Comprehensive Status Report: November 18, 2004

OTRC Project Title: FPSO Roll Motions
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MMS COTR: A. Konczvald

This report provides a comprehensive summary of the research completed in all prior Phases of this project (November 2000 – August 2004), and describes research being done in the present Phase (September 2004 – August 2005) to complete this project.

# FPSO Roll Motions

**Spyros A. Kinnas**

## Motivation:

FPSO (Floating Production systems for Storage and Off-loading) hulls have been reported to be subject to excessive roll motions (in some cases of 20 degrees amplitude), which may cause fatigue in mooring lines, disruption of operation, and discomfort to the crew. An economical solution to mitigate these roll motions is through the installation of bilge keels on these hulls.

## Objective:

Develop a robust, validated computational model to study the effects of bilge keel shapes (extent and orientation) on roll motions, and use this model to provide guidance on their design.

## Summary:

The work performed under this project since inception will be summarized in the next pages. The reader can find additional information (including copies of theses, conference papers, presentations, and movies of results) on the FPSO web site at UT Austin:
http://cavity.ce.utexas.edu/kinnas/fpso/

We have embarked on an effort to develop a computational technique for the prediction of the hydrodynamic coefficients of 2-D hulls subject to roll.

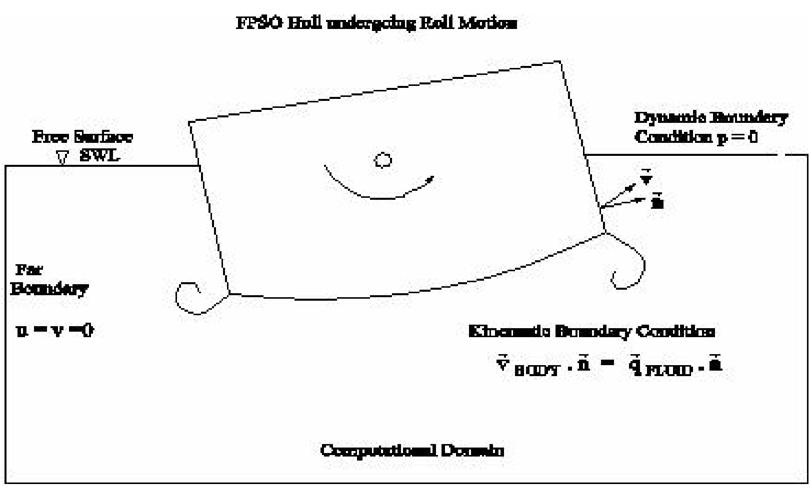



Figure 1: Computational domain and boundary conditions for a 2-D FPSO hull subject to roll motions.

In the initial phase of this work, a previously developed method at UT Austin (Choi, PhD, 2000, Choi and Kinnas, 2000, 2003) for the prediction of 3-D unsteady flows around marine propellers was extended to predict the flow first around a 2-D vertical flat plate subject to a sinusoidal horizontal gust, and then the flow past an FPSO hull (without or with bilge keels) subject to heave or roll motions. The work was performed by an MS student with the Ocean Engineering Group in the Civil Engineering Department at UT, under the supervision of Prof. Kinnas. It should be noted that the second half of the related thesis was supported by OTRC/MMS, while the first half was supported by an independent international consortium on high-speed propulsors, led by the PI. The geometry of the problem in the case of an FPSO hull in roll is depicted in Fig. 1, while the formulation is summarized in Appendix A. The main characteristics of this method were (Kakar, UT MS, 2002):

- Node based finite volume method, 2$^{nd}$ order accurate in space. The components of the velocity vectors and the pressure are defined at the nodes (corners) of the quadrilateral cells.
- Implicit Lax-Wendroff method, 2$^{nd}$ order accurate in time
- Solved for the Euler equations (i.e. we ignored the effects of viscosity) and implemented 2$^{nd}$ and/or 4$^{th}$ order artificial dissipation terms to stabilize the solution (this also forced separation at the tip of the plate or at the tip of the bilge keel). In the case of the vertical flat plate we also solved for the Navier-Stokes equations in laminar flow.
- Solved for the velocity flow-field at each new time-step using the momentum equations, while the pressure was determined using the SIMPLE method in order to enforce the continuity equation at each time step.

In the case of an FPSO hull we:

- Implemented linearized kinematic and dynamic boundary conditions on the free surface.
- Applied the kinematic boundary condition on the mean body position (by imposing a flow velocity normal to the mean position of the body equal to the normal component of the rigid body velocity at the same location and time step).
- Applied appropriate conditions at the far boundaries, which were placed sufficiently far from the body to ensure that the flow around the body was not affected by reflections of the radiated waves at those boundaries. It should be mentioned that the current method can also be applied in the case of *shallow* depth water, even though most of the presented results are in the case of deep water.

The major findings of this initial phase were:

- The current method was able to predict separated flow downstream of sharp corners (e.g. the tip of a flat plate or that of a bilge keel).
- In the case of the vertical plate the predicted force over one period from the Euler or the Navier-Stokes method was found to be very close to each other. In other words solving for the Euler equations seemed to be sufficiently accurate.
- In the case of 2-D FPSO hulls subject to heave motions the hydrodynamic coefficients were found to be well predicted over a wide range of Froude numbers by the current method, when compared to those using a boundary element approach of Newman or those measured by (Vugts 1968).
- In the case of FPSO hulls subject to roll motions, however, the present method seemed to over predict the added mass and under-predict the damping coefficients, especially in the case of a bilge keel. The results from the current method seemed to be comparable to those produced by boundary element methods.

- The present method was able to predict the expected trend on the hydrodynamic coefficients with increasing bilge keel length.

Work then focused on validating the numerics of the current method, primarily in the case of the vertical flat plate. It should be noted that at this point of our research the results in the case of an FPSO hull did not seem to converge (i.e. the error in the solution seemed to grow with time) with significant changes in the grid resolution, and it was thus decided to investigate the behavior of the method in the case of the vertical plate first. The results of this effort were presented in the papers by Kinnas et al (12th Offshore Symposium, 2003) and Kinnas et al (ISOPE, 2003). At this stage two new graduate students (Yu, PhD level, and Kacham, MS level) and a post-doctoral associate (Dr. H. Lee) performed this work under the supervision of Prof. Kinnas. The following changes were performed:

- The finite volume scheme was changed from a node based to a cell based. The components of the velocity vectors and the pressure were now defined at the centroids of the quadrilateral cells.

- Corrections were made in the treatment of the unsteady terms. In addition an implicit Crank-Nicolson scheme in time was implemented, and the pressure correction scheme was improved.

- The convective terms of the momentum equations were treated via an upwind 2nd order differencing scheme

- In the case of the vertical plate convergence studies in terms of grid resolution and time step were performed, and our results were compared with those from commercial Navier-Stokes solver (Fluent) and experiments (Sarpkaya and O'Keefe 1995).

