R}_i = -R_i$ when i = 1 or 2, and $\overline{R}_i = R_i$ when i = 3 or 4.

Note that we assume the sampling errors and model errors are uncorrelated across states and uncorrelated with each other. Therefore, $Cov(e_i, e_j)$ and $Cov(u_i, u_j)$ are diagonal matrices. There are 32 terms altogether in this sum.

5.4 Variances and Correlations Needed for Change Variance Estimates

In order to estimate equation (2) we must first estimate the individual variances and correlation parameters appearing in this expression. We do this in four steps:

- by estimating models for the sampling error in ASEC state estimates using direct estimates of ASEC sampling error variances and covariances;
- by averaging direct estimates of sampling error correlations;
- by estimating models for state ASEC estimates used to produce the state poverty ratio predictions; and
- by treating pairs of ASEC state equations (by age-group and year) jointly via Bayesian techniques to estimate the correlation between model errors in the two equations.

These steps are described in more detail below.

Sampling Error Variances

We estimated the sampling error variances, V_{ei}, for each age-group poverty ratio (0-4 and 5-17) by fitting sampling error models to directly-estimated ASEC sampling error covariance matrices for 2000 and 2001 for each state. We produced the latter using the VPLX program, as described in Fay and Train (1995). Otto and Bell (1995) discuss the type of sampling error models used. Since we had only two years of sampling error variance and covariance estimates, we simplified the sampling error model slightly (dropping the random effects discussed in Otto and Bell (1995)). Separately for each age-group poverty ratio, we fit the sampling error models to the

[5] The assignments for *R* shown in formula (2) differ from the assignments shown in the 3/7/05 posting of this document. In the previous version, the assignments for *R* had been shown as positive for all *i*, when in fact the assignments are negative when *i* = 1, 2. Computations for posted results have always used the correct assignments for *R* as given above.

directly-estimated state covariance matrices by maximum likelihood assuming a Wishart distribution for the covariance matrices. The models allow the sampling variances (nonzero elements of the diagonal matrices, V_{ei}) to differ across states and years through a generalized variance function that depends on the level of the poverty ratio estimates and on the ASEC state sample sizes. The models assume, however, that the sampling error correlations between years (ρ_{e13} and ρ_{e24}) are constant across states for a given poverty ratio. Also, note that because we use separate sampling error models for each age-group poverty ratio, the fitted sampling error models do not provide estimates of sampling error correlations between the poverty ratios for different age groups.

Sampling Error Correlations

We estimated the sampling error correlations between the poverty ratios (ρ_{e13}, ρ_{e24}, $\rho_{e12} = \rho_{e34}$, ρ_{e14}, ρ_{e23}) by averaging corresponding direct estimates over states and years. More specifically, we constructed correlation matrices from direct sampling covariance matrices for 2000-2002[6] and then averaged these over the 50 states and Washington, D.C. We then assumed stationarity of the sampling error correlations between different poverty ratios, which means assuming that between years t and j the correlation depends only on the lag, $t-j$. For example, the stationarity assumption implies that $\rho_{e12} = \rho_{e34}$ since these are both sampling error correlations between the ages 0-4 and ages 5-17 poverty ratios within a single year (i.e., at lag 0). Given this assumption, we averaged over years the state average correlations that corresponded to the same two poverty ratios and had a common lag. Thus, our estimate of $\rho_{e12} = \rho_{e34}$ averaged the directly-estimated sampling error correlations between the ages 0-4 and ages 5-17 poverty ratios for a given year and given state over the years 2000-2002 and over the 50 states and Washington, D.C. We used analogous averaging procedures to estimate ρ_{e13}, ρ_{e24}, ρ_{e14} and ρ_{e23}. In each case we used simple unweighted averages of the correlations.

Model Error Variance

We estimated the model error variance, σ^2_{ui}, when fitting the state models to the ASEC direct poverty ratio estimates. We used a Bayesian approach in estimation of the state model, and we can regard σ^2_{ui} as estimated by its posterior mean. We used a noninformative (flat) prior for all the model parameters.

