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# TORSIONAL STRENGTH OF LONGITUDINALS IN MARINE STRUCTURES

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16. Abstract (Limit: 200 words)

Lateral-torsional instability (tripping) of stiffeners is often considered in design of marine structures to be only a secondary mode of failure. However, analytical studies and some tests have demonstrated that this mode has a serious potential of being the primary mode. This report presents a review of the general analytical methods and of the more prominent current design recommendations. A comparison of the theories with the few available test results indicated an acceptably reasonable agreement. The following parameters were taken into account in developing the solutions:
(1)Rotational restraint provided by the plate. (2) Effective area of the attached plate. (3)Deformation of the cross section, specifically, the bending of the stiffener web plate. (4)An approximation of the nonlinear material and structural behavior. A sample comparison of the torsional strengths of an angle and of a tee stiffener with the flanges of the same size was made according to the proposed design methods. The results showed that an angle stiffener with a lower slenderness ratio (L/r) is stronger than a tee but is weaker for a higher slenderness ratio. However, when the effect of web deformations was included, the overall axial strength of angle stiffeners tended to fall below that of tee stiffeners.

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# Table of Contents

# List of Figures

# List of Tables

# ABSTRACT

Lateral-torsional instability (tripping) of stiffeners is often considered in design of marine structures to be only a secondary mode of failure. However, analytical studies and some tests have demonstrated that this mode has a serious potential of being the primary mode. Some design-oriented equations for axial and lateral loads have been proposed for both symmetrical and asymmetrical stiffeners.

For symmetrical cross sections (tees), torsional failure is characterized by the twisting of the stiffener about its line of attachment to the plating; for asymmetrical cross sections (angles) -- by the twisting coupled with flexural deformation.

This report presents a review of the general analytical methods and of the more prominent current design recommendations. A comparison of the theories with the few available test results indicated an acceptably reasonable agreement. The following parameters were taken into account in developing the solutions:

1. Rotational restraint provided by the plate.
2. Effective area of the attached plate.
3. Deformation of the cross section, specifically, the bending of the stiffener web plate.
4. An approximation of the nonlinear material and structural behavior.

A sample comparison of the torsional strengths of an angle and of a tee stiffener with the flanges of the same size was made according to the proposed design methods. The results showed that an angle stiffener with a lower slenderness ratio (L/r) is stronger than a tee but is weaker for a higher slenderness ratio. However, when the effect of web deformations was included, the overall axial strength of angle stiffeners tended to fall below that of tee stiffeners.

A short computer program (in BASIC) is included in the report for performing the otherwise tedious iterative procedure of analyzing angle stiffeners with web deformations.

# 1. INTRODUCTION

## 1.1 General Remarks

Stiffened plates are commonly used in marine structures, such as ships, superstructures of offshore platforms, as well as, in aircraft, bridges, etc. In these applications, stiffened plates are often loaded in compression and therefore the avoidance of buckling is an important consideration in design.

This report deals with the torsional buckling (tripping) of stiffeners under axial compression and end moments. The purpose was to improve the accuracy of the present design rules for this failure mode. Particular emphasis was put on asymmetrical stiffeners (angle, zee).

A stiffened plate can fail by instability in the following modes:

1. Overall buckling: the stiffener buckles together with the plate.
2. Torsional buckling (tripping) of stiffeners, often forming more than one half waves between the transverses.
3. Plate buckling between stiffeners.
4. A concurrent or sequential combination of all or some of the first three modes.

Although initial buckling may take place in the plate at a relatively low stress level, a significant amount of post-buckling strength may remain in the stiffened plate if the structure is carefully designed and fabricated.

Some recent publications have pointed out the significance of lateral-torsional (tripping) instability as a primary failure mode for stiffened plate structures [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The potential for such failure has important ramifications on the weight, fabrication costs, and structural reliability. Thus, the ability to predict tripping failure in the early stages of design can haveimportant consequences on the final design.

Review of literature and of available test results shows that relatively little material is available on the subject of tripping of stiffeners, especially, on the analysis and design of asymmetrical stiffeners under axial and bending loads. Also, there are some differences and contradictions in the available studies.

Critical appraisal is made here of the available theoretical solutions and design recommendations and a comparison is made with the available test results in order to find discrepancies among the assumptions, formulations and results and to suggest the more accurate and practical method(s).

## 1.2 Basic Concept of Torsional Buckling (Tripping) of Stiffeners

For the purpose of design and analysis, a wide stiffened plate may be treated as an assembly of connected flat plate strips, each with one stiffener in the middle (see Fig. 1). When a stiffened plate is subjected to compression, bending, shear or a combinaton of these stresses, its theoretical buckling load can be evaluated by analyzing one plate-stiffener combination as an individual member. However, it is also possible that a plate may buckle locally before the whole member becomes unstable or before the yield stress of the material is reached. The development of local buckles may result in an appreciable loss of strength, (See Fig. 2), and sometimes will initiate the failure of the entire structure.

Torsional buckling (tripping) is defined as the twisting of the stiffener about its line of attachment to the plate. This kind of lateral-torsional instability has often been ignored or considered as a "secondary" mode of failure in structural design[2]. There are two deformation modes possible during the failure of a stiffened plate. In the first, the consecutive stiffeners deform in the transverse direction by approximately equal but opposite amounts. This mode is called the ANTISYMMETRICAL mode. In the second mode, the stiffeners deform in the same direction, and this is called the SYMMETRICAL mode[12]. (See Fig. 4)

For the purpose of design, it is desirable to simplify the analytical procedure and to make the formulation as compact as possible.

The following factors can affect the strength of a stiffener:

1. Shape of cross section (Tee, Angle, Flat bar, Bulb-flat)
2. Properties of material (yield stress, modulus of elasticity)
3. Slenderness ratio
4. Intial imperfections
5. Residual stresses
6. Boundary conditions

Some of the factors, such as the boundary conditions, are not exactly known or cannot be measured quantitatively, and appropriate assumptions have to be made to obtain the best approximate results.

To minimize the weight of a stiffener for the sake of economy, the effective slenderness must be kept as small as possible in order that the material could be used at the greatest possible permissible stress. This design consideration is especially important for higher strength metals.

In the inelastic range, the tangent modulus approach has been used in the same manner as for columns. This is, generally, a conservative approximation to the solution of a complex problem. Although tangent modulus reflects only the effect of nonlinearity of the stress-strain properties, it has been widely used to take into account residual stresses, geometrical imperfections and edge restraint by lumping all these into an approximately equivalent tangent modulus $E_t$. Most commonly, this has been accomplished by introducing a transition curve between the yield and elastic buckling strengths.

The most common types of longitudinal stiffeners are: Flat-bar, Tee, Angle and Bulb-flat. Of these the flat-bar and tee are symmetrical about the web, and the angle and bulb-flat are asymmetrical. In the classical formulation of the differential equations in which the cross section is assumed not to deform, the only difference in the solution for the two groups is that for symmetrical stiffeners, only the torsional mode is needed since it is uncoupled from the other buckling modes, whereas for asymmetrical stiffeners, the coupling of the torsional and lateral modes must be considered.

Distortion of the cross section complicates the problem considerably, and the effect becomes quite significant, particularly for asymmetrical stiffeners.[10]

# 2. SYMMETRICAL STIFFENERS

## 2.1 Tripping Under Axial Load

Torsional buckling (tripping) usually involves both sideways and vertical flexural deformations (u and v), as well as rotation R of the stiffener. If the cross section is assumed not to distort, then, for small deformations, the displacements are coupled as shown in Fig. 3.[4, 13, 1, 2, 5] The buckling state can be formulated by applying the Principle of Minimum Total Potential which results in the condition that for all possible deformations, the total strain energy U is greater than or equal to the work done by the externally applied forces[4, 13, 14]. The strain energy stored in the structure in the buckled state is given by the following expression:

$$U = 1/2\int_0^a [E(I_y\bar{s}^2 + I_w)R''^2 + GJR'^2 + CR^2]dz \tag{2.1}$$

where u and R refer to the translation of the flange and the rotation about the toe of the stiffener. The total strain energy U is made up of the contributions from sideways bending, longitudinal warping, torsion, and rotation of the supporting plate modelled as an elastic spring. In this expression: a is the length of the stiffener between transverse supports; E and G are the Young and shear moduli of the material, respectively; C is the rotational spring constant (per unit length) of the plate. Terms $I_y$, s, $I_w$, and J are defined in Fig. 5 from the geometry of the stiffener.

When the axial thrusts and/or moments are applied at the stiffener ends, it is convenient to compute the external work by integrating over the cross section the product of the axial stress arising from these forces and the axial shortening $\delta$(x,y) of the stiffener fiber.

$$W = \int\int F_z\delta(x,y)dxdy \tag{2.2}$$

This integration is carried out over the area of the stiffener end. The relative displacement of the two ends of the stiffener due to its curvature for a fiber at location (x,y) is

$$\delta(x,y) = 1/2\int_0^a [u'^2 + v'^2]dz \tag{2.3}$$

If the stiffener is restrained against twisting but allowed to warp at its ends, and the restraint from the plating is not zero, and the cross section is assumed not to distort, it is logical to select a buckled shape which corresponds to simply supported end conditions because of the repeatability and continuity of typical plated structures.[4, 5, 2, 3]. The simplest choice is to assume a sinusoidal shape.

$$R = R_0 \sin(m\pi z/a) \tag{2.4}$$

where m is the number of half waves.

Substituting this mode shape into Eqs. (2.1) and (2.2) and performing the necessary operations, the elastic tripping stress is obtained after equating U and W.

$$F_{TA} = [E(I_y\bar{s}^2 + I_w)(m\pi/a)^2 + C(a/m\pi)^2 + GJ]/I_p \tag{2.5}$$

where $I_p = \int_A [x^2 + y^2] dxdy$ is the polar moment of inertia about the stiffener toe.

Denoting the warping constant about the toe by $I_{wn} = I_y\bar{s}^2 + I_w$, the minimum tripping stress is found by minimizing Eq. (2.5) with respect to m, that is, by setting $\partial F_{TA}/\partial m$=0. Then,

$$Min(F_{TA}) = [GJ + 2\sqrt{CEI_{wn}}]/I_p \tag{2.6}$$

and the critical wave number is

$$m = (a/\pi)\{C/EI_{wn}\}^{1/4} \tag{2.7}$$

This minimum value is valid only if m is treated as a continuous function. It is accurate enough for m>3. If m<3, Eq. (2.5) should be used to determine $F_{TA}$ with m being the lower adjoining integer of m from Eq. (2.7). The minimum value of m is 1.

A plot of Eq. (2.5) is shown in Fig. 6. As can be seen for span lengths less than $L_{cr}$, Eq. (2.6) can underestimate $F_{TA}$ because it assumes the number of buckled half-waves to be a continuous instead of a discrete function. For this case, Eq. (2.5) with C = 0 may give a better estimate.

When the restraint against rotation is taken to be zero (C=0), the lowest buckling stress can be seen to occur for one half wave, m=1.[11]

$$F_{TA} = [GJ + EI_{wn}(\pi/a)^2]/I_p \tag{2.8}$$

Equation (2.6) can be simplified to

$$F_{TA} = (E/I_p)[J/2.6 + 2\sqrt{CI_{wn}/E}] \tag{2.9}$$

and an estimate of the rotational restraint C can be made from the dimensions of the plate and stiffener web by

$$C = \frac{E}{3(1-\nu^2)} \frac{(t_p t_w)^3}{(bt_w^3 + 2d_w t_p^3)} \tag{2.10}$$

where $\nu$ is Poisson's Ratio, and $t_p$ and $t_w$ are the thickness of the plate and the stiffener web, respectively; b is the spacing between two adjacent stiffeners and $d_w$ is the depth of the stiffener web.