- The Navier-Stokes equations were implemented in the case of an FPSO hull

- The grid distribution was modified in order to improve the cell distribution at the free surface

- A boundary element method (with linearized free-surface conditions) was developed in order to validate the current method in the case of inviscid flow and in the absence of bilge keels, but also in order to assess the effect of the separated flow (which the boundary element method cannot model) on the predictions in the case of bilge keels.

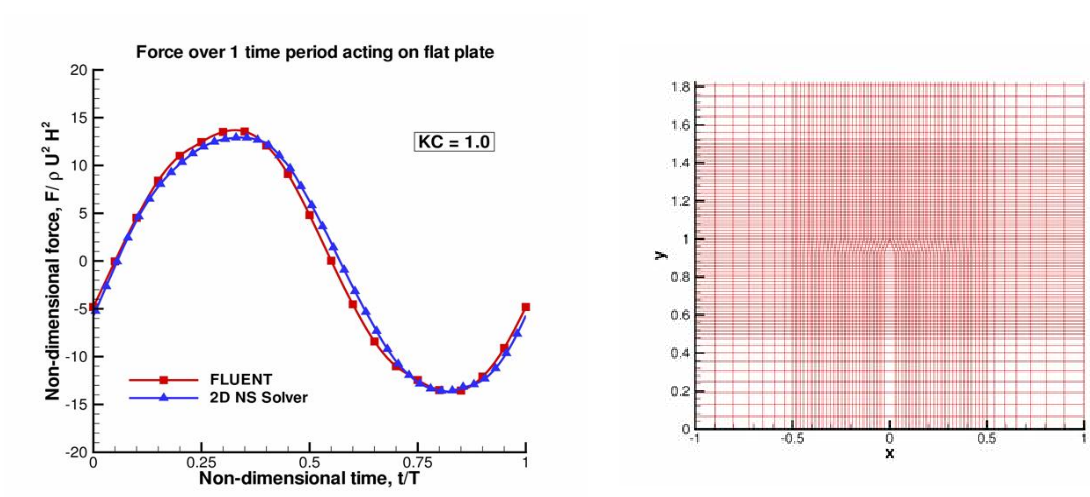



Figure 2: Predicted force over one period (left) and grid detail (right), in the case of a vertical plate.

The outcome of our work in this phase can be summarized in Figures 2, 3, and 4. Figure 2 shows that the results of our method compare well to those of a commercial code (Fluent). Figure 3 shows the comparisons of our predictions with the measurements of (Sarpkaya and O'Keefe1995).

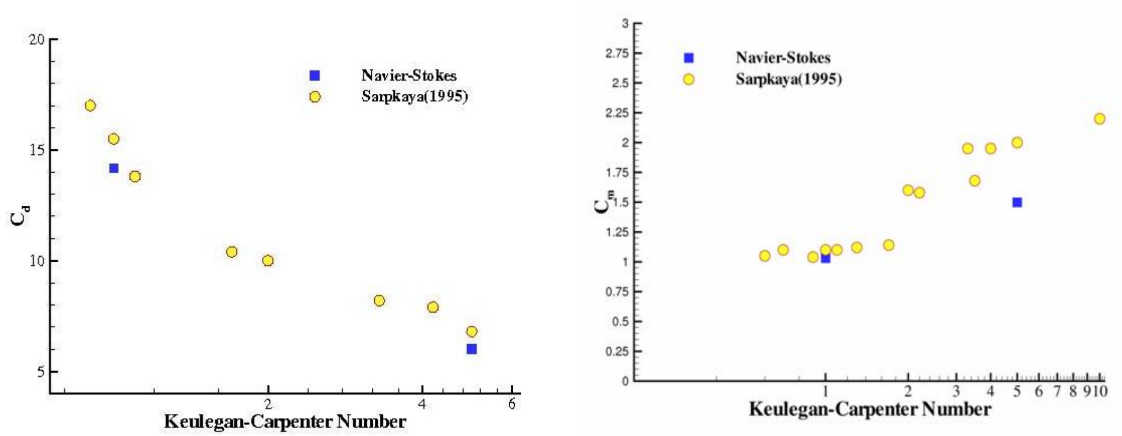



Figure 3: Predicted and measured drag and inertia coefficient on a vertical plate subject to a horizontal sinusoidal gust.

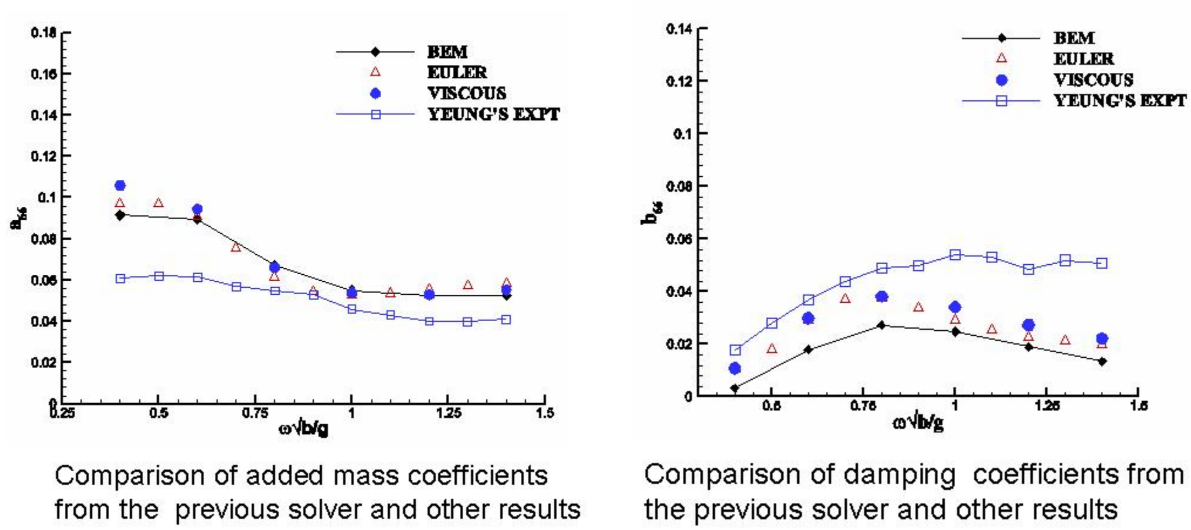



Figure 4: Added mass and damping coefficient as predicted by an *older version* of the current method in the case of an FPSO hull with 4% bilge keels.

As seen in the Figure 4, our method did not seem to predict as well the added mass or the damping coefficient in the case of an FPSO hull. This led us to the next phase of our research. At this stage an additional student (Vinayan, PhD level) was added to the research group. Some of the results of this phase our research have been reported in the MS Thesis of Kacham (2004) and more recent results will be reported in two papers by Kinnas et al to be presented at ISOPE 2005 and OMAE 2005. The following were accomplished in this phase, thus far:

- We studied the numerics of our method in the case of a fully submerged hull. In this way, the additional complexity of the free surface was factored out. In particular we compared the pressure distribution along the hull surface as predicted by the present method and the previously developed boundary element method. These comparisons led us to the introduction of the moving grid (see next item). One of these comparisons is shown in Figure 5 (left part). Please note that the very good agreement of the results from our method (in the absence of viscosity) with those of the potential method, as shown in Fig. 5, was only achieved after the incorporation of the moving grid. It should also be noted that this case was used in order to find the proper grid resolution.

