32 AI

Measurements of the surface flow above round bubble plumes

JEROME H. MILGRAM and JAMES J. BURGESS

Massachusetts Inst. of Technology, Cambridge, Mass. 02139, USA

When a bubble plume exists beneath a free surface, such as after a subsea gas well blowout, a generally horizontal flow occurs in the vicinity of the surface and this flow is influenced by the fact that the surface is free. Two very different theories for such surface flows have been developed in the past and the bases of these theories are reviewed here. The results of measurements of surface flows above plumes of relatively large scale are given. These are compared with both of the existing theories. One theory is found to be accurate at small radii from the plume centre and the other theory is found to be accurate at large radii. The needed boundary condition for the theory that is accurate at large radii is supplied by the results of the theory that is accurate at small radii.

INTRODUCTION

A bubble plume occurs when buoyant gas is released in the interior or at the bottom of a liquid. For an axisymmetric geometry the mean cross sections of the plume are round. Generally most of the vertical extent of the plume is in a region where the flow is not influenced by the details of the gas release or the upper surface. This region is called the zone of established flow (ZOEF) and the most comprehensive theories and experimental measurements for this zone have been presented by Fannelop and Sjoen,[1] Milgram and Van Houten[2] and Milgram.[3]

Two examples of bubble plumes are blowouts of subsea gas-containing hydrocarbon wells and the intentional release of air at the bottom of a slow-moving river to mix the otherwise stagnant water. For these examples the upper surfaces are free and the flows in their vicinities are of interest. From the surface down to a depth whose order of magnitude is equal to that of the plume radius the presence of the surface influences the flow so this region is called the zone of surface flow (ZOSF). Fannelop and Sjoen[1] and Van Houten[2] developed theories for the zone of surface flow. These two theories are very different; each one on being based on different presumptions about the flow. Fannelop and Sjoen also presented the results of flow speed measurements in the ZOSF for an airflow rate of 0.008 N m^3/s at a depth of 5.5 m and for an airflow rate of 0.010 N m^3/s at a depth of 9.9 m. The horizontal extent of these measurements was from twice the radius of the plume, as measured beneath the surface, to about 20 plume radii. Although the profiles of horizontal velocity vs depth varied considerably from the presumptions of the theory, it was found that the surface flow speed measurements were well described by the Fannelop and Sjoen theory.

In this paper the results of surface flow speed measurements for relatively large scale plumes having airflow rates of 0.047, 0.118 and 0.364 N m^3/s released at a depth of 51 m are presented. The measurements get as close to the plume center as one-half of a plume radius. These measurements and the prior measurements of Fannelop and Sjoen are compared with the two theories so the region of applicability of each one can be estimated. The Fannelop and Sjoen measurements and the new and large scale measurements presented in this paper are the only published bubble plume surface flow measurements known to the authors.

Received May 1983. Discussion closes March 1984.

SURFACE FLOW THEORIES

Complete details of the surface flow theories can be found in refs. 1 and 2. Here, only the underlying assumptions, a brief review and the resulting equations for each theory are given.

The Fannelop and Sjoen[1] theory

This theory considers the flow for radii that are large enough for the effects of mean vertical flow components to be negligible. In other words, it considers the flow after it has turned from the generally upward flow of the zone of established flow to a generally horizontal flow. The variation of horizontal velocity, v, with depth, z, below the surface is presumed to follow a Gaussian function.

$$v(r, z) = V(r) \exp\left[-z/h(r)\right]^2 \qquad (1)$$

where r is the radius and h is the 'depth' of the moving layer. For such a profile, the mass flux, m, through a cylinder of radius r is

$$m = \pi^{3/2}\rho_w hVr \qquad (2)$$

where ρ_w is the density of the water and the effect of the gas on the mean density of the fluid is neglected.