Because the second term $2\sqrt{CI_{wn}/E}$ of Eq. (2.9) is usually much larger than the first term J/2.6, one can conservatively simplify $F_{TA}$ to

$$F_{TA} = (E/I_p)2\sqrt{CI_{wn}/E} \tag{2.11}$$

Substitution of $I_{wn}$ and C in terms of the plate and stiffener dimensions results in

$$F_{TA} = (0.35E/I_p)\sqrt{(A_f b_f^2 d_w^2)/(b/t_p^3 + 2d_w/t_w^3)} \tag{2.12}$$

When m is a noninteger less than 3, a better estimate of $F_{TA}$ can be made by using Eq. (2.5) with C = 0 and m equal to the lower adjoining integer of Eq. (2.7). This gives

$$F_{TA} = (0.8A_f E/I_p)(b_f d_w/a)^2 \tag{2.13}$$

## 2.2 Rotational Restraint by Plate

The only term in Eq. (2.5) which is not a property of the stiffener cross section and which requires further attention is C, the rotational restraint by plate. As one can see from Eqs. (2.6) and (2.7), m and the tripping stress $F_{TA}$ increase as C increases. When C has a high value, the stiffener web may start bending, a condition which violates the assumption that the cross section is to remain undistorted. To bypass consideration of the effects of serious distortion, Faulkner set an arbitrary upper limit for C of $38EJ/a^2$[1, 2]. However, Adamchak showed that this value is overly optimistic[4].

An approximation of the rotational restraint can be computed by considering the plate to be made up of unit-wdth beam strips. Then, C becomes

$$C_0 = \frac{Et_p^3}{3b}$$

where $C_0$ is the value of C when the plate is not subjected to axial compression. Since the bending of the web is likely to occur for higher values of C, this analysis would seem to be too optimistic.

The difficulty of estimating the correct value of C is in establishing the effect of axial stresses on the response of the plate. Buckling of the plate may cause the wave length to be closer to the a/m value and thus reduce the C value. A suggestion was made by Faulkner and Adamchak to evaluate C with respect to the buckled wave length of the plate by linear interpolation[4, 2]:

$$C = \begin{cases} C_o(1-F_{TA}/F_{PE}) & \text{for} \quad F_{TA} \le F_{PE} \\ 0 & \text{for} \quad F_{TA} > F_{PE} \end{cases} \qquad (2.15)$$

where $F_{PE}$ is the elastic buckling stress of the plate assuming simple support along the edges. Then, $F_{TA}$ becomes

$$F_{TA} = [EI_{wn}(m\pi/a)^2 + GJ + C_0(a/m\pi)^2]/[I_p + C_0/F_{PE}(a/m\pi)^2] \qquad (2.16)$$

and the initial unloaded restraint $C_0$,

$$C_0 = \pi^2 D/2b[1+(mb/a)^2]^2 \qquad (2.17)$$

After plate buckling, the C value would no longer be uniform along the line of rotation. Sometimes it would even generate moments which would encourage tripping.

The General Dynamics Design Guidelines take into account the bending of the stiffener web by including its stiffness in the calculation of C, as shown in Fig. 7.[5] This simplified approach should give a relatively good approximation for the plate restraint (Eq. (2.10)) because the beam strip analysis gives an exaggerated value of C while the inclusion of the web stiffness should provide some correction to the C value.

When the cross section of the stiffener is sturdy or when the plate slenderness ratio is large, the stiffener would form fewer half-waves than the buckled plate and, thus, the plate would provide greater restraint against tripping.

A common approach is to ignore web deformations and to set the restraint equal to zero ("piano hinge connection"). This seems to be the most popular approach in "classical" solutions used for formulating many design formulas, particularly, for axial compression[8, 11].

Another conservative alternative suggested by Adamchak is to use the minimum value of $C_0$ for all mode numbers, i.e., let m=1 in Eq. (2.17).[4]

## 2.3 Effective Width of Plate

Under the effective width concept, only a portion of the plate width is considered to be effective in carrying axial loads after the local buckling stress of the plate has been exceeded. The effective plate regions are adjacent to the plate edges at the stiffeners. For an ideally flat plate, the effective width $b_e$ is equal to the total width b up to the point of buckling, then, the axial stress is redistributed in the plate, and the additional loading is carried by the post-buckling strength with $b_e$ becoming smaller and smaller. It is commonly assumed that the capacity of the plate increases until the edge stress reaches the yield stress level. Modifications of this behavior due to initial imperfections and residual stresses have been introduced by many researchers. A compilation of these studies is given by Ostapenko and Surahman in the form of an analytical model of the complete (pre- and post-ultimate ranges) load vs. shortening axial behavior of wide and long plates[15, 16].

A typical stress distribution and the idealized effective width $b_e$ in a stiffened plate are shown in Fig. 10.[17] A simple, yet reasonable, prediction for the effective width at ultimate strength of plates typical for marine structures is given below for the range $1<B^2$:

$$\frac{b_e}{b} = \frac{2}{B} - \frac{1}{B^2} \tag{2.18}$$

or by a somewhat more conservative formula based on a statistical study[16]

$$\frac{b_e}{b} = \frac{1.82}{B} - \frac{0.93}{B^2} \tag{2.19}$$

where B is the plate slenderness ratio parameter

$$B = (b/t)\sqrt{F_y/E} \tag{2.20}$$

These equations, Eqs. (2.18) and (2.19), represent a lower mean ultimate strength for plates having realistic initial distortions and residual stresses of the magnitude induced by welding.

Since the tripping stress at the stiffener toe may be less than the yield stress, the effective width equation, Eq. (2.18), can be modified by replacing $F_y$ with $F_{TA}$ in Eq. (2.20).

$$\frac{b_e}{b} = \frac{2}{B\sqrt{F_y/F_{TA}}} - \frac{1}{B^2(F_y/F_{TA})} \tag{2.21}$$

or in Eq. (2.19),

$$\frac{b_e}{b} = \frac{1.82}{B\sqrt{F_y/F_{TA}}} - \frac{0.93}{B^2(F_y/F_{TA})} \tag{2.22}$$

## 2.4 Buckling Under End Moments

Since a stiffened plate may be subjected to lateral loading and/or end moments, the torsional strength of the stifffeners (tripping) must be also considered for this case.

For a stiffener subjected to axial compression P, bending moment M and transverse loading q, the potential energy function for lateral-torsional buckling about an enforced axis of rotation is[13]:

$$U^* = 1/2 \int [EI_y u''^2 + EI_w R''^2 + GJR'^2 - Pu'^2 + 2Mu'' R - \tag{2.23}$$

$$- P(I_p/A + ez/I_x)R'^2 - \bar{s}qR^2 + CR^2]dz$$

To analyze a full plate-stiffener combination, one can take advantage of this equation in the following manner. When lateral loading is zero, q=0, and the structure is subjected only to end moments, a relationship between the axial load and the moment acting on the stiffener alone can be derived from the recognition that the stress at the stiffener toe due to the total end moments should be essentially zero. This should be so since the centroid of the total plate-stiffener section is approximately at the plate surface. Then,

$$F_n = Mc_1/I_s - P/A_s = 0 \tag{2.24}$$

This gives the required axial force on the stiffener to be

$$P = Mc_1A_s/I_s \tag{2.25}$$

where $I_s$ and $A_s$ are the moment of inertia and the area of the stiffener, $c_1$ is the distance between the stiffener centroid and the toe and $F_n$ is the stress at the toe.

By substituting Eq. (2.25) into Eq. (2.23) and setting q=0, the differential equation becomes a function of M only. The critical moment $M_{cr}$ on the stiffener can be found by assuming a sinusoidal function for rotation R

$$R = R_0 \sin(m\pi z/a) \tag{2.26}$$

and applying the Principle of Minimum Total Potential (Rayleigh-Ritz method), that is, by setting $\partial U^*/\partial R_0 = 0$.

$$M_{cr} = \frac{EI_{wn}(m\pi/a)^2 + GJ + C(a/m\pi)^2}{2\bar{s} + c_1(A_s\bar{s} + I_p)/I_s} \tag{2.27}$$

This moment and the axial force P from Eq. (2.25) produce flange stress $F_{TB}$,

$$F_{TB} = M_{cr}c_2/I_s - P/A_s \tag{2.28}$$

where $c_2$ is the distance from the centroid of the stiffener to the outer fiber of the flange.

The substitution of Eq. (2.25) into Eq. (2.28) gives $F_{TB}$ as a function of $M_{cr}$ only.

$$F_{TB} = M_{cr}c_2/I_s + M_{cr}c_1/I_s = M_{cr}d/I_s \tag{2.29}$$

where d is the depth of the tee stiffener. (See Fig. 8)

The full plate-stiffener combination is expected to buckle under the total moment $M_T$ which would produce the stress in the flange approximately equal to $F_{TB}$.

$$F_{TB} = M_T c_3/I_{ps} \tag{2.30}$$

where $c_3$ is the distance from the outer fiber of the flange to the plate-stiffener centroid and $I_{ps}$ is the moment of inertia of the plate-stiffener section. Then,

$$M_T = F_{TB}I_{ps}/c_3 = M_{cr}I_{ps}d/(I_s c_3) \tag{2.31}$$

Since $c_3$ is approximately equal to d (Fig. 8), the critical moment for the total plate-stiffener section can be given by

$$M_T = M_{cr}\frac{I_{ps}}{I_s} \tag{2.32}$$

where $M_{cr}$ is from Eq. (2.27) and is based on the properties of the stiffener alone.

Again, the minimum value with respect to the half-wave length can be found by applying the procedure used in the previous section. The equation for the critical wave number is identical to Eq. (2.7) and the minimum tripping stress due to end moments becomes

$$\mathrm{Min}(F_{TB}) = \frac{Ed(J/2.6 + 2\sqrt{CI_{wn}/E})}{2\bar{s}I_s + c_1(A_s\bar{s}^2 + I_p)} \tag{2.33}$$

Design guidelines of Reference[5] suggest the tripping stress due to bending to be given by

$$F_{TB} = \frac{0.17E}{d_w S}\sqrt{A_f b_f^2 d_w^2/(b/t_p^3 + 2d_w/t_w^3)} \tag{2.34}$$

and, for m less than 3, by

$$F_{TB} = \frac{0.4A_f E}{d_w S}(b_f d_w/a)^2 \tag{2.35}$$

These two equatons incorporate a number of simplifications, for the most part, conservative.

## 2.5 Torsional Buckling Under Combined Forces

Torsional buckling (tripping) under combined forces, that is, concurrent compression, end moments and lateral loading, is a very complex phenomenon. However, this type of loading is encountered in many practical cases. An approximate approach to compute the tripping stress in this case is to apply the interaction formula, such as,

$$(F/F_{TA})^\alpha + (M/M_{cr})^\beta + (q/q_{cr})^\gamma = 1 \tag{2.36}$$

where $F_{TA}$, $M_{cr}$ and $q_{cr}$ are the critical values when each loading is applied individually under the same boundary conditions, and each computed with the same mode number even though minimum buckling conditions for the individual cases may occur with different modes. Powers $\alpha$, $\beta$ and $\gamma$ can be established empirically on the basis of experimental and analytical results[4].

It has been suggested by Adamchak that a linear interaction relationship, as shown in Fig. 9, although conservative, can be used for design purposes until further evidence, either analytical or experimental, indicates a more realistic choice[4].

Since at present there is no solution for the critical lateral loading, $q_{cr}$, an approximation can be used by replacing this loading with end moments having the same value as the maximum moment caused by the loading.

## 2.6 Inelastic Range

In the inelastic range, the nonlinear behavior of stiffened plates makes the analysis much more complicated. It has been proposed to modify the elastic tripping stress of the previous sections to the inelastic range by substituting the tangent modulus for Young's modulus when the calculated tripping stress is above the proportional limit. This approach requires a trial and error solution because the calculated tripping stress must correspond to the assumed tripping stress used to select the tangent modulus. The popular approach for avoiding this iterative procedure is to use a parabolic approximaton.[4, 2, 18, 5, 8]

$$F'_E = F_y - (F_y - F_p)\frac{F_p}{F_E} \tag{2.37}$$

where

$F'_E$ = Buckling stress in the inelastic range

$F_y$ = Yield stress

$F'_p$ = Proportional limit of material

$F_p$ = Reduced proportional limit = $F'_p - F_R$

$F_E$ = Euler column buckling stress

$F_R$ = Representative residual stress due to fabrication and welding

Many tests on column buckling have shown that the effective stress-strain curve departs from linearity below the proportional limit. This is mainly due to the residual stresses $F_R$, especially in the flange plates. For hot-rolled steel shapes this is typically equal to $0.3F_y$, but no definite value or pattern of residual stresses has been established for plate stiffeners.[19, 20, 21, 22] A conservative approximation of the compressive residual stress of $F_R = 0.5F_y$ is usually made. Then, the following tripping strength expressions result:

$$F_{TA} = \begin{cases} F_{TA} & \text{for} \quad F_{TA} \le 0.5F_y \\ F_y(1 - F_y/4F_{TA}) & \text{for} \quad F_{TA} > 0.5F_y \end{cases} \tag{2.38}$$

Although the non-uniform stress distribution over the cross section due to constant moment or uniform lateral pressure renders the analysis less reliable, the above simplification has been recommended for these cases also.[4, 2, 5]

# 3. ASYMMETRICAL STIFFENERS

## 3.1 General Comments on Asymmetrical Stiffeners

Since asymmetrical stiffeners, such as angles, lack an axis of symmetry perpendicular to the plate (See Fig. 11), the three differential equations are interdependent. Thus, the sideways flexure and twisting coupled through the enforced axis of rotation interact with the overall flexure normal to the plane of the plate.