- We further improved the current method by including the effects of the moving grid in the Navier-Stokes equations, as described in Appendix A. The effects of the moving grid were also incorporated in the boundary element method. The effects of the Reynolds number on the pressure distribution are shown in Figure 5 (right part). Note that it is only for small Reynolds numbers (1000) that the predicted pressure distributions look different.

- We then applied the most recent method in the case of a hull with and without bilge keels and for various Froude numbers. The new grid (Kacham, 2004) is shown in Figure 6. The grid details and three different bilge keel orientations that we tested are shown in Figure 7. The newest results are shown in Figure 8, where the added mass and damping coefficients are compared with measurements and other numerical results, (Yeung et al 2000). Fig. 9 shows the predicted vorticity contours. Please note the significant improvement of the new results, in comparison to the results of the older version of our method, as shown in Fig. 4.

- The effect of the bilge keel orientation was studied and preliminary results are shown in Figure 10. It should be noted that our method seems to predict the same effect of bilge keel orientation on the results as those measured in Na et al (2002) or predicted by Seah and Yeung (2003), despite the fact that our bilge keel geometry is not the same to that tested.

- We finally modified our boundary element method (BEM) by incorporating the non-linear free-surface conditions. The formulation is summarized in Appendix B. This will help us quantify the effects of the currently used simplified linearized free surface conditions in our finite volume method. In fact some preliminary results have shown that at larger angles of roll (20 degrees amplitude) the effects of non-linear free surface conditions on the hydrodynamic coefficients can be important. Figures 11 and 12 show the predicted pressure distributions over the hull using linearized and non-linear free-surface conditions at two different roll amplitudes. Please note that the predicted pressures differ considerably even at small amplitudes of roll (even though the integrated roll moments are very close to each other), and that should certainly affect the behavior of the viscous solution and its effect on the pressure distribution. Finally, Fig. 13 shows comparisons of the predicted wave profiles using linear and non-linear free-surface conditions. Please note that the profiles starboard and portside of the FPSO hull are NOT anti-symmetric in non-linear theory, and this seems to be consistent with observations.

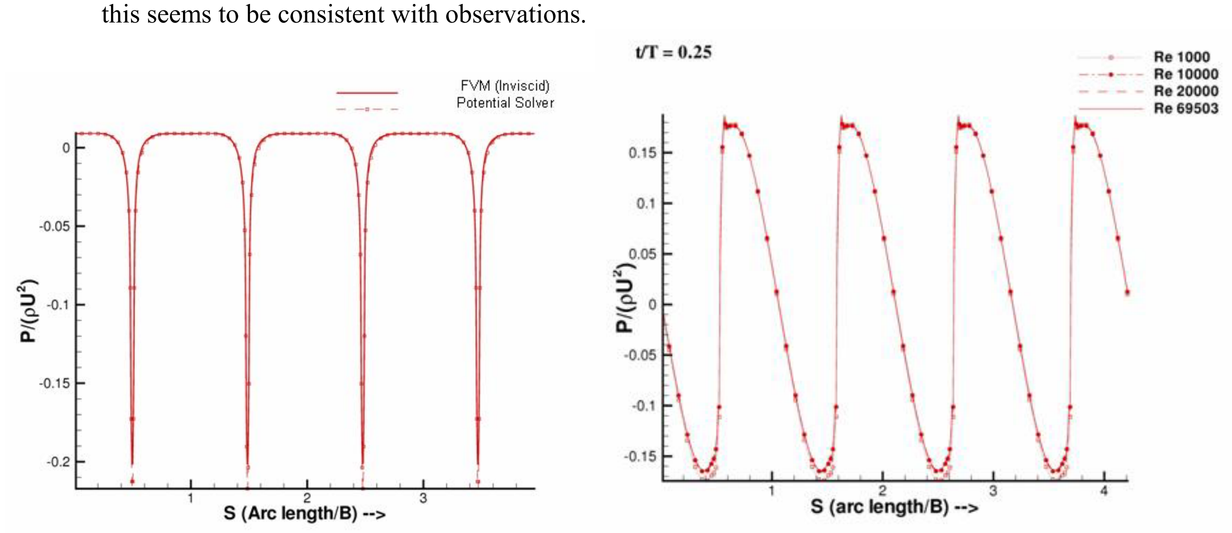



Figure 5: Validation of the current method using the pressures predicted from the boundary element method in the case of a submerged hull without bilge keels (left), and the effect of Reynolds number on predicted pressures along hull in the case of a submerged hull with bilge keels.

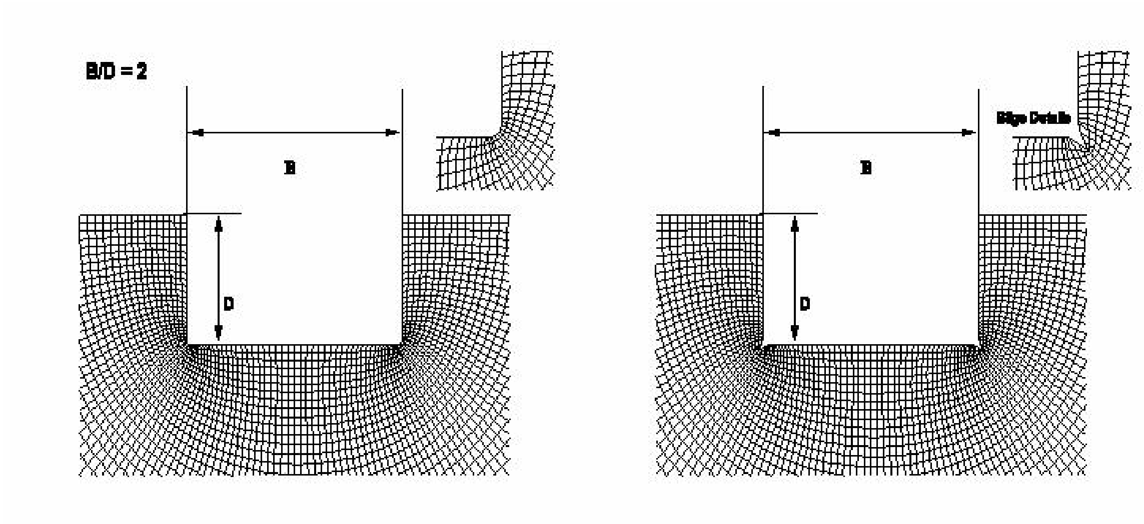



Figure 6: Improved grid utilized by the present method; without bilge keels (left) and with bilge keels (right) (Kacham, MS, UT 2004)