The horizontal flow is presumed to entrain additional fluid as it moves along with the rate of entrainment being proportional to the surface flow speed, V, the contact area

0309-1708/84/010040-5 $2.00

and an unknown entrainment coefficient, β. This leads to the following equation for conservation of mass:

$$\frac{d}{dr}(\sqrt{\pi}\rho_w hVr) = 2r\rho_w\beta V \qquad (3)$$

Tangential forces are neglected altogether so that the vertical integral of radial momentum flux in an infinitesimal azimuthal angular interval is a constant independent of radius. Integrating this over all azimuth angles yields what Fannelop and Sjoen call the 'horizontal momentum flux', M, given by:

$$M = \pi^{3/2}\rho_w hV^2 r/\sqrt{2} \qquad (4)$$

where M is considered to be independent of radius. The solution from the Fannelop and Sjoen theory is given as $V(r)$ and $h(r)$. For the initial conditions of V_i and h_i at a radius r_i at which the outward integration is started the solution is:

$$V(r) = V_i \Big/ \left[1 + \frac{2\beta}{\sqrt{\pi}}\frac{r_i}{h_i}\left(\frac{r^2}{r_i^2} - 1\right)\right]^{1/2} \qquad (5)$$

$$h(r) = h_i\frac{r_i}{r}\left[1 + \frac{2\beta}{\sqrt{\pi}}\frac{r_i}{h_i}\left(\frac{r^2}{r_i^2} - 1\right)\right] \qquad (6)$$

In the zone of established flow the vertical velocity, $u(r, z)$ is most commonly given (cf. refs. 1, 2, 3) as

$$u(r, z) = U(z)\exp\,[-r/b(z)]^2 \qquad (7)$$

In the upper part of the ZOEF,

$$\frac{b}{U}\frac{dU}{dz} \ll 1 \quad \text{and} \quad \frac{db}{dz} \ll 1$$

Therefore the center-line vertical velocity, U_i, and the plume radius, b_i, at the entrance of the ZOSF can be taken as these values in the ZOEF where the depth z_i, is equal to the plume radius.

Fannelop and Sjoen recommend taking r_i equal to $2b_i$. Furthermore, they recommend taking the radial mass flux, m, and the 'horizontal momentum flux', M, at r_i as equal to the vertical mass and momentum fluxes at z_i. This provides the initial conditions:

$$V_1 = \sqrt{2U_i} \quad \text{and } h_i = b_i/\sqrt{2\pi} \qquad (8a, b)$$

However, there is no unequivocal basis for these initial conditions so it is possible for the Fannelop and Sjoen theory to describe a portion of the flow with different initial conditions.

The Van Houten[2] theory

This theory determines an approximation to the actual mean flow by means of successive applications of the fundamental equations of fluid motion. The following initial approximations are made:

(1) Steady flow can be considered with the influence of turbulence accounted for through an eddy viscosity.
(2) The flow can be analysed as a single phase fluid with the effects of the gas bubbles approximated by a spatially varying fluid density.
(3) The fluid can be analysed as an incompressible fluid. The density of the fluid is assumed to vary from streamline to streamline, but is assumed to be constant on a given streamline. This ignores the expansion of the gas in the ZOSF with the resulting error minimized by taking the density on each streamline to be that at the mid-height of zone. It also ignores the percolation of the gas bubbles through the liquid and out into the atmosphere.
(4) The Reynolds number for the flow with the turbulent eddy viscosity is presumed to be large (much greater than unity).
(5) The boundary conditions are approximated as:
   (a) At the bottom of the ZOSF the flow is vertical with mean velocity and density distributions equal to those at the top of the ZOEF which can be approximated by the ZOEF theories given in refs. 1, 2 and 3.
   (b) At an outer radius, $r_0 \gg b_i$, the flow can be considered assumed to be horizontal with the pressure distribution being hydrostatic.
   (c) The upper surface is free.

Thus the ZOSF is modelled in this theory as the interaction of a steady, axisymmetric, rotational, non-homogenous, but incompressible, single phase plume with a free surface. The solution for the flow is determined by a set of successive approximations which have been found to be convergent. First, the flow is presumed to be inviscid and the free surface is presumed to be rigid. The flow velocities and pressures are calculated. Then the free surface is relaxed so that the hydrostatic pressure associated with the elevation of each point balances the previously calculated dynamic pressure. The velocities and pressures are calculated in this re-shaped domain and the free surface relaxed again. This process is continued until the solution for the inviscid flow converges. Then, for a presumed eddy viscosity, the head loss at each point in the flow is calculated. The iterative process is then repeated with the distribution of total head adjusted in accordance with the calculated head loss. By virtue of the presumed high Reynolds number, the head loss calculation only needs to be done once.