The coupled flexural-torsional buckling stress may either be associated with the local instability or may follow as a secondary mode with an increasing load on a panel buckled initially in a local mode.[8]

The suggested methods for estimating the tripping stress are based on the idea of modelling the plate-stiffener combination as a column.[23, 2, 3, 5, 8] Such a column is assumed to be restrained against twisting by the torsional spring provided by the attached plate (See Fig. 12). This analysis assumes that all distortions and local deformations of the cross section are negligible and, thus, admits only the flexural and torsional displacements. Assuming that the panel contains many identical and equidistant stiffeners, the following two possible buckling modes may develop as shown in Fig. 12:

1. Symmetrical mode in which the twisting and flexural deformations of all the stiffeners are identical in magnitude and direction.
2. Antisymmetrical mode in which the twisting and flexural deformations of adjacent stiffeners are equal but opposite.

In general, symmetrical mode is more usual for flexural buckling because in antisymmetrical mode, the resistance of the plate to its distortion acts as an elastic foundation on the stiffener and hence makes it more difficult for the buckling to occur. But antisymmetrical mode is more common with torsional buckling (tripping) since antisymmetrical mode requires less bending energy of the plate. Tripping buckling of the stiffeners is more sensitive to plate buckling because the restraint, C, provided by the plate then varies along the line of attachment. It is important to realize that none of the above modes can occur independently as they are coupled at buckling.

## 3.2 Buckling Under Axial Load

For the tripping of asymmetrical stiffeners under axial load, flexure and torsion are coupled and therefore the coupled axial tripping stress $F_{CA}$, which is always smaller than either the individual axial buckling or the tripping stress, can be calculated by substituting the corresponding terms into the equilibrium equations; this will provide two homogeneous simultaneous equations, and the vanishing of the determinant formed by their coefficients results in the following buckling condition:[23, 2, 5, 8]

$$(1 - F_E/F)(1 - F_{TA}/F) = (y_e/r_0)^2 \tag{3.1}$$

where $r_o$ is the radius of gyration of the stiffener about the enforced axis of rotation (the toe of the stiffener), and $y_e$ is the vertical coordinate of this axis relative to the principal axes through the plate-stiffener centroid (See Fig. 13). $F_E$ and $F_{TA}$ are the Euler buckling stress of the plate-stiffener gross section and the axial compression tripping stress, respectively. Solving these two equations, one obtains the critical buckling-tripping stress

$$F_{CA} = \frac{F_E + F_{TA}}{2(1-(y_e/r_o)^2)}\{1 - \sqrt{1 - \frac{4F_E F_{TA}(1-(y_e/r_o)^2)}{(F_E+F_{TA})^2}}\} \tag{3.2}$$

where

| | |
|---|---|
| $F_{CA}$ | Coupled axial buckling-tripping stress |
| $F_E$ | Euler buckling stress of the plate-stiffener gross section |
| $F_{TA}$ | Tripping stress due to axial compression (Eq. (2.5)) |
| $r_o$ | Polar radius of gyration about the toe = $\sqrt{I_p/A_{ps}}$ |
| $y_e$ | Vertical coordinate from plate-stiffener centroid to the axis of rotation (stiffener toe) |
| $I_p$ | Polar moment of inertia of stiffener about toe |
| $A_{ps}$ | Area of plate-stiffener cross section |

The difference between $F_{TA}$ values for symmetrical and asymmetrical stiffeners is due to the difference in their twisting properties and modes of deformation. One may expect a transverse warping effect, $I_x x^2$, to appear in asymmetrical stiffeners due to the eccentricity of the shear center to the y axis. This consideration requires special attention when adopting this equation.

Because the torsional and flexural effects are coupled, the associated critical stress is smaller than for either of the component modes taken separately. The coupling between the modes may be neglected if the critical stress corresponding to one component mode is much smaller than the stress corresponding to the other mode.

Another approach is to treat the coupling of torsion and flexure as pure torsion about an axis which is parallel to the toe line and lies in the plane of the plate. Small displacements of this axis from the middle plane of the plate are then considered to be of a secondary effect and are usually ignored.

The effective plate area to form the cross section of an idealized column is considered to be the same as the effective plate area for symmetrical stiffeners discussed in the previous section. Before plate buckling, the effective area is equal to the actual plate area. After the plate buckles, the membrane stress is redistributed and the effective area is reduced.

When the stiffeners deform in the symmetrical mode, the determination of flexural buckling stress $F_E$ is straightforward and involves only the cross-sectional properties. But the stress for torsional buckling $F_{TA}$ can only be estimated reliably if the stiffness of the restraining spring is known. This stiffness is estimated by considering the plate as a series of transverse beams under bending and hence depends on the end load and the wave length. Formulations of this type have been obtained, for example, by Bijlaard and General Dynamics.[8, 5]

## 3.3 Torsional Buckling Under Bending

Torsional buckling of asymmetrical stiffeners due to end moments is analogous to the behavior of an eccentrically loaded column. When the end moments are applied, the stiffener immediately begins to twist (trip) due to the eccentricity of the shear center with respect to the stiffener web. At first, the twist increases gradually; when the bending stress $F_B$ approaches the lateral-torsional tripping stress $F_{CB}$, the twisting increases at a faster rate until at $F_B = F_{CB}$ the twisting increases without limit. The larger the eccentricity of the shear center to the centroid of the cross section, the sooner the infinite twisting is approached. This description is valid for elastic conditions.

An analytical method has not yet been developed for calculating the coupled tripping stress $F_{CB}$ for an asymmetrical stiffener under end moments. But one can use the beam-column approach to approximate this critical stress. The differential equations are derived in Appendix I. The critical stress can be obtained by solving the two homogeneous equations, Eqs. (A.31) and (A.32), whose coefficient determinant gives an expression for the solution of the critical moment.(Eqs. (A.34) and (A.35))

Since there are no test data to verify the accuracy of this solution and there is no simplified solution available for this load case, we must wait until more sophisticated formulations are derived or until more tests for this loading case are conducted before accepting the above solution as a valid one.

## 3.4 Torsional Buckling Considering Distortion of Cross Section

Inclination of the principal axes of an asymmetrical stiffener, such as an angle or zee, with respect to the plate surface leads to an immediate sideways bending of the stiffener flange as soon as the stiffener-plate column starts deflecting under an axial load. The lateral forces developing in the flange tend to bend the web and, by distorting the cross section, reduce the capacity of the stiffener. The immediate consequence of this effect is that for many cases, the angle stiffeners become weaker than the tee stiffeners, and this is contrary to the conclusions reached from the classical buckling solutions described above.

A method for considering these second-order deformations of the cross section of asymmetrical stiffeners in the elastic range was provided by van der Neut[10].

The stiffener web is treated as a plate subjected to longitudinal compression and undergoing large deflections. A simplification was made of the differential equation of the web plate, however, without affecting the accuracy. The top edge is attached to the flange which provides rotational and flexible lateral support. The bottom edge is rotationally restrained by the plate.

Presented next is a streamlined procedure of the method proposed by van der Neut.[10] This procedure is suitable for calculator or microcomputer operations.

First of all, some definitions:

$$p = \frac{F_{CA}}{F_E}, \text{ the ratio of the tripping to the column buckling stress} \quad (3.3)$$

$$m = a/\ (\text{Length of half-wave}) \tag{3.4}$$

(Here, m may have a value which is non-integer and different from 1.0.)

and parameters f, A and Q;

$$f = 1 - pm^2 \tag{3.5}$$

$$A = [12(1-\nu^2)p]^{0.25}\,(\frac{\pi d}{a})\sqrt{r/mt_w} \tag{3.6}$$

$$Q = (bt_p + A_w/2)/A_{ps} \tag{3.7}$$

With p and m set to some specific values, the following two simultaneous equations are to be solved for constants $C_1$ and $C_2$.

$$\begin{aligned} &C_1[X\cos A - (1-Y)\sin A] + C_2[X\cosh A + (1-Y)\sinh A] \\ &\quad = \cos A + \cosh A + X(\sin A + \sinh A) + Y(\cos A - \cosh A) \end{aligned} \tag{3.8}$$

$$\begin{aligned} &[(Y-1)\cos A + Z\sin A]C_1 + [(Y+1)\cosh A + \sinh A]C_2 \\ &\quad = (Y+1)\sinh A + (Y-1)\sin A + Z(\cosh A - \cos A) \end{aligned} \tag{3.9}$$

where

$$X = \frac{A^3}{A_w d^2}\{\frac{J_f G}{pF_E} - I_y - (1/f-1)\frac{Q_f^2}{A_{ps}}\}$$

$$Y = \frac{A^2 Q_f^2 Q}{f I_{ps} A_w}$$

$$Z = \frac{A}{A_w}\{\frac{1}{r^2(1-f)}[\frac{fr^2 + (Qd)^2}{fI_{ps}}Q_f^2 - I_y] + A_f\}$$

Then, the rotational restraint by the plate, C, needed to maintain equilibrium, is computed from Eq. (3.10).

$$C = -\frac{2A}{C_1 + C_2} \tag{3.10}$$

To find a solution for a specific value of restraint by plate, C, several sets of p and m are tried out and the desired value of p is interpolated.

Application of the method requires an iterative procedure, and a computer program (in BASIC) is given in Appendix B. The program extends the van der Neut method outlined above to incorporate consideration of the effective width of the plate and of a parabolic transition for the inelastic range.

# 4. COMPARISON WITH TEST RESULTS

Apparently because the tripping of stiffeners is often considered to be a side check in structural design, the experimental data on tripping failure are very limited. The only test that could be found for the tee stiffener failure is the grillage test 1A reported by Smith. The ultimate failure of this specimen may have been caused primarily by the stiffener tripping.[6]

Horne conducted several tests on angle stiffeners in which he mainly emphasized the effect of imperfections and residual stresses on the strength of the stiffened plates. The only two tests known to have failed by tripping of angle stiffeners are specimens AS2 and AF2.[24].* The width of the flange of specimen AF2 was double that of specimen AS2; otherwise both specimens had the same dimensions. From the test results, it appears that AF2 specimen having stiffeners with wider flanges exhibited only marginally higher strength.

Table 1 shows the pertinent scantlings (nominal values) of these three test specimens. The tripping stress for these specimens was computed by using the methods described in this report. The results are listed in Table 2.

For specimen 1A, all three methods give values which are quite close to the test load. The Adamchak method gives a better estimate of the tripping stress because it considers the effect of the plate restraint varying along the member due to the axial load and due to the effect of web deformations as well as due to the effective width of the attached plate.[4]

The General Dynamics Design Guidelines also give realistic values, and it is suggested that this method be used for design purposes to get a fast and simple prediction on the tripping stress.[5]

The Faulkner interim solution using the lower and upper bounds in predicting the tripping stress also contains the effect of plate restraint and gives a fairly good prediction, but the arbitrarily assigned upper limit for the torsional restraint has been demonstrated by Adamchak to be unconservative.[1, 4]

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*The tripping type failure was also observed in the tests described by Scheer, but these dealt only with predeformed bulb-flat stiffeners and, thus, are not included in the current comparison.[25]

For angle stiffener tests, the Guidelines suggest that when $F_E > F_{TA}$, i.e., Euler buckling stress $F_E$ does not govern, and one may conservatively take 75 percent of $F_{TA}$ as the coupled tripping stress $F_{CA}$. This approach does not consider the length of the stiffener a and is valid only when the slenderness ratio of the column is low because, when the slenderness ratio is high, $F_E$ will become the controlling stress.