Model Error Correlations

We estimated the model error correlations (ρ_{u12}, ρ_{u13}, ρ_{u14}, ρ_{u23}, ρ_{u24}, ρ_{u34}) by using the Bayesian approach to treat each pair of ASEC state equations jointly. For each of the six possible distinct

[6] Covariance matrices for just the two years 2000-2001 are used in fitting the sampling error models discussed earlier because of concerns that the variances might change in 2002 due to some changes in the ASEC sample weighting in that year. Effects of such changes on estimates of sampling error correlations, however, are believed to be small, and hence all three years 2000-2002 are used for this purpose.

pairs of the four ASEC state equations for 2000 and 2001 and two age groups (ages 0-4 and ages 5-17 poverty ratios), we specified flat prior distributions for the regression coefficients and the model variances, as was done when fitting the models one equation at a time. The prior for the model error correlation was taken to be uniform on the interval [-1,1]. We then took the posterior mean of the model error correlation as its point estimate. Note that although this model-fitting procedure produced new estimates of the other model parameters involved in each pair of equations (the regression parameters and model error variances), for calculation of the change estimates and their variances we left these model parameters at their original Bayesian estimates obtained from fitting the single ASEC equations separately. This was done so that the results would remain consistent with the SAIPE program's published estimates, which were produced by fitting only one ASEC state equation at a time. This joint Bayesian treatment of two ASEC equations at a time was done using the WinBUGS package (Spiegelhalter, et al. 1996).

References

Bell, William R. (1999), "Derivation of Dependence of Prediction Errors on Model and Sampling Errors," Unpublished U.S. Census Bureau report.

Fay, Robert E. and Train, George F. (1995), "Aspects of Survey and Model-Based Postcensal Estimation of Income and Poverty Characteristics for States and Counties," American Statistical Association, Proceedings of the Section on Government Statistics.

Huang, Elizabeth T. and Bell, William R. (2002), "Comparing Alternative Models for Using Decennial Census Data in SAIPE State Poverty Estimates," American Statistical Association, Proceedings of the Survey Research Methods Section.

Otto, Mark C. and Bell, William R. (1995), "Sampling Error Modelling of Poverty and Income Statistics for States," American Statistical Association, Proceedings of the Section on Government Statistics, 160-165.

Spiegelhalter, David, Thomas, Andrew, Best, Nicky, and Gilks, Wally (1996), "BUGS 0.5: Bayesian Inference Using Gibbs Sampling Manual (version ii)," MRC Biostatistics Unit, Institute of Public Health, Cambridge, U.K.