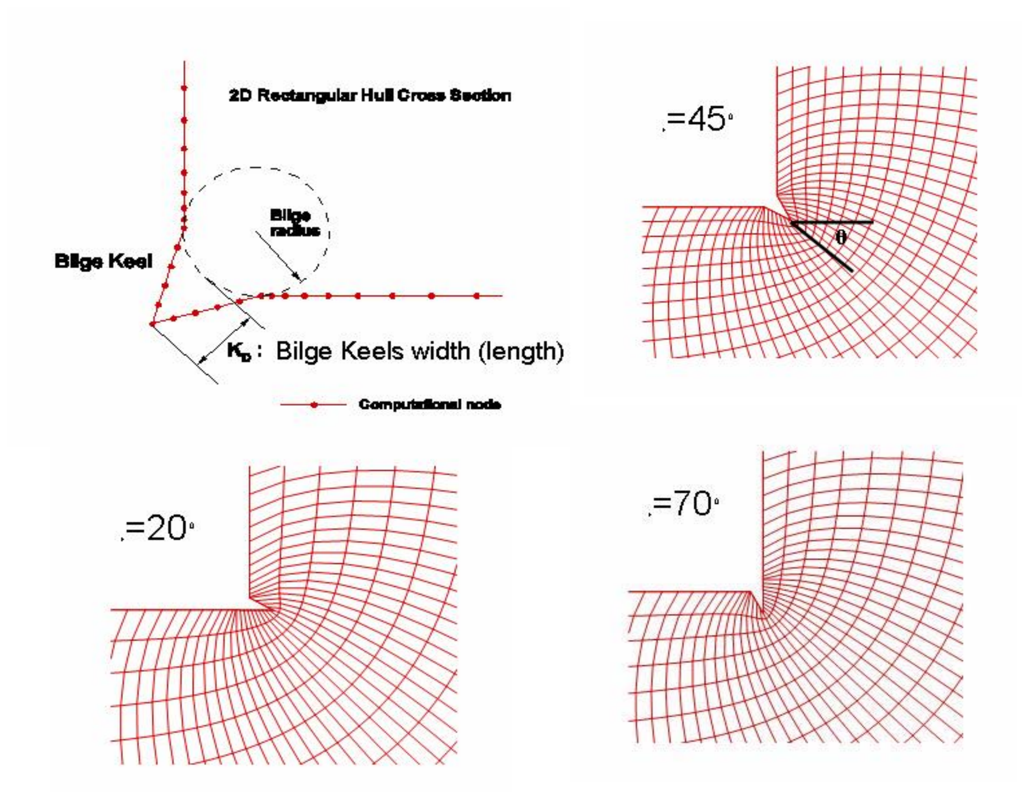
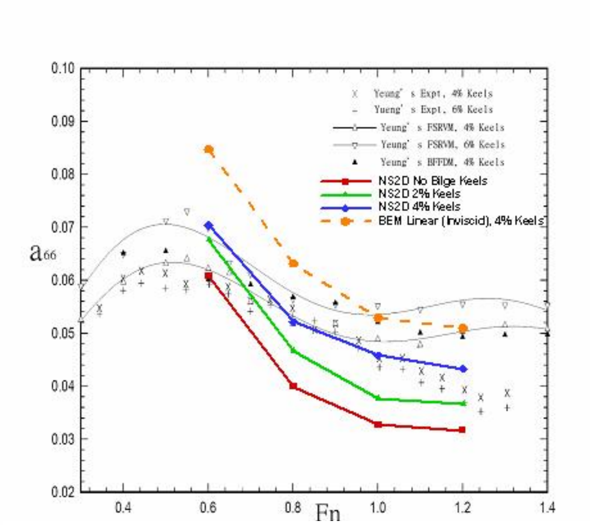
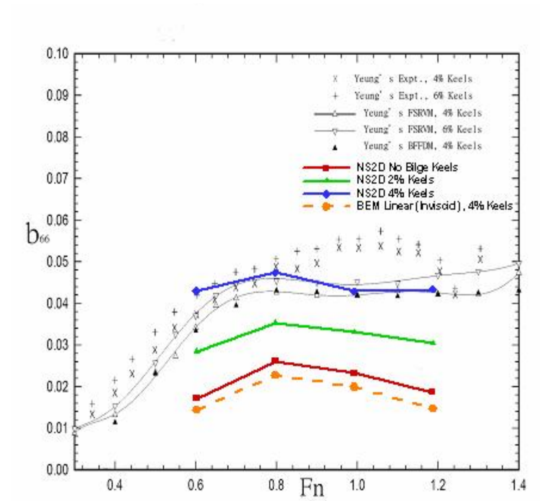



Figure 7: Definition of bilge keel length and three different orientations




Comparison of add mass coefficients from the present solver and other results




Comparison of damping coefficients from the present solver and other results

Figure 8: The latest predictions from our most recent method, compared with those measured and those predicted by other methods

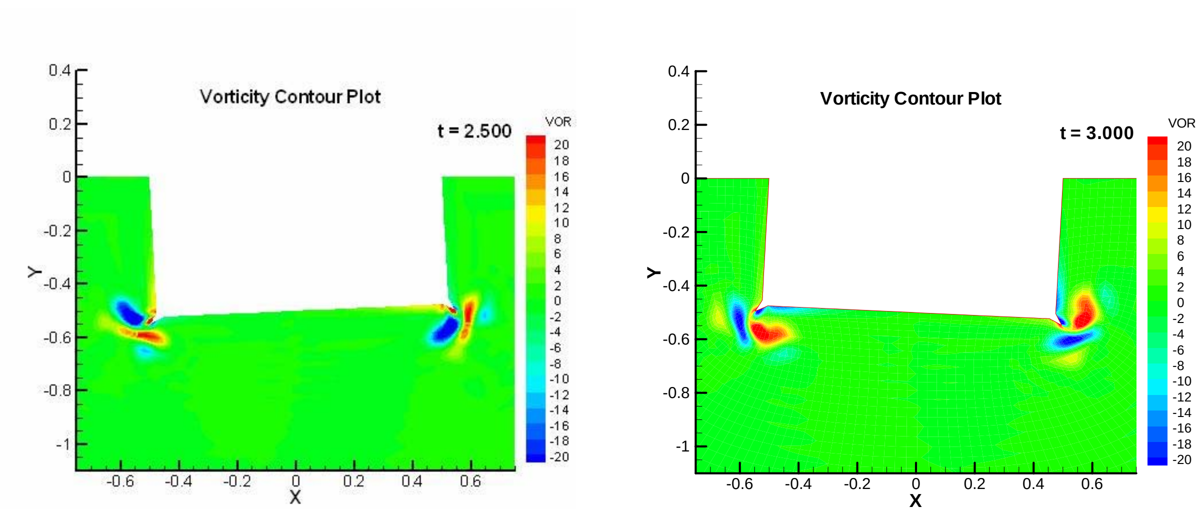



Figure 9: Predicted vorticity contour plots of a FPSO hull with 4% bilge keels subject to roll motion at Fn=0.6, and at two different time steps