The Argyris method provides quite accurate predictions for specimens AF2 and AS2, but this method requires the use of a computer for iterative solution of the minimum critical load. Therefore, it is not really a suitable method for design purposes.

In order to find the difference between the strengths of the tee and angle stiffeners when they have flanges of the same size, a trial angle stiffener section with the same dimensions as specimen 1A was analyzed by the Argyris method to find the coupled tripping stress $F_{CA}$. As shown in Table 3, the tripping stress $F_{CA}$ of the angle stiffener comes out to be greater than the tripping stress of the tee stiffener. A similar analysis was conducted on specimens AS2 and AF2. Two trial tee stiffener sections with the same dimensions as specimens AS2 and AF2 were analyzed by the Adamchak method to evaluate the tripping stress $F_{TA}$. The results are also shown in Table 3.

As can be seen in Table 3, specimen 1A becomes stronger when it is treated as if it had an angle stiffener rather than a tee stiffener with the same-sized flange. But for test specimens AF2 and AS2, the tee stiffener gives a greater strength than the angle stiffener. In order to get a better understanding of this apparent inconsistency, two other specimens, AS1 and AF1, were analyzed. These two specimens were of the same cross-sectional dimensions as AS2 and AF2, but the span was only one third. Again, the results show that the angle stiffener is stronger than the tee stiffener by up to 10 percent.

Figure 15 is plotted to show the relationship between $F_{CA}$ and $F_{TA}$ for angle and tee stiffeners with respect to the half-wave length. The tee stiffener is stronger than the angle stiffener for larger slenderness ratio values $L/r_o$, but weaker for smaller $L/r_o$. The critical value of $L/r_o$ is approximately 20. This transition can be explained as follows. The warping rigidity of the angle is significantly larger than of the tee. Also, for shorter stiffeners (low slenderness), the coupling effect for the eccentricity of the shear center of the angle stiffener does not significantly reduce the column strength with respect to the effect of warping rigidity. When the slenderness ratio increases, the effect of coupling becomes more significant, and this reduces the tripping strength of the angle stiffeners with respect to the tee stiffeners.

# 5. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## 5.1 Summary

The principal methods proposed in literature for predicting the tripping instability of longitudinal stiffeners in marine structures were reviewed, and they were shown to be sufficiently accurate for design purposes. The solutions were compared with the few available test results and were found to be in relatively good agreement.

Comparative solutions for angle and tee stiffeners illustrated the difference in the tripping strength for these sections.

The effect of including web deformations in the evaluation of the tripping strength of symmetrical stiffeners (tees) was shown to be important unless the rotational restraint along the stiffener line of attachment to the plate was very small. For a larger degree of rotational restraint, the corresponding error was appreciably higher.

The major handicap encountered in this study was the lack of experimental data to use for validating the analytical methods.

## 5.2 Conclusions

The following conclusions can be drawn from the study in this report:

1. Torsional strength (tripping) of stiffeners should be an important consideration in the design of marine structures.
2. The Design Guidelines prepared by General Dynamics[5] for evaluating the torsional strength of stiffeners may be used as a simple and realistic check. However, this method becomes less reliable for angle stiffeners with higher slenderness ratio values, $L/r_o$.
3. A comparison according to the classical buckling theory of the tee and angle stiffeners with flanges of the same size showed that an angle stiffener resists tripping due to axial compression better than a tee when the slenderness ratio $L/r_o$ is in a lower range. The situation reverses when the slenderness ratio becomes higher. However, when web deformations are included in the analysis, the overall axial strength of angle stiffeners tends to fall below that of the tee stiffeners.

4. Although lateral loading is unlikely to cause significant effect on the tripping strength for symmetrical (tee) stiffeners, it has a serious reducing influence for the angle stiffeners since it introduces bi-axial bending in them.

## 5.3 Recommendations

Until more satisfactory general solutions are developed, semi-empirical methods based on classical theories or numerical techniques will have to be used in order to obtain more accurate design rules.

The areas for which further study is required are considered to be the following:

1. Tests on symmetrical and asymmetrical stiffeners, especially under concurrent action of axial and lateral loading.
2. An improved definition of the influence of the plate behavior in the buckling and post-buckling ranges on the rotational restraint by the plate.
3. Consideration of residual stresses and large deformations. It appears, this would be realistically possible only in numerical solutions (Finite Element, etc.).

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# Appendix A. DERIVATION OF DIFFERENTIAL EQUATIONS

## A.1 General Differential Equations

The classical differential equations of torsional buckling (tripping) of longitudinals are derived here for use in the main body of the report. The principal assumptions are that the cross section does not deform, that there are no initial imperfections, and that the elastic range is not exceeded. The consequences of not satisfying these assumptions are discussed in the report where appropriate.

Under the assumptions stated above, the stiffener, being attached to the plate, has the following modes of buckling deformation: overall lateral buckling perpendicularly to the plate surface and without twisting, torsional buckling without lateral motion, and a combined mode of lateral and torsional motions. Torsional motion of the stiffener is restrained by the plate, but this effect is often neglected or taken into account in a simplified manner. Typically, a stiffener is taken with its tributary width of the plate and treated as a separate column without interaction with other stiffeners.

As shown in Figs. 16, 17, 18, the centroidal axes x and y of a stiffener are taken parallel and perpendicular to the plate, and they are not necessarily the principal axes when the cross section is asymmetrical. Bending moments of the stiffener column about these axes are given by

$$\begin{aligned} M_x &= EI_x v'' + EI_{xy} u'' \\ M_y &= EI_y u'' + EI_{xy} v'' \end{aligned} \qquad (A.1)$$

In these expressions, $I_{xy}$ is the product of inertia of the stiffener cross section. Furthermore,

$$\begin{aligned} q_x &= d^2 M_y / dz^2 \\ q_y &= d^2 M_x / dz^2 \end{aligned} \qquad (A.2)$$

where $q_x$ and $q_y$ are positive in the positive direction of the x and y axes. Substitution of Eqs. (A.1) into Eq. (A.2) results in the following differential equations:

$$q_x = EI_y u^{iv} + EI_{xy} v^{iv} \qquad (A.3)$$

$$q_y = EI_x v^{iv} + EI_{xy} u^{iv} \tag{A.4}$$

Consider a column with an arbitrary open cross section as shown in Fig. 16, and assume that a longitudinal fiber N with coordinates $(h_x, h_y)$ is prevented from deflecting in the x direction. Denoting the deflections of the shear center O $(x_o, y_o)$ by u and v, we find the deflections of N to be

$$u_n = u + (y_o - h_y) = 0 \tag{A.5}$$

$$v_n = v - (x_o - h_x) \tag{A.6}$$

Assume that the column is subjected to the equal end moments $M_x$ and $M_y$ and the axial thrust P as shown in Fig. 17. Then, the normal stress $F_z$ at any point in the column is independent of z and is given by the following equation if x and y axes are the principal axes:

$$F_z = P/A + M_x y/I_x + M_y x/I_y \tag{A.7}$$

or by

$$\begin{aligned} F_z = P/A &+ (M_y I_x - M_x I_{xy}) x/(I_x I_y - I_{xy}{}^2) \\ &+ (M_x I_y - M_y I_{xy}) y/(I_x I_y - I_{xy}{}^2) \end{aligned} \tag{A.8}$$

when x and y are not the principal axes.

The intensities of the lateral forces and distributed torque induced by the axial compressive force in the longitudinal fibers are:

$$\begin{aligned} q_x &= - {}_A(F_z tds) d^2/dz^2[u + (y_o - y)R] \\ q_y &= - {}_A(F_z tds) d^2/dz^2[v - (x_o - x)R] \\ m_z &= - {}_A(F_z tds)(y_o - y) d^2/dz^2[u + (y_o - y)R] \\ &\qquad + {}_A(F_z tds)(x_o - x) d^2/dz^2[v - (x_o - x)R] \end{aligned} \tag{A.9}$$

Because of the restraint at axis N, there will be reactions of intensity $q_o$ distributed continuously along this axis and acting in the direction parallel to the x axis. Assuming x and y axes to be the principal axes of the plate-stiffener and substituting expression (A.7) for $F_z$ in Eqs. (A.9) and then integrating, we obtain the following differential equations which include the effect of $q_o$:

$$q_x = -Pu'' - (Py_o - M_x)R'' + q_o \quad (A.10)$$

$$q_y = -Pv'' + (Px_o - M_y)R'' \quad (A.11)$$

$$m_z = -(Py_o - M_x)u'' + (Px_o - M_y)v'' + q_o(y_o - h_y) - (M_x B_x + M_y B_y + I_o P/A)R'' - CR \quad (A.12)$$

where $I_o$ is the polar moment of inertia with respect to the shear center O, and $B_x$ and $B_y$ are the bi-moments defined by

$$B_x = \left(\int_A y(x^2 + y^2)dA\right)/I_x - 2y_o \quad (A.13)$$

$$B_y = \left(\int_A x(x^2 + y^2)dA\right)/I_y - 2x_o$$

Combination of Eqs. (A.11) and (A.4) and the elimination of u by solving for u from Eq. (A.5) gives the first differential equation for the independent variables v and R:

$$EI_x v^{iv} - EI_{xy}(y_o - h_y)R^{iv} - (Px_o - M_y)R'' + Pv'' = 0 \quad (A.14)$$

Another differential equation for v and R is obtained by considering torsion of the column

$$EI_w R^{iv} - GJR'' = m_z \quad (A.15)$$

Substitution of Eq. (A.12) into Eq. (A.15) results in

$$EI_w R^{iv} - GJR'' + (Py_o - M_x)u'' - (Px_o - M_y)v'' + (M_x B_x + M_y B_y + I_o P/A)R'' + q_o(y_o - h_y) - CR = 0 \quad (A.16)$$

Again, using u from Eq. (A.5) and substituting it into Eq. (A.16), we obtain the second differential equation for the problem at hand:

$$E(I_w + I_y(y_o - h_y)^2]R^{iv} - [GJ - 2M_x(y_o - h_y) - M_x B_x - M_y B_y - I_o P/A + Py_o^2 - Ph_y^2]R'' + CR - (y_o - h_y)EI_{xy}v^{iv} - (Px_o - M_y) = 0 \quad (A.17)$$

Equations (A.14) and (A.17) can be used to find the lateral torsional buckling load for any member restrained to move in a prescribed plane and rotate about a line and subjected to an axial load and equal end moments.

## A.2 Symmetrical Stiffeners

For a tee stiffener, x and y axes are the principal axes because the stiffener has an axis of symmetry which is perpendicular to the attached plate as shown in Fig. 18. We know that for such a stiffener

$$I_{xy} = x_o = h_x = 0$$

$$h_y - y_o = \dot{s}$$

where $\dot{s}$ is the distance from the shear center of the stiffener to the toe of the stiffener (the axis of rotation). When the stiffener is subjected to axial load P only, i.e., $M_x = M_y = 0$, Eqs. (A.14) and (A.17) become:

$$EI_x v^{iv} + Pv'' = 0$$

$$EI_{wn} R^{iv} - (GJ - I_p P/A) R'' + CR = 0 \qquad \text{(A.18)}$$

where $I_p$ is the polar moment of inertia of the stiffener about the toe and $I_{wn}$ is the warping constant with respect to the axis of rotation N; and

$$I_p = I_o - Ay_o^2 + Ah_y^2 \qquad \text{(A.19)}$$

$$I_{wn} = I_w + I_y \dot{s}^2$$

These two differential equations are independent of each other. Assuming the stiffener ends to be simply supported and prevented from twisting but free to warp, one can assume the flexural and torsional displacement shapes to be sinusoidal:

$$v = A_2 \mathrm{Sin}(m\pi z/a)$$

$$R = A_3 \mathrm{Sin}(m\pi z/a)$$

Substituting them into Eq. (A.18) and solving, one obtains from the second equation of Eqs. (A.18) the tripping stress under uniaxial compression

$$F_{TA} = [EI_{wn}(m\pi/a)^2 + GJ + C(a/m\pi)^2]/I_p \qquad \text{(A.20)}$$

When the column is subjected to bending moment $M_x$ and no axial load, $M_y = P = 0$, the differential equations become:

$$EI_x v^{iv} + Pv'' = 0$$

$$EI_{wn} R^{iv} - (GJ - 2M_x \dot{s} - M_x B_x) R'' + CR = 0$$

where (A.21)

$$B_x = (\int_A y(x^2+y^2)dA)/I_p - 2y_o \quad (A.22)$$

$$= \{[h_y{}^4 - (y_o + t_f/2)^4]t_w/4 + A_f y_o(y_o{}^2 + b_f{}^2/12)\} - 2y_o$$

Again, assuming $R = A_3 \mathrm{Sin}(m\pi z/a)$ and substituting it into Eq. (A.21), we obtain the critical moment for the stiffener alone:

$$M_{cr} = [EI_{wn}(m\pi/a)^2 + GJ + C(a/m\pi)^2]/(2\bar{s}+B_x) \quad (A.23)$$

To modify this critical moment to include the effect of the plate, one can introduce an axial load P at the centroid of the stiffener section so that the stress distribution would be similar to the stress distribution of the plate-stiffener combination. Following the procedure described in Section 2.4, the critical moment M for the tee stiffener with stress at the toe equal to zero is obtained

$$M_{cr} = [EI_{wn}(m\pi/a)^2 + GJ + C(a/m\pi)^2]/(2\bar{s}+B_x+c_1 I_p/I_s) \quad (A.24)$$

Thus, the tripping stress in the flange under constant moment can be obtained by applying Eq. (2.28):

$$F_{TB} = d[EI_{wn}(m\pi/a)^2 + GJ + C(a/m\pi)^2]/(2\bar{s}I_s + B_x I_s + c_1 I_p) \quad (A.25)$$

Then, the total moment for the plate-stiffener combination can be obtained by applying Eq. (2.32) with $I_{ps}$ being the moment of inertia of the gross section and $I_s$ being the moment of inertia of the stiffener alone.