Table 1. Point Estimates for Children Ages 0-17

stfips	state	poverty rate 2000	poverty rate 2001	% change poverty rate[1] 00-01	change estimate[2] 00-01	1.05 change estimate[3] 00-01
00	U S A	16.2	16.3	0.6	0.1	-0.7
01	Alabama	20.5	22.1	7.8	1.6	0.6
02	Alaska	11.5	11.5	0.0	0.0	-0.6
04	Arizona	18.7	19.0	1.6	0.3	-0.6
05	Arkansas	21.8	23.7	8.7	1.9	0.8
06	California	18.5	17.6	-4.9	-0.9	-1.8
08	Colorado	12.2	11.1	-9.0	-1.1	-1.7
09	Connecticut	10.1	9.3	-7.9	-0.8	-1.3
10	Delaware	12.6	11.6	-7.9	-1.0	-1.6
11	District of Columbia	26.4	27.4	3.8	1.0	-0.3
12	Florida	17.7	18.8	6.2	1.1	0.2
13	Georgia	17.5	18.3	4.6	0.8	-0.1
15	Hawaii	14.3	14.8	3.5	0.5	-0.2
16	Idaho	15.2	15.2	0.0	0.0	-0.8
17	Illinois	14.6	14.6	0.0	0.0	-0.7
18	Indiana	12.1	12.2	0.8	0.1	-0.5
19	Iowa	10.8	9.8	-9.3	-1.0	-1.5
20	Kansas	11.9	12.7	6.7	0.8	0.2
21	Kentucky	19.3	19.8	2.6	0.5	-0.5
22	Louisiana	24.4	25.6	4.9	1.2	0.0
23	Maine	12.9	12.8	-0.8	-0.1	-0.8
24	Maryland	10.7	9.4	-12.1	-1.3	-1.8
25	Massachusetts	11.5	10.6	-7.8	-0.9	-1.5
26	Michigan	13.7	13.4	-2.2	-0.3	-1.0
27	Minnesota	8.7	8.7	0.0	0.0	-0.4
28	Mississippi	24.9	26.6	6.8	1.7	0.5
29	Missouri	14.8	15.1	2.0	0.3	-0.4
30	Montana	18.8	19.0	1.1	0.2	-0.7
31	Nebraska	11.9	12.6	5.9	0.7	0.1
32	Nevada	13.6	12.5	-8.1	-1.1	-1.8
33	New Hampshire	6.9	7.2	4.3	0.3	-0.1
34	New Jersey	10.5	10.1	-3.8	-0.4	-0.9
35	New Mexico	25.5	25.9	1.6	0.4	-0.9
36	New York	19.1	19.5	2.1	0.4	-0.6
37	North Carolina	16.5	16.4	-0.6	-0.1	-0.9
38	North Dakota	13.1	14.5	10.7	1.4	0.7
39	Ohio	14.1	14.8	5.0	0.7	0.0
40	Oklahoma	20.0	20.6	3.0	0.6	-0.4
41	Oregon	15.1	15.1	0.0	0.0	-0.8
42	Pennsylvania	13.1	13.8	5.3	0.7	0.0
44	Rhode Island	15.0	13.5	-10.0	-1.5	-2.3
45	South Carolina	18.2	20.0	9.9	1.8	0.9
46	South Dakota	15.1	14.6	-3.3	-0.5	-1.3
47	Tennessee	17.8	18.7	5.1	0.9	0.0
48	Texas	20.7	21.2	2.4	0.5	-0.5
49	Utah	11.1	11.3	1.8	0.2	-0.4
50	Vermont	11.6	11.3	-2.6	-0.3	-0.9
51	Virginia	12.2	11.2	-8.2	-1.0	-1.6
53	Washington	13.2	13.0	-1.5	-0.2	-0.9
54	West Virginia	21.9	23.2	5.9	1.3	0.2
55	Wisconsin	11.0	11.1	0.9	0.1	-0.5
56	Wyoming	13.9	13.4	-3.6	-0.5	-1.2

[1] 100 × [(2001 Poverty Rate Estimate – 2000 Poverty Rate Estimate)/(2000 Poverty Rate Estimate)]
[2] 2001 Poverty Rate Estimate – 2000 Poverty Rate Estimate
[3] 2001 Poverty Rate Estimate – 1.05 × (2000 Poverty Rate Estimate)

Source: U.S. Census Bureau, SAIPE program, www.census.gov/hhes/www/saipe.html.