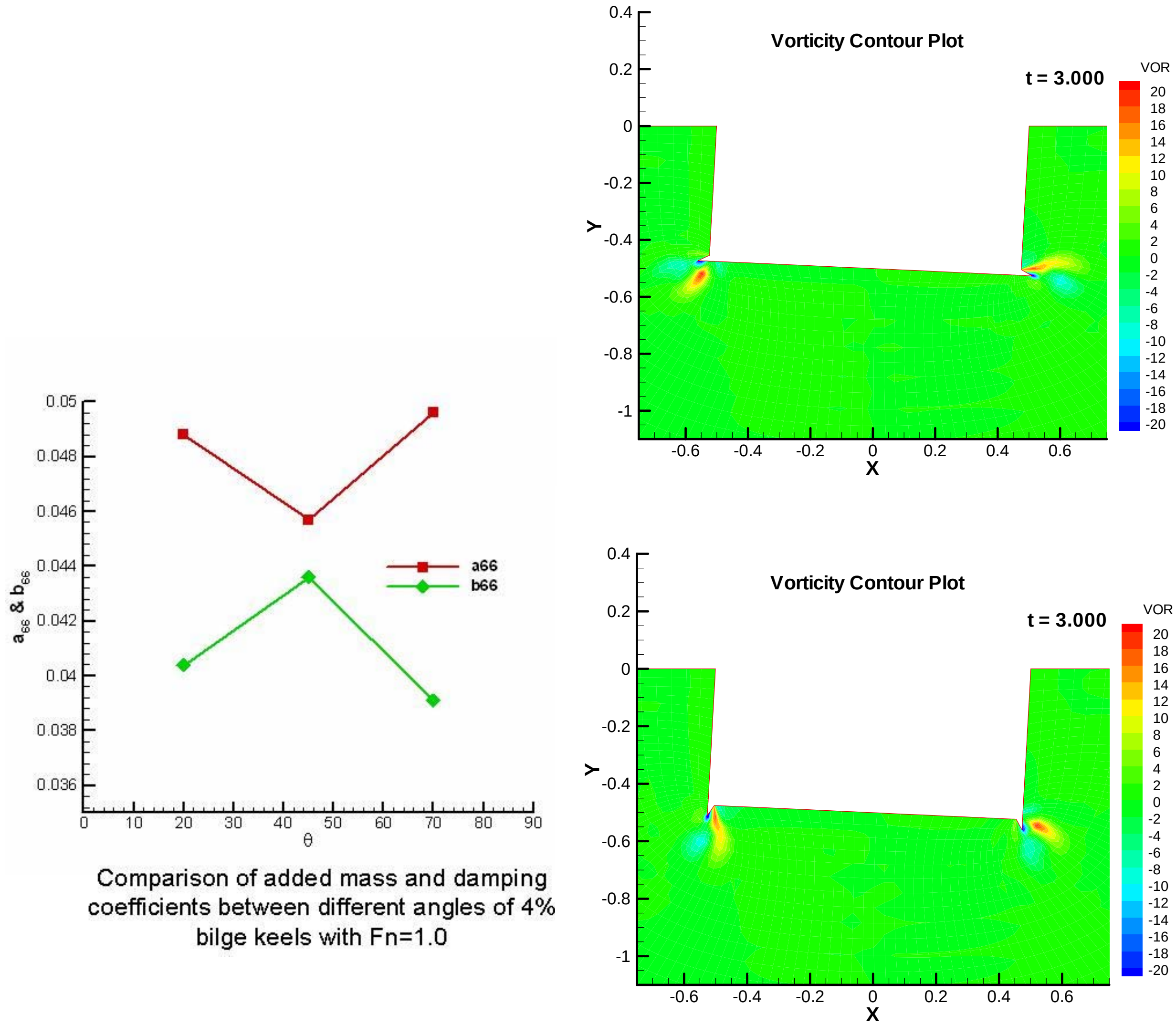



Figure 10: Effect of bilge keel orientation on results; hydrodynamic coefficients in roll (left), and vorticity contours (right)

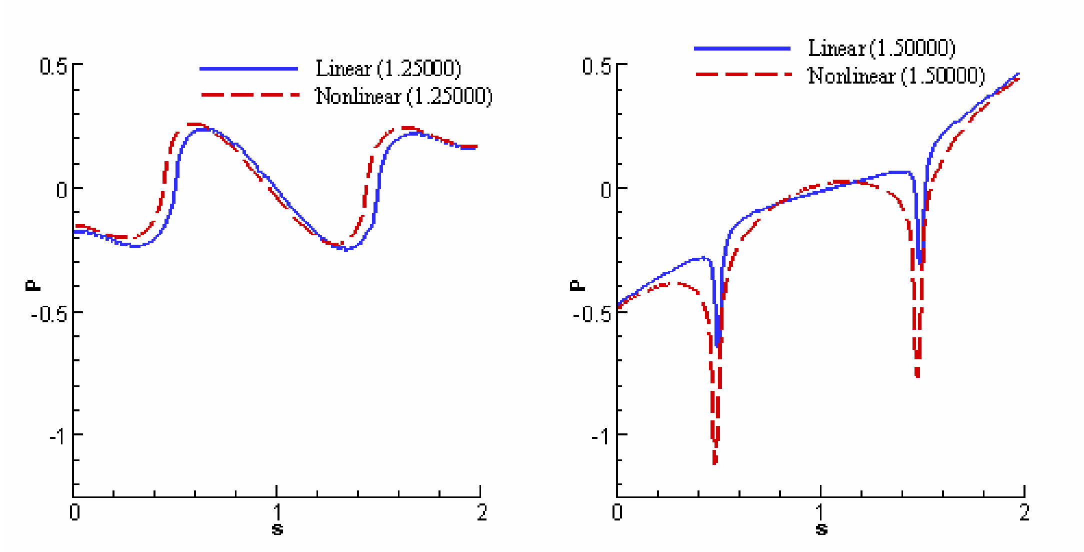



Figure 11: Pressure distribution vs. length of submerged part of hull, as predicted from linear and non-linear theories for a FPSO hull in roll with amplitude of 5 degrees.