## A.3 Asymmetrical Stiffeners

In asymmetrical cross sections, the x and y axes are not the principal axes, and therefore the effect of biaxial bending must be taken into account. Substitution of Eq. (A.8) into Eq. (A.9) gives:

$$q_x = -Pu'' - [Py_o - M_1 I_x - M_2 I_{xy}] + q_o$$

$$q_y = -Pv'' + [Px_o - M_1 I_{xy} - M_2 I_y] \quad (A.26)$$

$$m_z = -(Py_o - M_1 I_x - M_2 I_{xy})u'' + (Px_o - M_1 I_{xy} - M_2 I_y)v''$$

$$-(M_1 B_1 + M_2 B_2 + I_o P/A)R''$$

where

$$M_1=(M_xI_y-M_yI_{xy})/(I_xI_y-I_{xy}^2) \quad (A.27)$$
$$M_2=(M_yI_x-M_xI_{xy})/(I_xI_y-I_{xy}^2)$$

and

$$B_1=\int\int y(x^2+y^2)dA-2(y_oI_x+x_oI_{xy}) \quad (A.28)$$

$$B_2=\int\int x(x^2+y^2)dA-2(x_oI_y+y_oI_{xy})$$

Following the same procedure of derivation as before, the differential equations for asymmetrical cross sections are obtained.

$$EI_xv^{iv}-Pv^{iv}-EI_{xy}(y_o-h_y)R^{iv}-(Px_o-M_1I_{xy}-M_2I_y)R''=0 \quad (A.29)$$

$$E[I_w+(y_o-h_y)^2I_y]R^{iv}-[GJ-2(y_o-h_y)(M_1I_x+M_2I_{xy})$$
$$-(M_1B_1+M_2B_2+I_oP/A-Py_o^2+Ph_y^2]R''+CR \quad (A.30)$$
$$-(y_o-h_y)EI_{xy}v^{iv}-(Px_o-M_1I_{xy}-M_2I_y)v''=0$$

For the special case of angle stiffeners, (see Fig. 19), we can conservatively assume the column to be simply supported at both ends. When there is only an axial load applied on the stiffener, $M_1=M_2=0$, and the differential equations become:

$$EI_xv^{iv}-Pv''-EI_{xy}(y_o-h_y)R^{iv}-Px_oR''=0 \quad (A.31)$$
$$EI_{wn}R^{iv}-[GJ+I_pP/A]R''+CR-(y_o-h_y)EI_{xy}v^{iv}-Px_ov''=0 \quad (A.32)$$

where $I_p$ is the polar moment of inertia about the stiffener toe and $I_{wn}=I_w+I_y(y_o-h_y)^2$ is the warping constant about the toe (axis of rotation).

Assuming $v = A_2\text{Sin}(m\pi z/a)$ and $R = A_3\text{Sin}(m\pi z/a)$ and then substituting them into the differential equations (Eqs. (A.31) and (A.32)), we obtain the following two equations:

$$[EI_x(m\pi/a)^2-P]A_2+[-EI_{xy}(y_o-h_y)(m\pi/a)^2+Px_o]A_3=0$$
$$[-EI_{xy}(y_o-h_y)(m\pi/a)^2+Px_o]A_2+[EI_{wn}(m\pi/a)^2 \quad (A.33)$$
$$+GJ-I_pP/A+C(a/m\pi)^2]A_3=0$$

The vanishing of the determinant of the coefficients of these equations gives an expression for computing the critical stress for lateral-torsional buckling of angle stiffeners. Because the analytical solution for the minimum value of the critical stress is quite difficult, a computer program was used to solve for the critical stress.

When the angle stiffener is subjected only to equal end moments, then $P = M_y = 0$ in the differential equations. Again, using the same procedure, we come to the following two homogeneous equations. By setting the coefficient determinant equal to zero, an expression for solving for the critical moment is derived.

$$[EI_x(m\pi/a)^2]A_2+[EI_{xy}(h_y-y_o)(m\pi/a)^2]A_3 = 0 \tag{A.34}$$

$$[EI_{xy}(h_y-y_o)(m\pi/a)^2]A_2+[EI_{wn}(m\pi/a)^2+GJ-2(y_o-h_y)M_x$$
$$-(M_xI_yB_1+M_xI_{xy}B_2)/(I_xI_y-I_{xy}^2)+I_pP/A)+C(a/m\pi)^2]A_3 = 0$$

where

$$B_1 = \int y(x^2+y^2)dA-2(y_oI_x+x_oI_{xy}) \tag{A.35}$$

$$B_2 = \int x(x^2+y^2)dA-2(x_oI_y+y_oI_{xy})$$

In the above derivation, the cross section is assumed not to deform. A method for considering deformation of the cross section is described in Section 3.4.[10]

# Appendix B. COMPUTER PROGRAM FOR ANGLE STIFFENERS

The computer program is based on the method proposed by van der Neut for determining the elastic buckling load of asymmetrical stiffeners.[10] The principal extensions beyond the method are the considration of the effective width of plate (according to Eq. (2.19)), parabolic transition for the inelastic range and an automation of the iterative process by introducing an exponential extrapolation. The program is interactive and allows a number of options through prompting. The stiffener data is read from a separate file ANGDAT.DO a sample of which is listed after the program.

```
Listing of **    vdnta12.bas    **      , 03-31-1986,  15:11:18    Page 1
***************************************************************

10 'VDNTA12 Direct Comp or Automated convergence,
12 'by exponent. interpolat; y=A+B(1-exp(-x)),printing,
14 ' A. Ostapenko,9/8/85,1/15/86
16  PI=3.1415926536899
18 INPUT"Results to store for printg<Y/N>";Q1$
     : IF Q1$="Y" OR Q1$="y" THEN INPUT"File to store results in";F$
     : OPEN F$ FOR APPEND AS 2
     : Q1=1
     : ELSE Q1=0
20 OPEN"angdat.do" FOR INPUT AS 1
30 INPUT#1,S$,D$,N,E,FY,L,B,TP,D,TW, BF,TF
     : IF S$="'" THEN 30 ' To bypass data not to be used.
45 PRINT S$;D$;N;E;FY;L;B;TP;D;TW; BF;TF
48 IF Q1=1 THEN PRINT#2,S$;D$;N;E;FY;L;B;TP;D;TW; BF;TF
     : PRINT#2,
50    ' Compute sect properties
60 BE=TP*SQR(E/FY)*(2-TP/B*SQR(E/FY)) 'Modif for B-effective acc to AO
70 AF=BF*TF
     : AW=D*TW
     : AS=AF+AW
     : AT=AS+B*TP
     : AE=AS+BE*TP
     :  Y=(AF*D+AW*D/2)/AE
80 I=(AW/3+AF)*D*D-AE*Y*Y  ' I total effective
90 R=SQR(I/AE)             ' r (rad of gyr)
100 Q2=(AF*BF/2)^2         ' Qf^2
110 JF=AF*BF*BF/3          ' If abt web
120 Q=(BE*TP+AW/2)/AE       ' Q-parameter
130 FE=PI*PI*E/((L/R)^2)   ' Column Euler stress
140 T=E*AF*TF*TF/7.8       ' T=GJ of flge
150 INPUT"Direct comput or Extrapolat <D/E>";QQ$
     :  IF QQ$="D" OR QQ$="d" THEN 190
160 INPUT"To use default p (.05,.15)<Y/N>";Q$
170 IF Q$="Y" OR Q$="y" THEN P1=.05
     : P2=.15
     : GOTO 220
180 INPUT"New pe1,pe2";P1,P2
     : GOTO 220
185 ' Direct computation of C for p,lines 190-200
190 INPUT"p value (For extrapol, type E)";P
     :  ON ERROR GOTO 210
192 GOSUB 400
```

```
    : FA=P*FE
    : IF FA>FY/2 THEN PY=1-FY/4/FA ELSE PY=FA/FY
195 IF Q1=1 THEN PRINT#2,"Soln:"D$", 1/m=n="N", p="P",C=CE="CE",FA="FA",FS
   ="FA/FY",Py="PY",FC="PY*FY;",Feff="PY*FY*AE/AT
    : PRINT#2,
200 PRINT"Soln:"D$", 1/m=n="N", p="P",C=CE="CE",FA="FA",FS="FA/FY",Py="PY"
   ,FC="PY*FY;",Feff="PY*FY*AE/AT
    : PRINT"Next ";
    : GOTO 190
210 RESUME 150
220 INPUT"Reqd plate restraint CR";XR
    : TL=.001
    : P=P1
    : GOSUB 400
    : X1=CE
    : Y1=P1
    : P=P2
    : GOSUB 400
    : X2=CE
    : Y2=P2
260  ' Exponential Approx; Y=A+Bexp(-X)
270 N1=1-EXP(-X1)
    : N2=1-EXP(-X2)
    : DT=N2-N1
    : AA=(Y1*N2-Y2*N1)/DT
    : BB=(Y2-Y1)/DT
    : Y4=AA+BB*(1-EXP(-XR))
    : P=ABS(Y4)
    : GOSUB 400
    :  X4=CE
    : PRINT"X1,X2,X4;Y1,Y2,Y4,p=/Y4/="X1;X2;X4;Y1;Y2;Y4;P
340 ' Check tolerance
350 IF ABS(X4-XR)<=TL THEN FA=Y4*FE
    : ELSE GOSUB 850
    : GOTO 260
360 IF FA>FY/2 THEN PY=1-FY/4/FA ELSE PY=FA/FY   ' compute Py due to yldg
370 IF Q1=1 THEN PRINT#2,"Meth:VDNTA8- Spec:"D$,"1/m=n=";N;",p=";Y4
   : PRINT#2,"(Req CE,C)=",XR;X4;",FA="FA",FS="FA/FY",Py="PY",FC="PY*FY
   ;",Feff="PY*FY*AE/AT
    : PRINT#2,
380 PRINT"Soln: "D$,"1/m=n=";N;",p=";Y4F
   :PRINT"(Req CE,C)=",XR;X4;",FA="FA",FS="FA/FY",Py="PY",FC="PY*FY;",F
   eff="PY*FY*AE/AT
385 INPUT"DONE. Another C <Y/N>";Q$
390 IF Q$="Y" OR Q$="y" THEN 150 ELSE INPUT"Another Specim <Y/N>";Q1$
    : IF Q1$="Y"OR Q1$="y" THEN 30
395 CLOSE
    : SYSTEM
400 ' Constants=f(n,p)
410 F=1-P/N/N
    : A=(10.92*P)^.25*SQR(N*R/TW)*PI*D/L
    : X=A*A*A/(AW*D*D)*(T/P/FE-JF-Q2*(1/F-1)/AT)
    : Y=A*A*Q2*Q/(F*I*AW)
    : Z=A/AW*(1/(R*R*(1-F))*((Q*Q*D*D+F*R*R)*Q2/(F*I)-JF)+AF)
460  ' Coeffs for simult eqs
470 CS=COS(A)
    : CH=(EXP(A)+EXP(-A))/2
    : SN=SIN(A)
    : SH=(EXP(A)-EXP(-A))/2
480 A1=X*CS-(1+Y)*SN                'a11
490 A2=X*CH+(1-Y)*SH                'a12
500 A3=CS+CH+X*(SN+SH)+Y*(CS-CH)  'c=a13
520 A4=(Y-1)*CS+Z*SN                'a21
```

```
530 A5=(Y+1)*CH+Z*SH                      'a22
540 A6=(Y+1)*SH+(Y-1)*SN+Z*(CH-CS)  'a23
550 ' Solve eqs for C1,C2
560 DT=A1*A5-A2*A4
      : C1=(A3*A5-A6*A2)/DT
      : C2=(A1*A6-A4*A3)/DT
590 ' Solve for RE=rotat restr
600 CE=-2*A/(C1+C2)
610 RETURN
850 ' Select closest 2 out of 3;X1,X2,X4
860 D1=ABS(X1-XR)
      : D2=ABS(X2-XR)
      : D4=ABS(X4-XR)
      : IF D1>D2 THEN IF D1>D4 THEN X1=X4
      : Y1=Y4
      : GOTO 890
880 IF D2>D4 THEN X2=X4
      : Y2=Y4
      : GOTO 890
885 P=ABS(Y2+Y4)/2
      : GOSUB 400
      : X2=CE
      : Y2=P
890  RETURN
```