Because the flow is presumed to be incompressible, the theory is best developed in terms of the stream function, ψ, from which the radial and vertical (positive downwards) velocity components, v and w are given as:

$$v = \frac{1}{r}\frac{\partial\psi}{\partial z} \quad \text{and} \quad w = -\frac{1}{r}\frac{\partial\psi}{\partial r} \qquad (9a, b)$$

The governing equation for the stream function is obtained by integrating Euler's momentum equation along the streamlines as:

$$\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} = r^2\left\{\left(\frac{\omega}{r}\right)_0 + \frac{1}{\rho^2}\frac{\partial\rho}{\partial\psi}(P - P_0) + \Delta\frac{\partial H}{\partial\psi}\right\} \qquad (10)$$

ρ is the fluid density and it is a function of ψ since it varies from streamline to streamline. ω is the vorticity which is azimuthally directed and the subscript o refers to values on the lower boundary for each value of ψ. P is the pressure. $\Delta(\partial H/\partial\psi)$ is the effect of head loss due to viscosity. Since the pressure, which is obtained from Bernoulli's equation, depends on the solution, the numerical procedure for solving equation (10), even for a prescribed boundary shape and a prescribed $\Delta(\partial H/\partial\psi)$ distribution must be an iterative one. Details can be found in ref. 2.

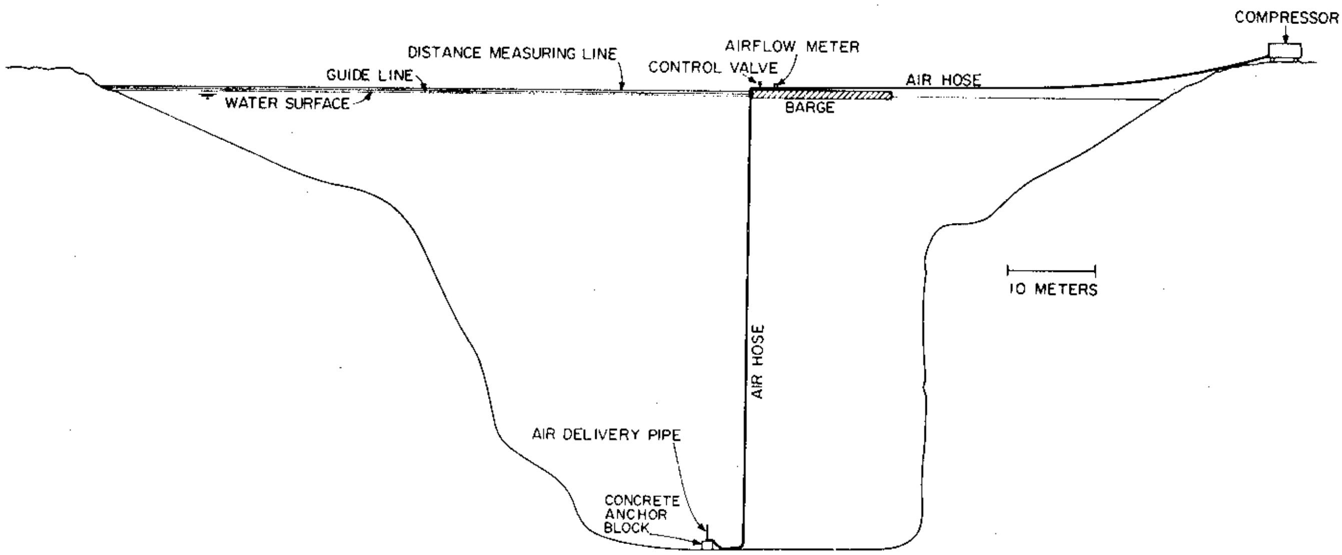

Figure 1. Experimental arrangement for bubble plume generation. An accurately scaled depth profile of Bugg Spring is shown.