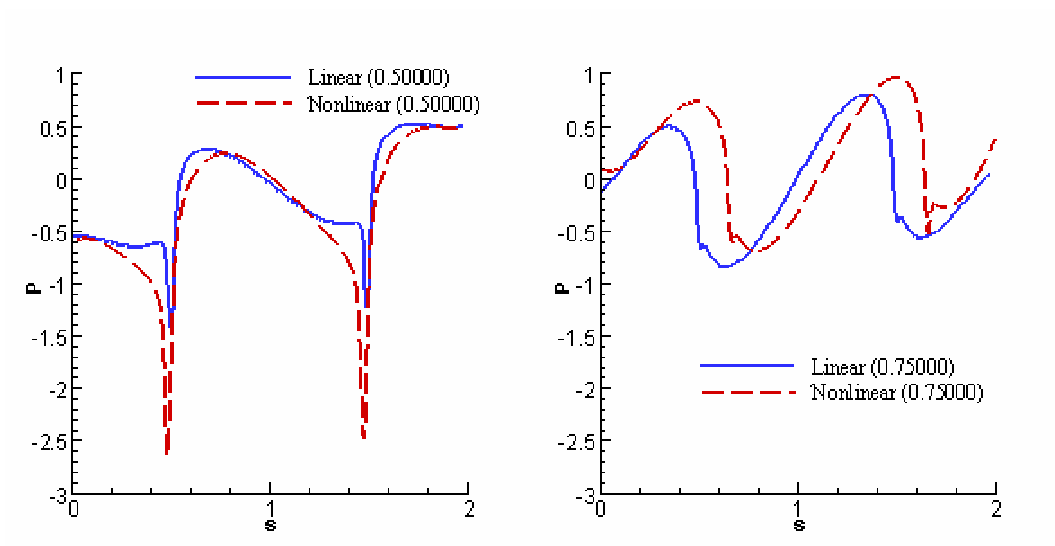



Figure 12: Pressure distribution vs. length of submerged part of hull, as predicted from linear and non-linear theories for a FPSO hull in roll with amplitude of 20 degrees.

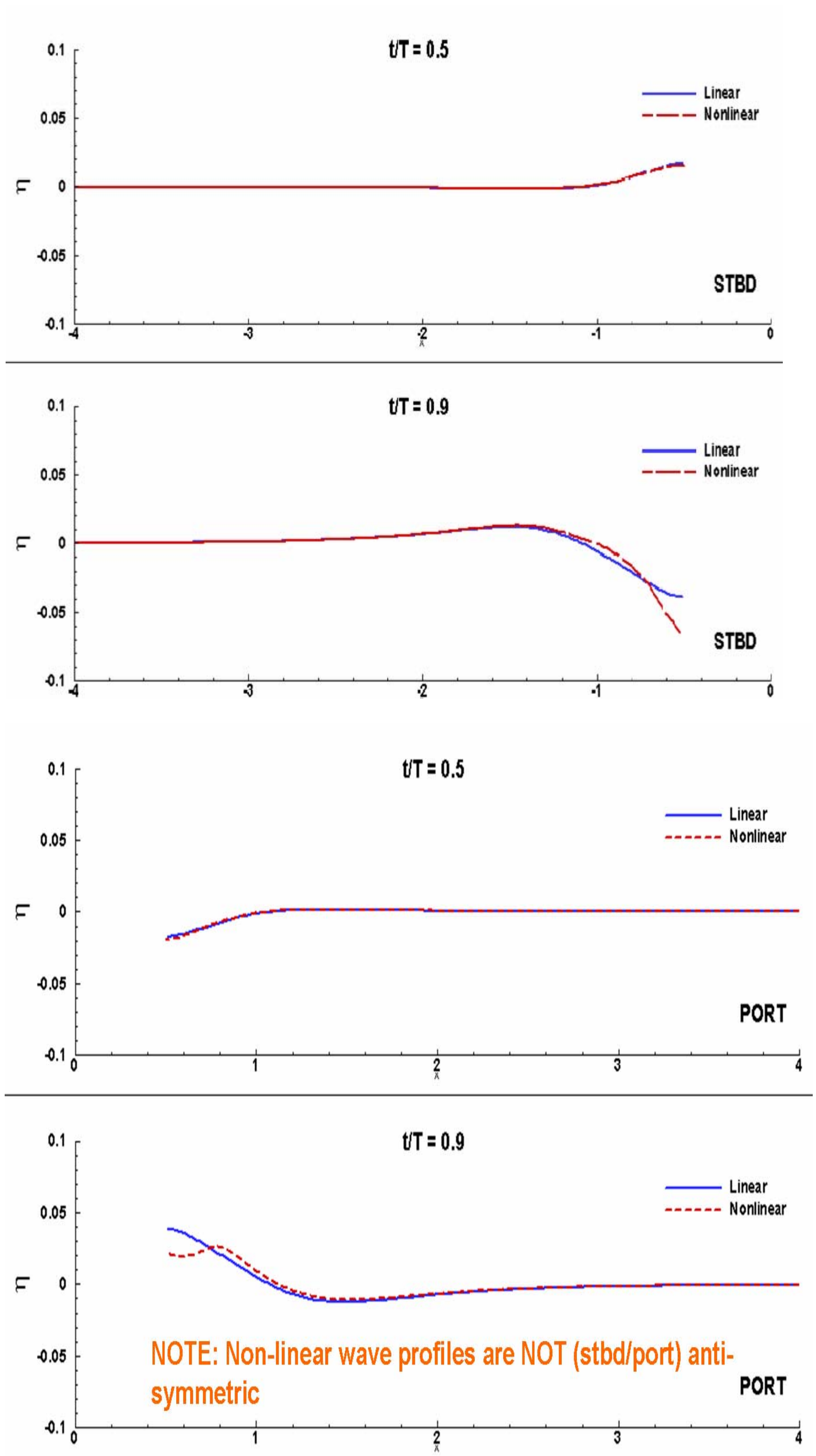



Figure 13: Wave profiles predicted by the boundary element method using linear and non-linear free-surface boundary conditions for a 2D FPSO hull in roll with amplitude of 20 degrees.

## Conclusions Based on Work to Date

The current version of our finite volume method (FVM) seems to predict the hydrodynamic coefficients in roll quite well. The method allows us to study the effect of frequency, Reynolds number, and bilge keel shape (extent and orientation) on the results. At the same time our boundary element method (BEM) was extended to include the effects of non-linear free surface conditions, which were found to be more important in the case of larger roll angles. The two approaches (FVM and BEM) have been very useful in helping us assess the numerical accuracy of both of our methods, and also quantify the effect of separated flow on the predictions.

## Present Work

The focus for 2004 -2005 is addressing the following issues:

- Complete convergence studies, especially sensitivity studies in terms of grid, time step, and domain size over the whole range of Froude numbers.
- Apply our method for larger amplitudes of roll motion and compare our predictions with existing measurements and other numerical results.
- Use the same geometry as that in experiments (Na, 2002) in the case of vertical or horizontal bilge keels, in order to validate the method.
- Incorporate the effects of the non-linear free-surface conditions in the finite volume method.
- Apply the model on a 3-D FPSO hull (using a strip theory type of approach at first) and compare with experiments (some of which are being performed at OTRC's Wave Basin by Drs. Ward & Mercier) and other numerical results

## Future Topics

- Use the method to study existing or develop new optimum bilge keel shapes. Study the effect of bilge keel length (along the longitudinal direction) on the hull hydrodynamic coefficients.
- Use the hydrodynamic coefficients (without or with bilge keels) as determined by the present method to solve the equations of motion and determine the response of an FPSO hull subject to a given wave environment.
- Apply a fully 3-D model on an FPSO hull. Due to the fact that this approach will be computer intensive we plan on solving the Navier-Stokes equations on parts of the hull along its length, which will interact with each other using a parallel computer cluster technology. The PI is in the process of purchasing a computer cluster using other funds.