Table 2. Standard Errors and *z*-statistics for Children Ages 0-17

stfips	state	change estimate[1] 00-01	S.E. of change est. 00-01	z-statistic[2] 00-01	1.05 change estimate[3] 00-01	S.E. of 1.05 change est. 00-01	1.05 z-statistic[4] 00-01
00	U S A	0.1	0.36	0.39	-0.7	0.38	-1.77
01	Alabama	1.6	1.45	1.14	0.6	1.48	0.43
02	Alaska	0.0	1.33	0.03	-0.6	1.36	-0.40
04	Arizona	0.3	1.58	0.24	-0.6	1.61	-0.35
05	Arkansas	1.9	1.57	1.21	0.8	1.60	0.51
06	California	-0.9	1.02	-0.90	-1.8	1.04	-1.78
08	Colorado	-1.1	1.27	-0.85	-1.7	1.29	-1.30
09	Connecticut	-0.8	1.25	-0.62	-1.3	1.27	-1.01
10	Delaware	-1.0	1.32	-0.82	-1.6	1.35	-1.28
11	District of Columbia	1.0	2.41	0.43	-0.3	2.46	-0.12
12	Florida	1.1	1.18	0.91	0.2	1.20	0.15
13	Georgia	0.8	1.45	0.53	-0.1	1.47	-0.07
15	Hawaii	0.5	1.37	0.34	-0.2	1.40	-0.18
16	Idaho	0.0	1.38	0.03	-0.8	1.40	-0.51
17	Illinois	0.0	1.10	0.07	-0.7	1.13	-0.58
18	Indiana	0.1	1.27	0.03	-0.5	1.29	-0.44
19	Iowa	-1.0	1.24	-0.81	-1.5	1.27	-1.22
20	Kansas	0.8	1.26	0.64	0.2	1.29	0.16
21	Kentucky	0.5	1.41	0.35	-0.5	1.44	-0.33
22	Louisiana	1.2	1.70	0.66	0.0	1.74	-0.05
23	Maine	-0.1	1.29	-0.06	-0.8	1.31	-0.55
24	Maryland	-1.3	1.28	-1.02	-1.8	1.30	-1.41
25	Massachusetts	-0.9	1.31	-0.66	-1.5	1.34	-1.08
26	Michigan	-0.3	1.16	-0.21	-1.0	1.18	-0.78
27	Minnesota	0.0	1.22	0.05	-0.4	1.24	-0.30
28	Mississippi	1.7	1.72	1.03	0.5	1.75	0.30
29	Missouri	0.3	1.31	0.17	-0.4	1.33	-0.39
30	Montana	0.2	1.55	0.10	-0.7	1.58	-0.49
31	Nebraska	0.7	1.30	0.55	0.1	1.32	0.09
32	Nevada	-1.1	1.37	-0.85	-1.8	1.40	-1.32
33	New Hampshire	0.3	1.25	0.28	-0.1	1.27	0.00
34	New Jersey	-0.4	1.15	-0.41	-0.9	1.18	-0.85
35	New Mexico	0.4	1.79	0.23	-0.9	1.83	-0.47
36	New York	0.4	1.12	0.36	-0.6	1.14	-0.49
37	North Carolina	-0.1	1.28	-0.01	-0.9	1.30	-0.65
38	North Dakota	1.4	1.39	1.05	0.7	1.42	0.57
39	Ohio	0.7	1.17	0.62	0.0	1.19	0.02
40	Oklahoma	0.6	1.47	0.39	-0.4	1.49	-0.29
41	Oregon	0.0	1.38	0.04	-0.8	1.41	-0.50
42	Pennsylvania	0.7	1.14	0.65	0.0	1.16	0.08
44	Rhode Island	-1.5	1.43	-1.00	-2.3	1.46	-1.50
45	South Carolina	1.8	1.40	1.24	0.9	1.42	0.58
46	South Dakota	-0.5	1.47	-0.35	-1.3	1.50	-0.85
47	Tennessee	0.9	1.42	0.63	0.0	1.44	0.00
48	Texas	0.5	1.19	0.46	-0.5	1.21	-0.41
49	Utah	0.2	1.39	0.18	-0.4	1.41	-0.22
50	Vermont	-0.3	1.30	-0.21	-0.9	1.33	-0.64
51	Virginia	-1.0	1.27	-0.81	-1.6	1.29	-1.27
53	Washington	-0.2	1.26	-0.12	-0.9	1.28	-0.63
54	West Virginia	1.3	1.59	0.82	0.2	1.62	0.13
55	Wisconsin	0.1	1.24	0.08	-0.5	1.26	-0.36
56	Wyoming	-0.5	1.43	-0.30	-1.2	1.46	-0.77

[1] 2001 Poverty Rate Estimate – 2000 Poverty Rate Estimate

[2] $((2001\ \text{Poverty Rate}) - (2000\ \text{Poverty Rate}))/\sqrt{\text{Var}((2001\ \text{Poverty Rate}) - (2000\ \text{Poverty Rate}))}$

[3] 2001 Poverty Rate Estimate – $1.05\times$ (2000 Poverty Rate Estimate)

[4] $((2001\ \text{Poverty Rate}) - 1.05\times(2000\ \text{Poverty Rate}))/\sqrt{\text{Var}((2001\ \text{Poverty Rate}) - 1.05\times(2000\ \text{Poverty Rate}))}$

Source: Author calculations See text for discussion of critical values.