EXPERIMENTS

The experiments took place in Bugg Spring which is a natural sinkhole spring located at Okahumpka, Florida, and which is part of the United States Naval Research Laboratory. The spring is 53 m deep and has a mean surface diameter of 110 m. A catamaran barge which is tightly moored to anchors on shore by five cables floats with one of its edges over the deepest part of the spring. A diagram of the experimental arrangement is shown in Fig. 1. A 2.5 m tall vertical air delivery pipe having a 5 cm inside diameter was secured to a concrete anchor block so that the vertical axis of the pipe, when projected to the surface, was 4.6 m away from the edge of the barge and the open end of the pipe was 50 m beneath the surface. Air was supplied to the bottom of the pipe through a hose from an airflow meter on the barge which in turn was connected by a hose to a rotary screw air compressor on the shore. A distance measuring line, about 50 m long was strung about 80 cm above the surface from the barge to a large tree on the shore such that the line passed over the axis of the pipe. The line had a diameter of approximately 3 mm and had a core made of very strong aramid fiber. This allowed the line to be set up with a tension of the order of 400 Newtons so that it had very little sag. Starting with an origin immediately above the wellhead marking tapes were put on the line at intervals of 1 m.

A second line, called the guide line, made of the same material was strung about 50 cm above the surface and was horizontally displaced from the distance line by about 10 cm. The guide line did not have distance marking tapes so its surface was smooth.

A ballasted float as shown in Fig. 2 was constructed. The bottom ballast steel plate had dimensions of 51 cm × 51 cm × 0.95 cm thick. This was bonded to a polystyrene foam block having dimensions of 51 cm × 51 cm × 25 cm thick on top of which was bonded a piece of plywood having dimensions of 51 cm × 51 cm × 0.6 cm thick. A 90 cm long wood rod having a diameter of 1.3 cm was attached to the center of the float so that its axis was vertical. A slack rope restraining line was attached to the rod with one end near the top and the other near the bottom. The guide line passed between the restraining line and the rod so the float was restrained to move along the guide line.

While the bubble plume was forming the float was held fixed at a distance of 5–6 m from the edge of the barge. Then the float was released. While the float moved along in the surface flow a small rowing boat was manoeuvered along a path parallel to the distance line and about 7 m from it. A time lapse 35 mm film camera driven by a quartz oscillator controlled clock at a frame rate of one picture per second was used to take photographs of the vertical wood rod and the distance line. Figure 3 is one of the photographs. From the rod locations in each pair of successive photographs the float speed at the mid-point