## Related Publications and References:


- Choi, J.-K. and Kinnas, S.A., *An Unsteady 3-D Euler Solver Coupled with a Cavitating Propeller Analysis Method*, Twenty-third Symposium on Naval Hydrodynamics, Val de Reuil, France, September 17-22, 2000.
- Choi, J.-K. *Vortical Inflow – Propeller Interaction Using an Unsteady Three-Dimensional Euler Solver*, PhD thesis, Ocean Engineering Group, Department of Civil Engineering, The University of Texas at Austin, August 2000.
- Choi, J.-K. and Kinnas, S.A., *Prediction of Unsteady Effective Wake by a Euler Solver/Vortex-Lattice Coupled Method*, Journal of Ship Research, Vol. 47, pp. 131-144, June 2003.
- Kinnas, S.A., Yu, Y.-H., Vinayan, V., Kacham, K., *Modeling of the Unsteady Separated Flow over Bilge Keels of FPSO Hulls under Heave and Roll Motions*, The 15th International Offshore and Polar Engineering Conference, 2005, (Abstract accepted , Paper under preparation).
- Kinnas, S.A., Vinayan, V., Yu, Y.-H., *Modeling of the Viscous Flow Around FPSO Hull Sections under Heave and Roll Motions*, OMAE 2005, (Abstract accepted , Paper under preparation).
- Kacham, B., *Inviscid and Viscous 2D Unsteady Flow Solvers Applied to FPSO Hull Roll Motions*, MS thesis, UT Austin, Ocean Engineering Group, Department of Civil Engineering, December 2004 (also UT-OE Report 04-7) .
- Kinnas, S.A., Yu, Y.-H., Lee, H., Kakar, K., *Modeling of Oscillating Flow Past a Vertical Plate*, The 13th International Offshore and Polar Engineering Conference, Honolulu, Hawaii, May 25-30, 2003, pp.218-226.
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- Kakar, K., *Computational Modeling of FPSO Hull Roll Motions and Two-component Marine Propulsion Systems* , MS thesis, UT Austin, Ocean Engineering Group, Department of Civil Engineering, August 2002 (also UT-OE Report 02-3).
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- Sarpkaya, T. and O'Keefe, J. (1995). *Oscillating flow about two and three dimensional bilge keels*. In Proceedings of the 14$^{th}$ International Conference on Offshore Mechanics and Arctic Engineering, pages pp. 263-270, Copenhagen, Denmark.
- Seah, R. K. M. and Yeung, R. W., *Sway and Roll Hydrodynamics of Cylindrical Sections*, International Journal of Offshore and Polar Engineering, Vol. 13, No. 4, pp. 241-248, December 2003.
- Vugts, J. *The hydrodynamic coefficients for swaying, heaving and rolling cylinders in a free surface*, In International Shipbuilding Progress, pp. 251-276, 1968.
- Yeung, R. W., Roddier, D., Liao, S.-W, Alessandrini, B., and Gentaz, L. *On roll hydrodynamics of cylinders fitted with bilge keels*, In proceedings of Twenty –Third Symposium on Naval Hydrodynamics, Val De Reuil, France, 2000.
- Young, Y.L and Kinnas, S.A., *A BEM Technique for the Modeling of Super-cavitating and Surface-Piercing Propeller Flows*, 24th Symposium on Naval Hydrodynamics, Fukuoka, JAPAN, 8-13 July, 2002.
- Young, Y.L., *Numerical Modeling of Supercavitating and Surface-Piercing Propellers*, PhD thesis, UT Austin, Ocean Engineering Group, Department of Civil Engineering, May 2002 (also UT-OE Report 02-1).

## Appendix A: Formulation and Numerical Implementation of the Present Finite Volume Method

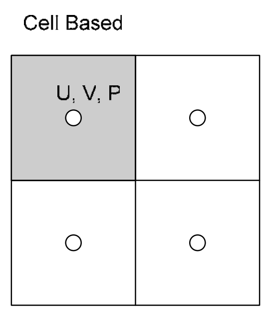
- Governing Equations

Non-Dimensional Governing Equation (Navier-Stokes Equation & Continuity Equation)

$$\frac{\partial \vec{U}}{\partial t} + \nabla \cdot (\vec{U}\vec{U}) = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2 \vec{U}, \quad \nabla \cdot \vec{U} = 0$$

where U represents the velocity and P the pressure; the Reynolds number is defined as $Re = U_m h/\nu$ ; and the length scale, h, is a representative length in the problem being solved.

- Cell Based Finite Volume Method




Collocated variable, non-staggered grid arrangement, non-orthogonal grids.

- Crank-Nicolson Method for Time Marching

$$\vec{U}_{i,j}^{n+1} = \vec{U}_{i,j}^{n} + \Delta t \cdot \frac{f^{n+1} + f^{n}}{2}$$

where f represents the summation of the convective terms, the viscous terms and the pressure terms at the present time step n and the next time step n+1.

- Pressure-correction Method
  *SIMPLE method (Patankar 1980)*

$$p = p^* + p',$$

$$V_{face} = V_{face}^* + V'_{face} = V_{face}^* - \Delta t \frac{\partial p'}{\partial n}$$

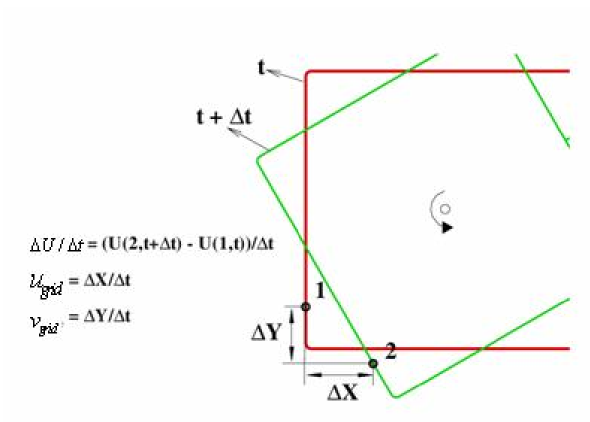
$$\sum_{face} \Delta t \frac{\partial p'}{\partial n} ds = \sum_{face} V_{face}^* ds$$

where $p'$ is the pressure correction, $V'_{face}$ is the velocity correction term, $\partial p'/\partial n$ is the pressure correction derivative with respect to the normal direction of the cell face, $V^*_{face} = (u^*; v^*)$ is the predicted velocity vector obtained from the momentum equation.