## ANGDAT.DO, input data file for the BASIC Program VDNTA12.BAS

```
The lines starting with an apostrophy (') are by-passed.
To use a data line for input, some character is to be inserted
in front of the apostrophy ("@" is used in the sample file below).

',"   DATA needed: S,Problem label" ,m,E,SY,L,B,TP,D,TW,BF,TF
@',"AS2(Angl)",1,29500,59.42,118.11,7.874,.407,5.512,.26,1.516,.472
',"AS2(Angl)", .8,29500,59.42,118.11,7.874,.407,5.512,.26,1.516,.472
',"AS2(Angl)", .6,29500,59.42,118.11,7.874,.407,5.512,.26,1.516,.472
@',"AF2(Angl)",1,29500,59.50,118.11,7.874,.406,5.598,.256,2.992,.386
',"AF2(Angl)", .9,29500,59.50,118.11,7.874,.406,5.598,.256,2.992,.386
',"AF2(Angl)", .7,29500,59.50,118.11,7.874,.406,5.598,.256,2.992,.386
@',"AS1(Angl)", 1,29500,60.21,47.244,7.874,.398,5.526,.26,1.506,.495
',"AS1(Angl)", .9,29500,60.21,47.244,7.874,.398,5.526,.26,1.506,.495
@',"AF1(Angl)",1,29500,60.21,47.244,7.874,.398,5.587,.26,2.992,.398
@',"AF1(Angl)",.8,29500,60.21,47.244,7.874,.398,5.587,.26,2.992,.398

',"Examples from other publications",N,E,SY,L,B,TP,D,TW,BF,TF

',"VDN-83Exm,thkn mdf",1,70000,250,540,70,3.14,38.08,1.92,18.08,2.667
',"GenDyn-81,Ex.2, Angle", 1,29500,36,48,24,.3,5.87,.28,3,.56
',"GenDyn-81,Ex.2,Angl w/ Tw=0.18",1,29500,36,48,24,.3,5.87,.18,3,.56
',"GenDyn-81,Ex.2,Angl, w/ L=1.5L",1,29500,36,72,24,.3,5.87,.28,3,.56
',"GenDyn-81 Ex.2,Angl w/Lngth=2L",1,29500,36,96,24,.3,5.87,.28,3,.56
',"GnDyn-81 Ex.3 mdfd to Angl",1,29500,36,48,40,.25,11.94,.25,3,.375
',"GnDyn-81, Ex.3,Angle,L=2L", 1,29500,36,96,40,.25,11.94,.25,3,.375
```

# NOTATION

| | |
|---|---|
| $A$ | A parameter |
| $a$ | Length of stiffener between transverse supports |
| $A_f$ | Area of stiffener flange |
| $A_{ps}$ | Area of plate-stiffener section |
| $A_s$ | Area of stiffener |
| $A_w$ | Area of stiffener web |
| $B$ | Plate slenderness ratio |
| $b$ | Spacing between stiffeners |
| $b_e$ | Effective width of the plate between stiffeners |
| $b_f$ | Flange width |
| $C$ | Rotational spring constant provided by supporting plate |
| $C_o$ | Rotational spring constant of unloaded supporting plate |
| $c_1$ | Distance between stiffener toe and its centroid |
| $c_2$ | Distance between stiffener centroid and the outer fiber of the flange |
| $c_3$ | Distance between the plate-stiffener centroid and the outer fiber of the flange |
| $d$ | Stiffener height |
| $d_c$ | Depth of stiffener from plate to mid-thickness of flange |
| $d_w$ | Depth of stiffener web |
| $D$ | Flexual rigidity of plate |

| | |
|---|---|
| $E$ | Young's modulus of material |
| $E_t$ | Tangent modulus of material |
| $f$ | A parameter |
| $F_{CA}$ | Coupled tripping stress of asymmetrical stiffener |
| $F_E$ | Euler plate-stiffener buckling stress |
| $F'_E$ | Buckling stress in inelastic range |
| $F_e$ | Edge stress of plate-stiffener |
| $F_n$ | Axial stress at stiffener toe |
| $F_{PE}$ | Buckling stress of elastic plate |
| $F_{TA}$ | Axial tripping stress of symmetrical stiffener |
| $F_{TB}$ | Lateral bending tripping stress |
| $F_p$ | Modified proportional limit $= F'_p - F_R$ |
| $F'_p$ | Material proportional limit |
| $F_R$ | Residual stress due to fabrication and welding |
| $F_y$ | Yield stress of material |
| $G$ | Shear modulus of material |
| $I_{ps}$ | Effective vertical moment of inertia of plate-stiffener section |
| $I_o$ | Polar moment of inertia about the shear center of the stiffener |
| $I_p$ | Polar moment of inertia about the toe of the stiffener |
| $I_s$ | Moment of inertia of a stiffener about its centroidal axis |
| $I_w$ | Warping constant of stiffener |

$I_{wn}$ Warping constant of stiffener about axis of rotation (toe)

$I_y$ Moment of inertia of stiffener about web plane

J St. Venant torsional constant of stiffener

$J_f$ St. Venant torsional constant of stiffener flange

M Vertical bending moment

$M_{cr}$ Elastic buckling moment

$M_T$ Total moment of the plate-stiffener section at tripping

m Number of half waves in panel length

P Axial end load

p $= F_{CA}/F_E$

Q A parameter

$Q_f$ First area moment of flange about web

q Uniform lateral loading

$q_{cr}$ Uniform lateral loading at elastic tripping

R Rotation of stiffener shear center

r Radius of gyration $= \sqrt{I_{ps}/A_{ps}}$

$r_o$ Polar radius of gyration of plate-stiffener with respect to stiffener toe

S Minimum plate-stiffener section modulus

$\bar{s}$ Distance from shear center of stiffener to toe

$t_f$ Flange thickness

$t_p$ Plate thickness

| | |
|---|---|
| $t_w$ | Stiffener web thickness |
| $U^*$ | Total potential energy of structure |
| U | Total strain energy of structure |
| u | Horizontal displacement of point (x,y) on stiffener cross section |
| v | Vertical displacement of point (x,y) on stiffener cross section |
| X,Y,Z | Parameters |
| W | Total external potential of applied forces |
| $y_e$ | Vertical coordinate from plate-stiffener centroid to axis of rotation (stiffener toe) |
| $\delta(x,y)$ | Axial shortening of longitudinal fiber at location (x,y) of a stiffener |
| $\nu$ | Poisson's ratio |

# TABLES

**Table 1:** Properties of Stiffener Sections in Test Specimens

| Reference | [6] | [7] | [7] | [7] | [7] |
|---|---|---|---|---|---|
| Specimen | 1A | AS2 | AF2 | AS1 | AF1 |
| $t_f$ | 0.560 | 0.472 | 0.386 | 0.495 | 0.398 |
| $t_w$ | 0.284 | 0.260 | 0.256 | 0.260 | 0.260 |
| $t_p$ | 0.315 | 0.407 | 0.406 | 0.398 | 0.398 |
| $b_f$ | 3.110 | 1.516 | 2.992 | 1.506 | 2.992 |
| $d_w$ | 6.050 | 5.512 | 5.598 | 5.526 | 5.587 |
| b | 24.000 | 7.874 | 7.874 | 7.874 | 7.874 |
| a | 48.000 | 118.110 | 118.110 | 47.244 | 47.244 |
| $S_{ys}$ | 36.740 | 59.420 | 59.495 | 60.206 | 60.510 |
| $S_{yp}$ | 36.080 | 53.171 | 51.271 | 56.217 | 56.493 |
| Type | Tee | Angle | Angle | Angle | Angle |
| P | 27.280 | 45.250 | 49.081 | 58.465 | 60.235 |

**Table 2:** Comparison of Predicted Values and Test Results

| Ref. | Test | Type | Faulkner | Guidelines | Adamchak | Argyris | Test |
|---|---|---|---|---|---|---|---|
| [6] | 1A | Tee | 24.700 | 24.275 | 26.890 | – | 27.280 |
| [7] | AS2 | Angle | – | 48.630 | – | 45.638 | 45.250 |
| [7] | AF2 | Angle | – | 52.962 | – | 45.745 | 49.081 |
| [7] | AS1 | Angle | – | 49.539 | – | 57.319 | 58.465 |
| [7] | AF1 | Angle | – | 54.864 | – | 56.572 | 60.235 |

**Table 3:** Comparison of Tripping Strength Between Angle and Tee with Same Size of Flange

| Test | Type | Faulkner | Guidelines | Adamchak | Argyris |
|---|---|---|---|---|---|
| 1A-MA | Angle | – | 25.861 | – | 25.318 |
| AS2-MT | Tee | 51.886 | 44.293 | 56.361 | – |
| AF2-MT | Tee | 51.923 | 50.744 | 57.313 | – |
| AS1-MT | Tee | 51.016 | 45.115 | 48.528 | – |
| AF1-MT | Tee | 52.945 | 51.955 | 50.948 | – |

MA: Modified from original tee to angle with same dimensions
MT: Modified from original angle to tee with same dimensions