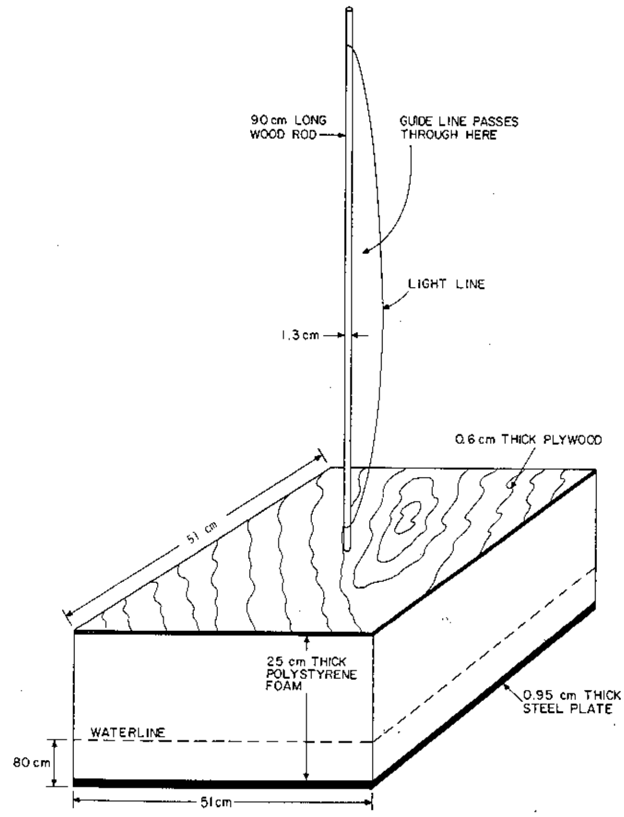

Figure 2. The surface speed following float

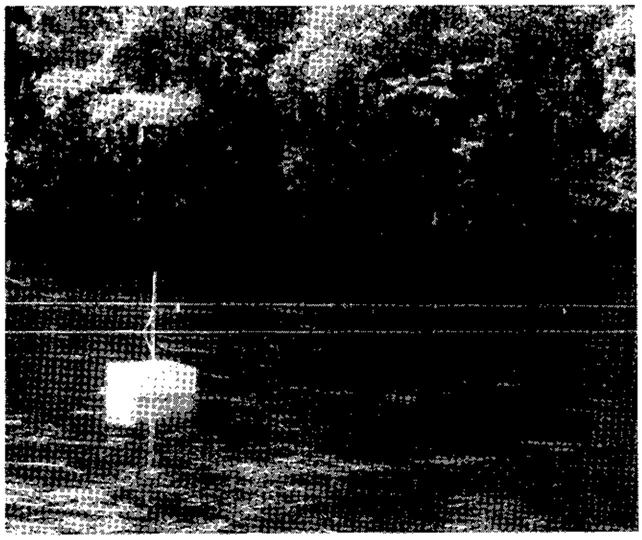

Figure 3. Photograph of the float moving in the surface flow. The distance line marked at 1 m intervals, the guide line, and the restraining line attached to the float can be seen

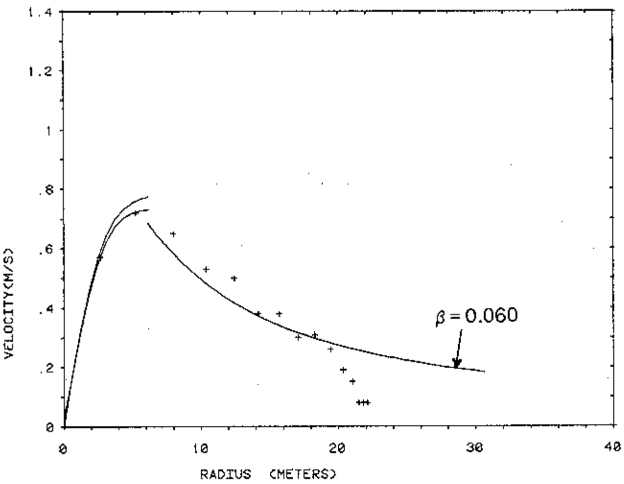

Figure 4. Surface flow speeds for a gas flow rate of 0.047 N m^3/s and a source depth of 50 m
+ measurement of velocity averaged over upper 8 cm
Upper curve at left is surface speed from Van Houten theory
Lower curve at left is average speed over upper 83 cm from Van Houten theory
Curve at right is surface speed from Fannelop and Sjoen theory with β as shown

between the two locations was calculated. The float had a draft of 8 cm so the float speed was taken as the speed of the flow averaged over the upper 8 cm. These speed distributions are shown in Figs. 4, 5 and 6 for airflow rates of 0.047, 0.118 and 0.364 N m^3/s.

COMPARISON BETWEEN EXPERIMENTS AND THEORIES

The numerical calculation procedure described in ref. 2 for the Van Houten theory was applied to the conditions of the Bugg Spring experiments and for a range of Reynolds numbers. It was found that for $r < 2b$ (radius less than twice the plume radius at the top of the ZOEF) the best agreement with experiment was given by the inviscid theory

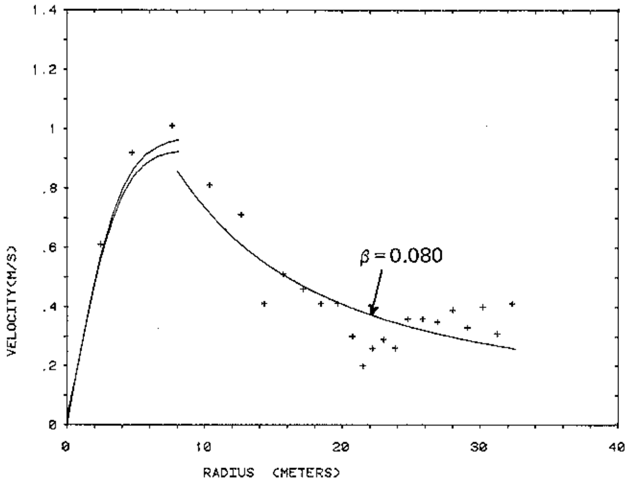

Figure 5. Surface flow speeds for a gas flow rate of 0.118 N m^3/s and a source depth of 50 m. Legend as in Figure 3