- Moving Grid
  When the grid is moving, additional terms need to be taken into account.

$$\frac{\partial u}{\partial t} = \frac{\Delta u}{\Delta t} - u_{grid}\left(\frac{\partial u}{\partial x}\right) - v_{grid}\left(\frac{\partial u}{\partial y}\right)$$

where ($u_{grid}$, $v_{grid}$) is the velocity of the moving grid; and represents the total change in the value of u with both increment in time and the corresponding change in the location of the point. When the above equation is substituted into the momentum equation.




$$\frac{\Delta \vec{U}}{\Delta t} - u_{grid}\left(\frac{\partial \vec{U}}{\partial x}\right) - v_{grid}\left(\frac{\partial \vec{U}}{\partial y}\right) + \nabla \cdot (\vec{U}\vec{U}) = -\nabla P + \frac{1}{\mathrm{Re}}\nabla^2 \vec{U}$$

## Appendix B: Formulation and Numerical Implementation of the Boundary Element Method

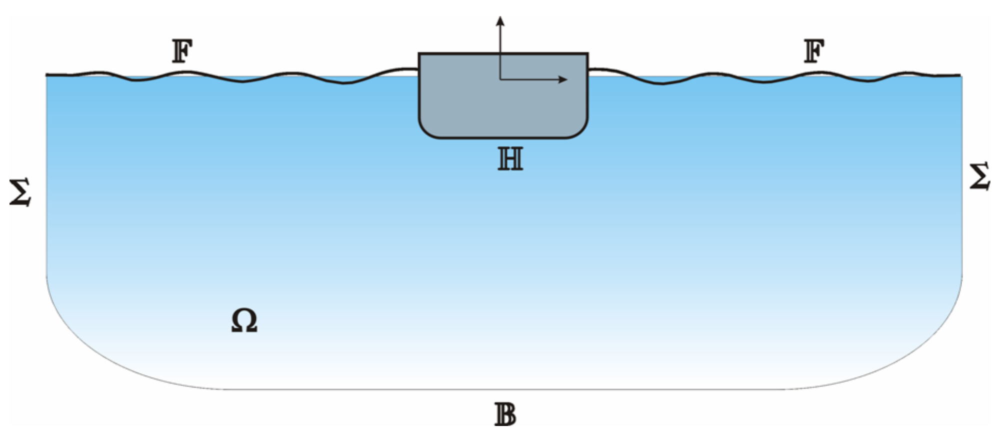



Figure 14: Domain of FPSO hull in roll and its boundary over which the boundary element method is applied.

Green's 3rd identity is solved over the boundary of the flow domain, S, as shown in Figure 14

$$\frac{\varphi}{2} + \frac{1}{2\pi}\iint_S \left[\varphi \frac{\partial(\log r)}{\partial n}\right] dS = \iint_S \left[\log r \frac{\partial \varphi}{\partial n}\right] dS$$

$$S \equiv F \cup H \cup \Sigma \cup B$$

**Mathematical Background – Boundary Conditions**

- **Free-surface Kinematic Boundary Condition**

$$\frac{\partial \eta}{\partial t} = \frac{\partial \varphi}{\partial y} - \frac{\partial \eta}{\partial x}\frac{\partial \varphi}{\partial x} \; ; y = \eta \qquad \text{Nonlinear}$$

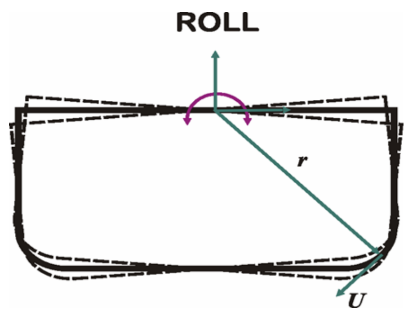
$$\frac{\partial \eta}{\partial t} = \frac{\partial \varphi}{\partial y} ; y = 0 \qquad \text{Linear}$$

- **Free-surface Dynamic Boundary Condition**

$$\frac{\partial \varphi}{\partial t} = - g\eta - \frac{1}{2}\left|\nabla \varphi\right|^2 ; y = \eta \qquad \text{Nonlinear}$$

$$\frac{\partial \varphi}{\partial t} = - g\eta \; ; y = 0 \qquad \text{Linear}$$

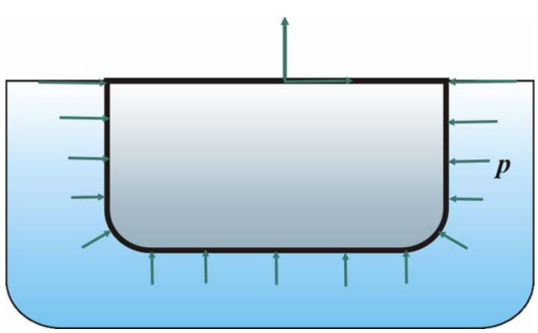
- **Hull/Body Kinematic Boundary Condition**




$$\left.\frac{\partial \varphi}{\partial n}\right|_H = \vec{U}.\vec{n}$$

$$\vec{U} = (\alpha_0 \sin \omega t)\vec{k} \times \vec{r}$$

**Roll Moment**




$$M_Z = \int p(xn_y - yn_x)dS_H$$

$$M_Z(t) = -\hat{a}_{66}\,\ddot{\alpha} - \hat{b}_{66}\,\dot{\alpha}$$

$$\ddot{\alpha} = \alpha_0 \omega \cos(\omega t) : Angular\ acceleration$$

$$\dot{\alpha} = \alpha_0 \sin(\omega t) \quad : Angular\ Velocity$$

***Hydrodynamic Coefficients***

$$\hat{a}_{66} = -\frac{1}{\pi \alpha_0 \omega}\int_0^T M_Z(t)\cos(\omega t)\,dt\,; a_{66} = \frac{\hat{a}_{66}}{4\rho b^2 \nabla}$$

$$\hat{b}_{66} = -\frac{1}{\pi \alpha_0}\int_0^T M_Z(t)\sin(\omega t)\,dt\,; b_{66} = \frac{\hat{b}_{66}}{4\rho b^2 \nabla}\sqrt{\frac{b}{g}}$$

**Numerical Formulation – Time Stepping Method**

The Mixed Eulerian-Lagrangian (MEL) method of Longuet-Higgins and Cokelet (1976) is applied through which the trajectories of the particle on the free surface are tracked by marching in time the following equation:

$$\frac{D\vec{F}}{Dt} = \vec{G} \qquad \vec{F} = \begin{Bmatrix} \xi \\ \eta \\ \varphi \end{Bmatrix} \;;\; \vec{G} = \begin{Bmatrix} \dfrac{\partial \varphi}{\partial x} \\ \dfrac{\partial \varphi}{\partial y} \\ \dfrac{1}{2}\left|\nabla \varphi\right|^2 - g\eta \end{Bmatrix}_{y=\eta(x,t)}$$

We have applied the following schemes in time:

- Euler Explicit
- Fourth-order Runge-Kutta
- Young and Kinnas 2002, Young (PhD), 2002