# FIGURES

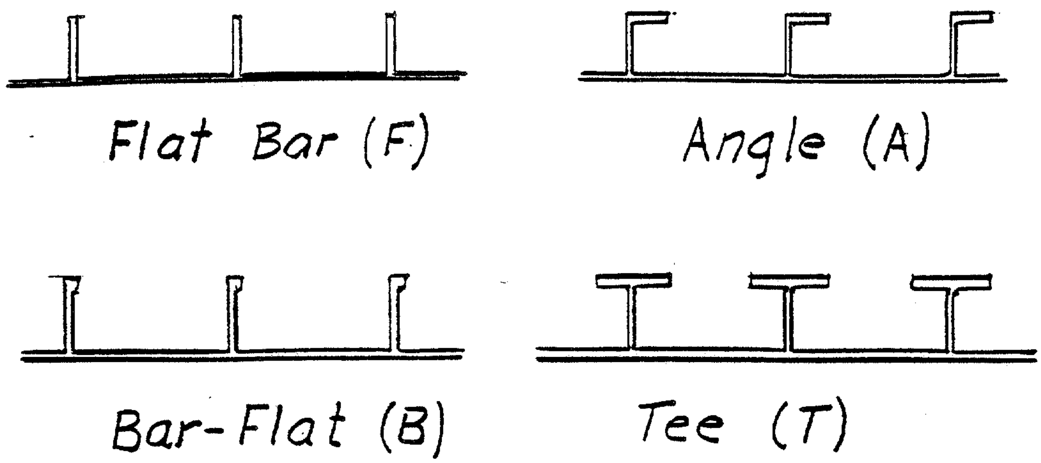



**Figure 1:** Plate with Different Stiffener Cross Sections

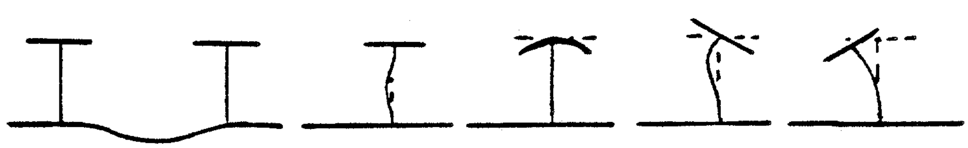
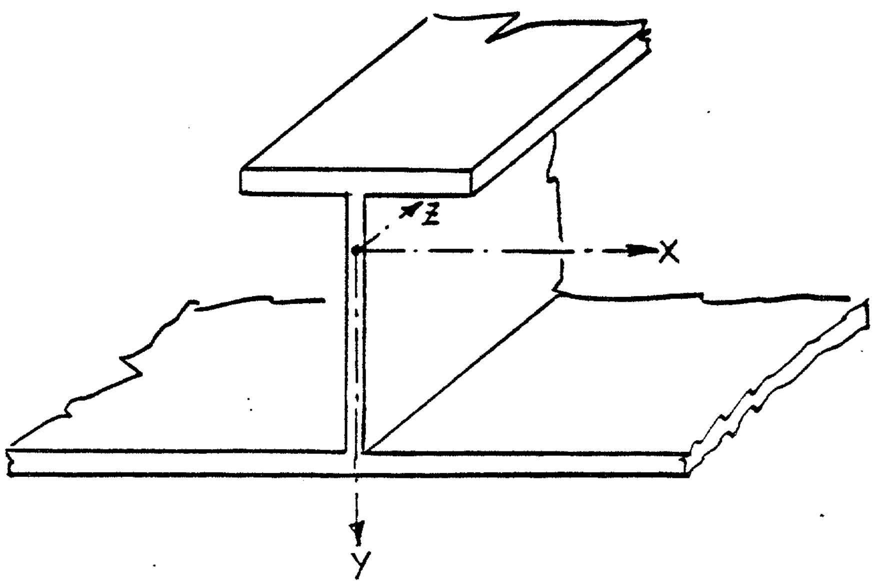


**Figure 2:** Local Deformations of Stiffened Plate



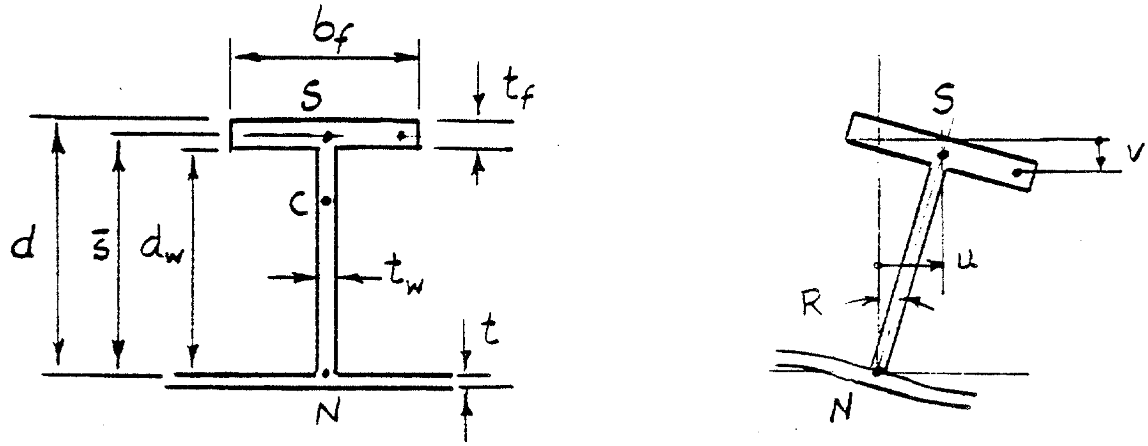
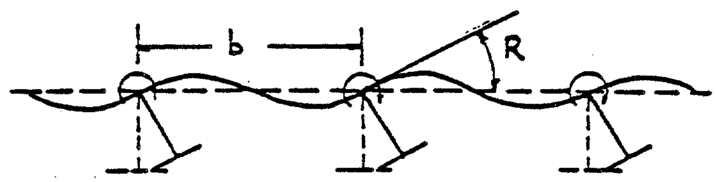
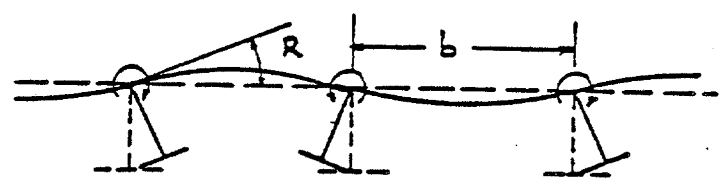




**Figure 3:** Definitions for Tee Stiffener




$C = 12EI/L = Et_p^3/d(1\text{-}\nu^2)$

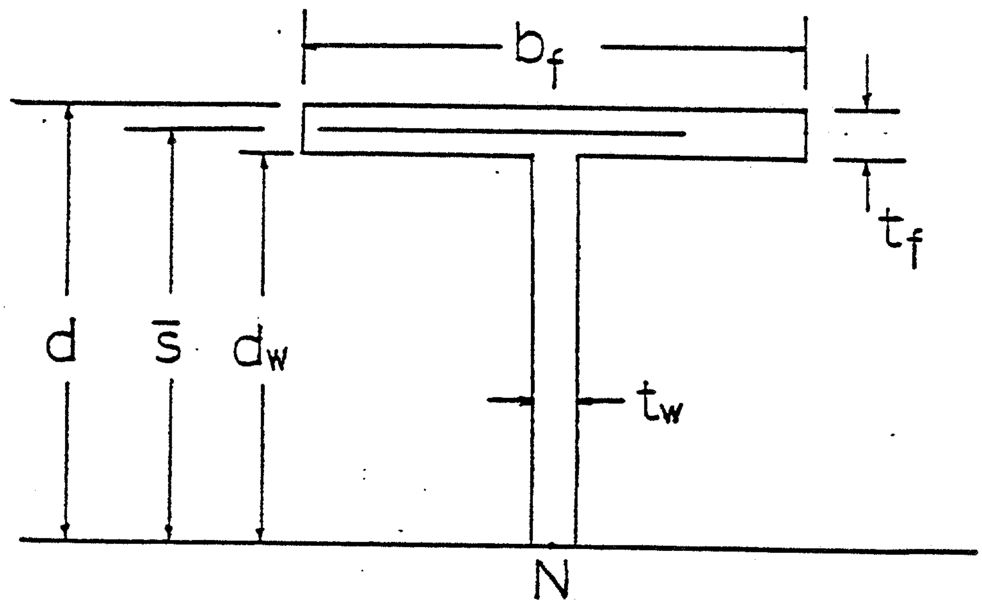
A. ANTISYMMETRICAL TRIPPING OF STIFFENERS




$C = 4EI/L = Et_p^3/3b(1\text{-}\nu^2)$

B. SYMMETRICAL TRIPPING OF STIFFENERS

**Figure 4:** Modes of Deformation (From Reference[5])




$$I_y = (t_f b_f^3 + d_w t_w^3)/12$$

$$I_w = (t_w^3 d_w^3 + t_f^3 b_f^3/4)/36$$

$$J = (d_w t_w^3 + b_f t_f^3)/3$$

$$I_p = I_x + I_y$$

**Figure 5:** Geometrical Tripping Parameters for Tee Stiffener

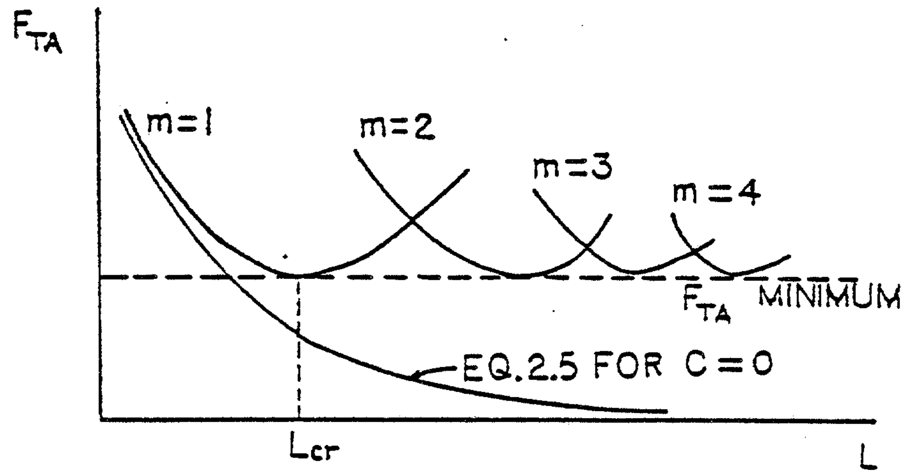
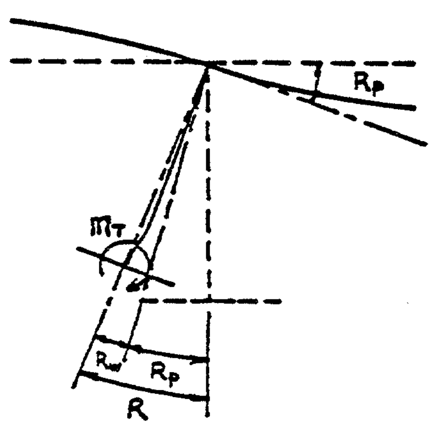



**Figure 6:** Tripping Stress Curve with Integer m




$$C_p = 4EI_{plate}/b = Et^3/(3b)$$

$$C_w = 2EI_{web}/d = Et_w^3/6$$

$$1/C = 1/C_p + 1/C_w$$

**Figure 7:** Evaluation of Rotational Restraint Including the Effect of Bending Stiffness of Stiffener Web (From Reference[5])