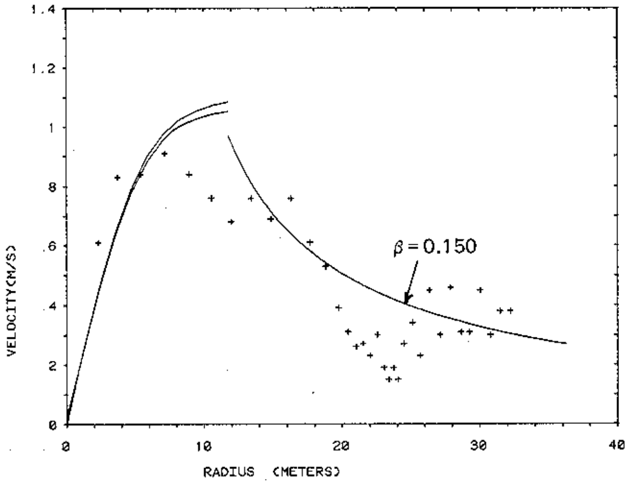

Figure 6. Surface flow speeds for a gas flow rate of 0.364 N m^3/s and a source depth of 50 m. Legend as in Figure 3

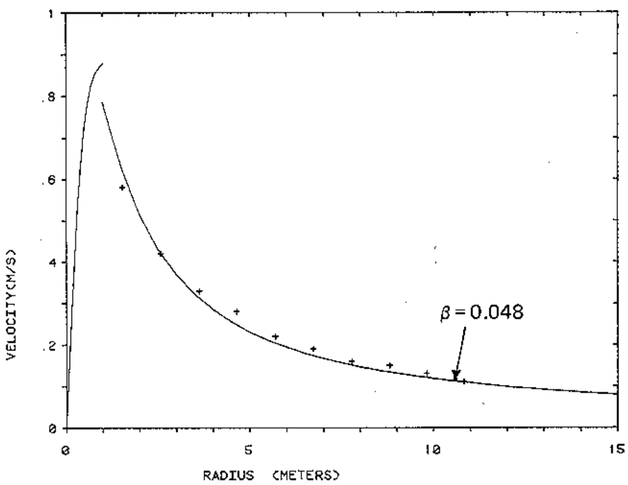

Figure 7. Surface flow speeds for a gas flow rate of 0.008 N m^3/s and a source depth of 5.5 m
+ measurements of Fannelop and Sjoen[1]
Curve at left is from Van Houten theory
Curve at right is from Fannelop and Sjoen theory with β as shown

result. Since the influence of the molecular viscosity is negligible, this means that the best agreement occurs when the effect of eddy viscosity is negligible in the theory. It was also found that good agreement for larger radii was not obtained for any reasonable constant eddy viscosity.

The Fannelop and Sjoen theory cannot describe the flow at small radii, but comparing it with the Bugg Spring experiments showed that it could describe the surface velocities for $r > 2b$. Furthermore, it was found that good results were obtained by using a Gaussian fit (minimum mean square error) to the Van Houten theory velocity vs depth profile at $r = 2b$ as the initial conditions for the Fannelop and Sjoen theory.

The results of applying the above procedure to the Bugg Spring conditions are shown in Figs. 4, 5 and 6. Results of the Van Houten theory are shown for both the surface flow speeds and the speed averaged over the upper 8 cm to allow a comparison with the experiments in which the speeds averaged over the upper 8 cm were measured. For the Gaussian profiles used by the Fannelop and Sjoen theory the difference between the surface speed and the speed averaged over the upper 8 cm is negligible in the figures. It was found that increasing values of the entrainment coefficient, β, were required for increasing gas flow rates.

The same procedure was applied to the conditions of the Fannelop and Sjoen experiments and the results are shown in Figs. 7 and 8. Although these experiments did not include measurements for $r < 2b$, good results were obtained at larger radii. Again, it was found that higher gas flow rates required larger entrainment coefficients.

DISCUSSION

For the data in Figs. 4 and 5, the Van Houten theory predicts the flow speeds for $r < 2b$ with an average error of about 5%. The agreement is not as good in Fig. 6, but the data in that figure are quite irregular so it is likely that they were influenced by unsteadiness of the plume. Apart from irregularities in the data, the Fannelop and Sjoen theory gives results of similar accuracy for $r > 2b$ with the initial conditions at $2b$ provided by the Van Houten theory.

The biggest question in the use of the theories for predicting the surface flow in bubble plumes involves the choice of the entrainment coefficient for the Fannelop and Sjoen theory. It is tempting to discount the 0.364 N m^3/s Bugg Spring data because of its irregularities and find the average of values of β for the other four conditions considered here. This yields a value for β of 0.061. However, trends towards higher values for higher gas flow rates and lower values for greater water depths are unmistakably clear.

The velocity vs depth profiles measured by Fannelop and Sjoen[1] do not have a self-similar form as is presumed by their theory. Near $r = 2b$ the profiles have slopes which increase with height throughout as is predicted by the inviscid Van Houten theory. Near $r = 10b$ the measured profiles resemble the Gaussian forms presumed for the Fannelop and Sjoen theory. For larger radii the profiles have changed form further such that they have maxima that lie beneath the surface. Therefore, in using the Fannelop and Sjoen theory to predict surface velocities, which it has been found to be able to do in spite of its velocity profile errors, β must be considered as a parameter that accounts for both entrainment and profile form change.

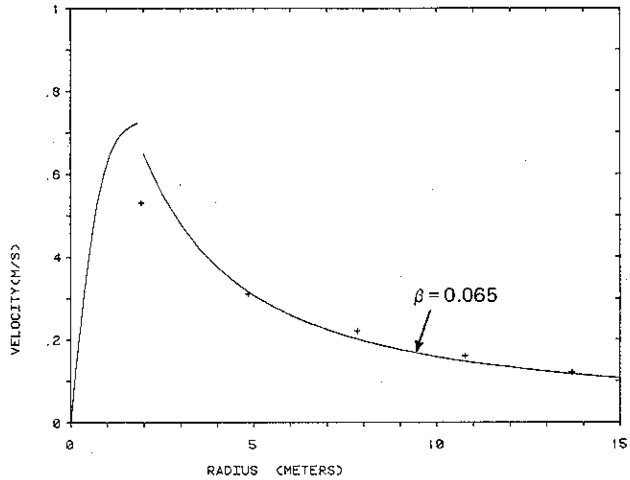

Figure 8. Surface flow speeds for a gas flow rate of 0.010 N m³/s and a source depth of 9.9 m. Legend as in Figure 7

One approach to improving our capability for predicting the surface flow above blowouts is to develop a theory that accounts for the velocity profile form change at large values r/b and then relating the entrainment coefficient to the profile form by comparison with experiments. Equivalently, the appropriate spatially dependent eddy viscosity required to make the Van Houten theory accurate at large radii could be determined by comparison with experiment. In either case far more detailed experiments than those which have already been done would be required.

Alternatively a strictly empirical approach for finding appropriate values of β could be used. The experiments that have already been done include small and medium scales. If large scale data were obtained, β would then be known as a function of plume parameters for the entire parametric range of interest. The most appropriate large scale plumes upon which to make the required measurements are those coming from blowouts of subsea gas-containing hydrocarbon wells. Experimental determinations of values of β is quite straightforward. Let V_1 and V_2 be the measured surface velocities at radii r_1 and r_2 respectively. Then, from equation (5)

$$\beta = \frac{\sqrt{\pi}}{2} h_i r_i \frac{V_2^2 - V_1^2}{V_1^2(r_1^2 - r_i^2) - V_2^2(r_2^2 - r_i^2)} \qquad (11)$$

r_i can be taken as $2b$ where b can be obtained theoretically by the methods of ref. 3 and the needed value of h_i can be obtained from the Van Houten theory.

ACKNOWLEDGEMENTS

This work was supported in part by the US Department of the Interior under contract number 14-08-001-20460 and in part by the Office of Naval Research under contract number N00014-81-K-0785.

REFERENCES

1. Fannelop, T. K. and Sjoen, K. Hydrodynamics of underwater blowouts, AIAA 18th Aerospace Sciences Meeting, Pasadena, California (1980)
2. Milgram, J. H. and Van Houten, R. J. Plumes from subsea well blowouts, Proc. of Third Intl Conf. on Offshore Structures, Cambridge, Mass. (1982)
3. Milgram, J. H. The mean flow in round bubble plumes, *Jl of Fluid Mechanics* 1983, **133**, 345