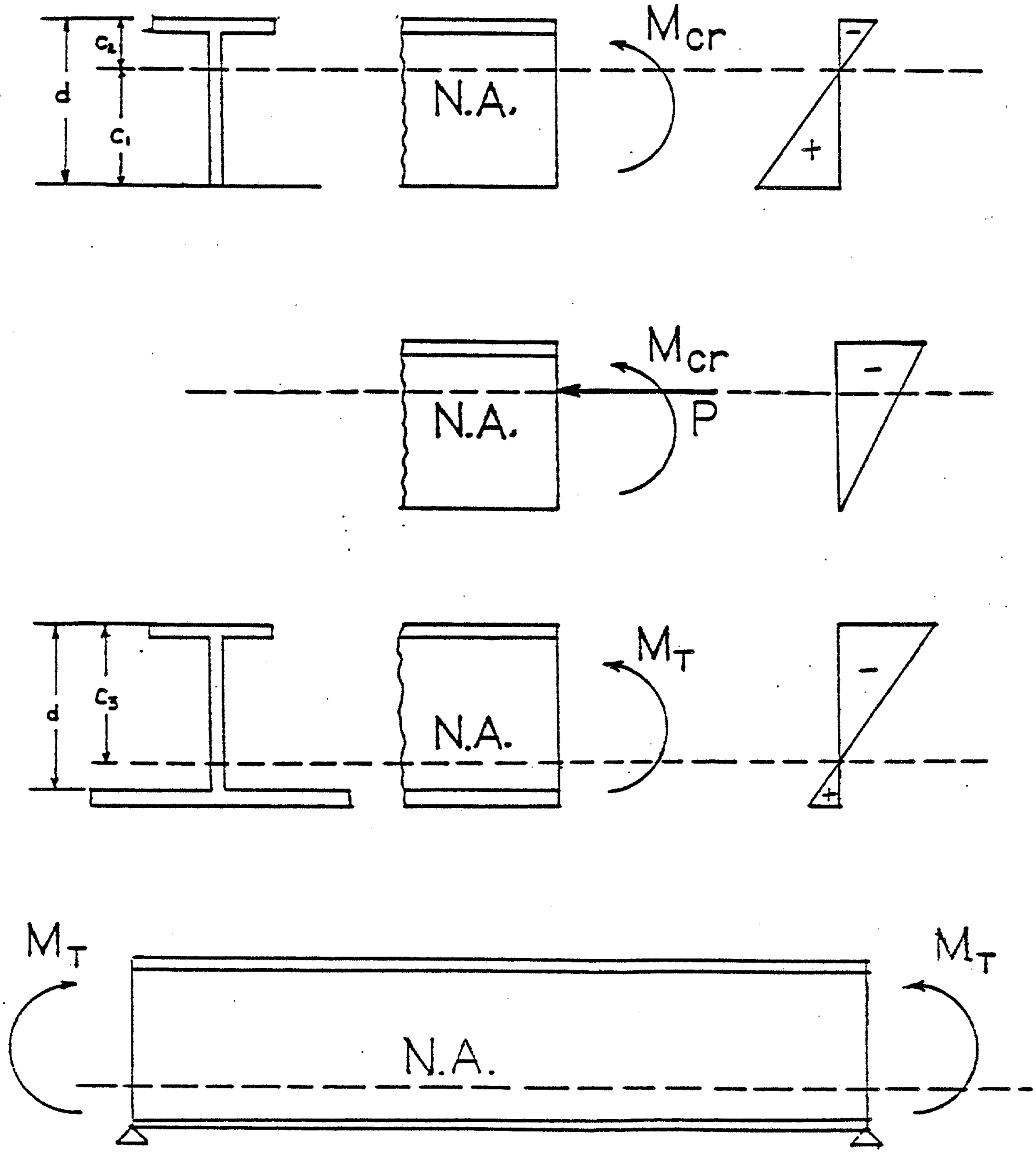



**Figure 8:** Stresses in Tee Stiffener under Bending

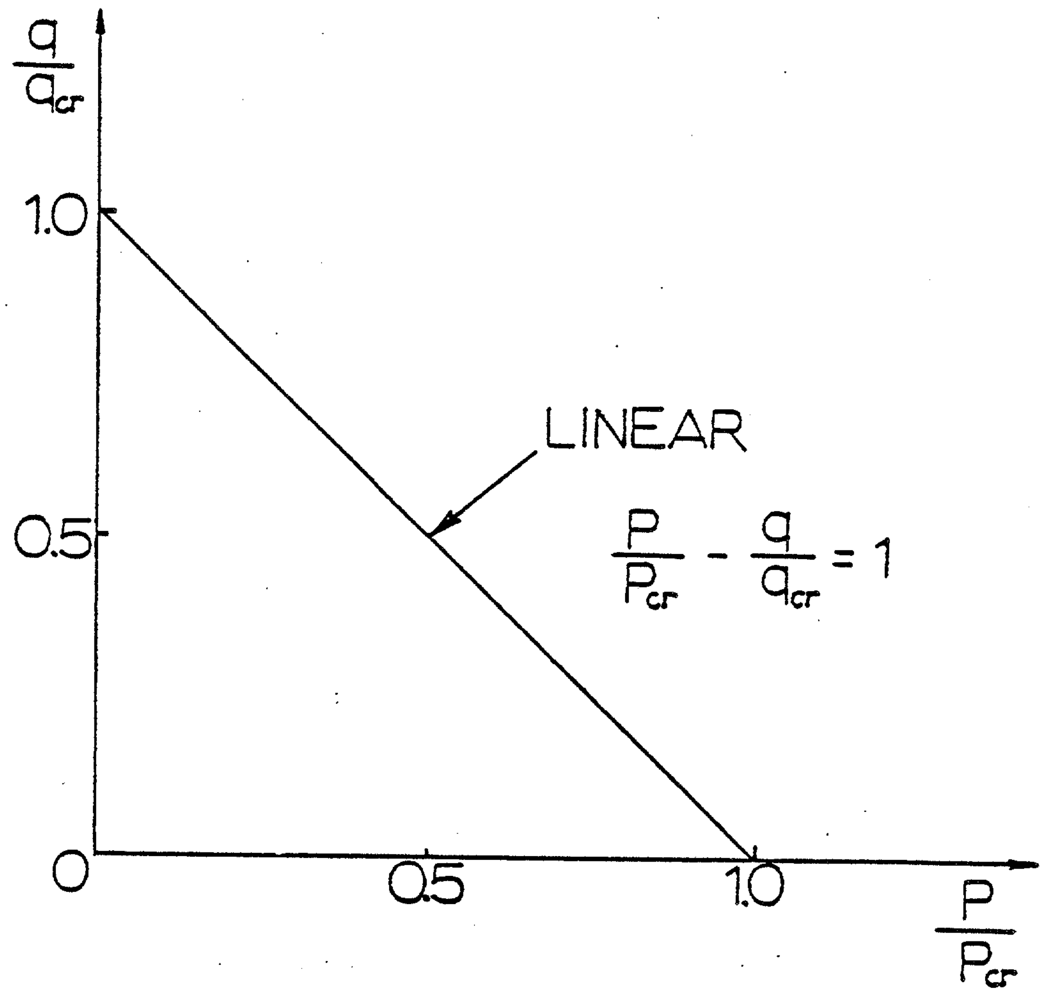



**Figure 9:** Interaction Curve for Combined Loads

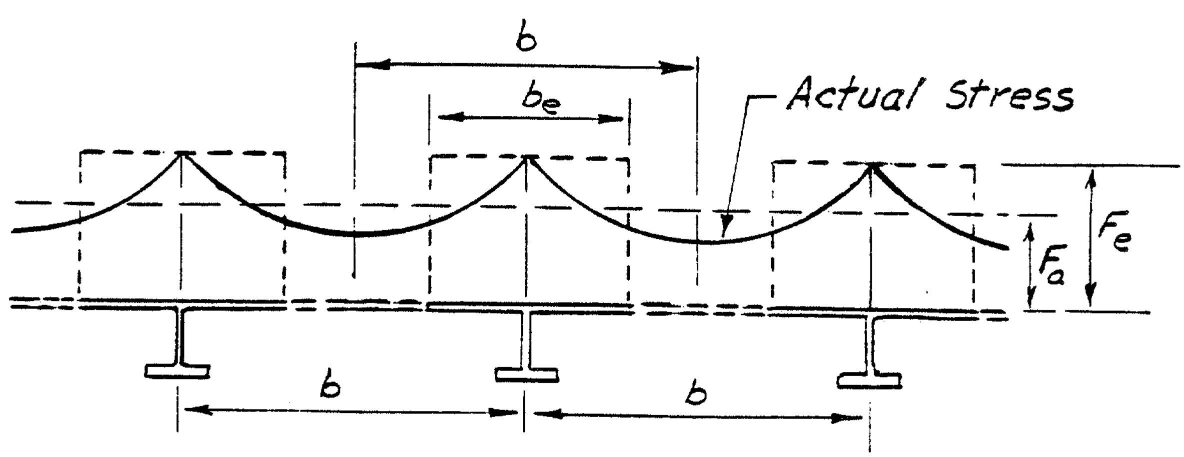



**Figure 10:** Stress Distribution and Effective Width of Plate

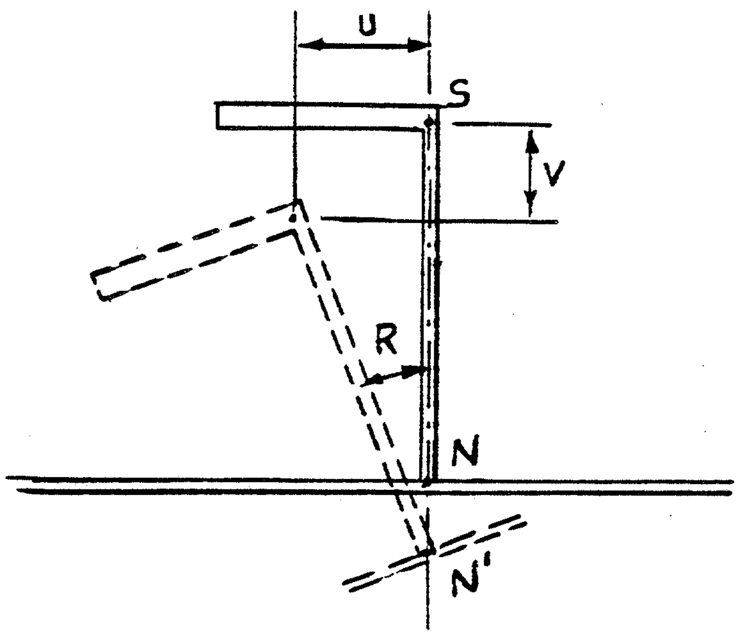



**Figure 11:** Coupled Displacements of Angle Section

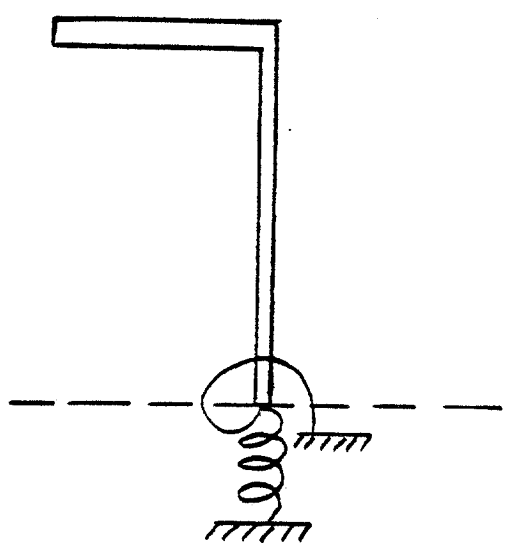


**Figure 12:** Angle Column with Idealized Support Conditions

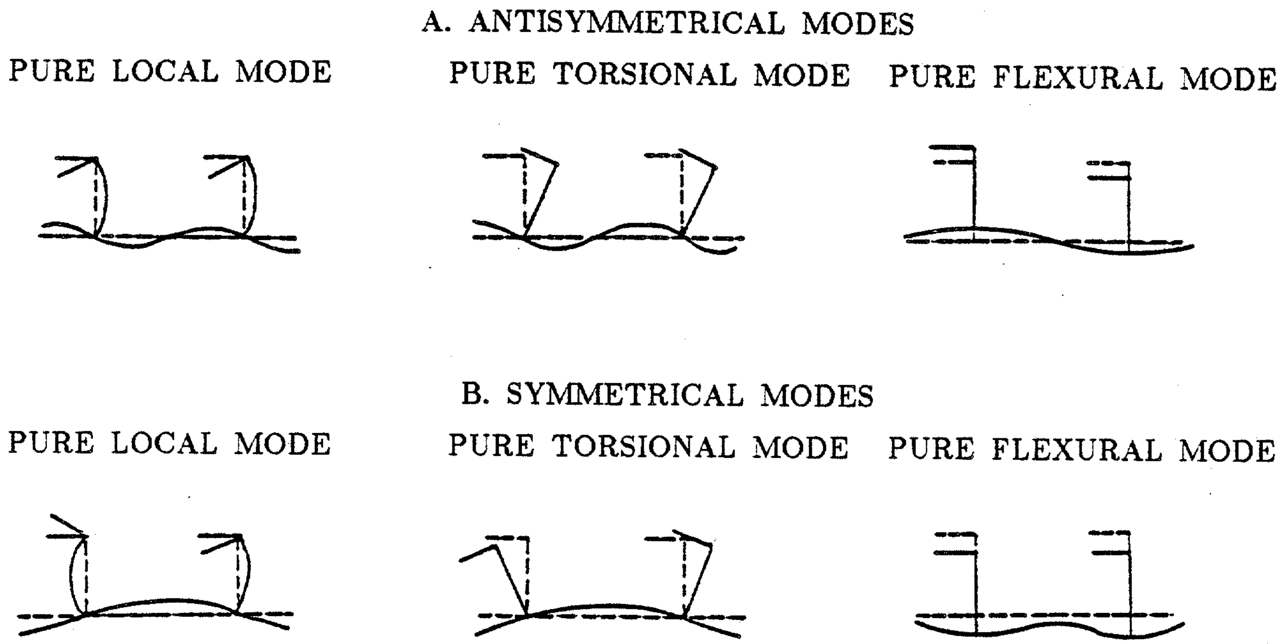



**Figure 13:** Modes of Instability of Plate with Angle Stiffeners

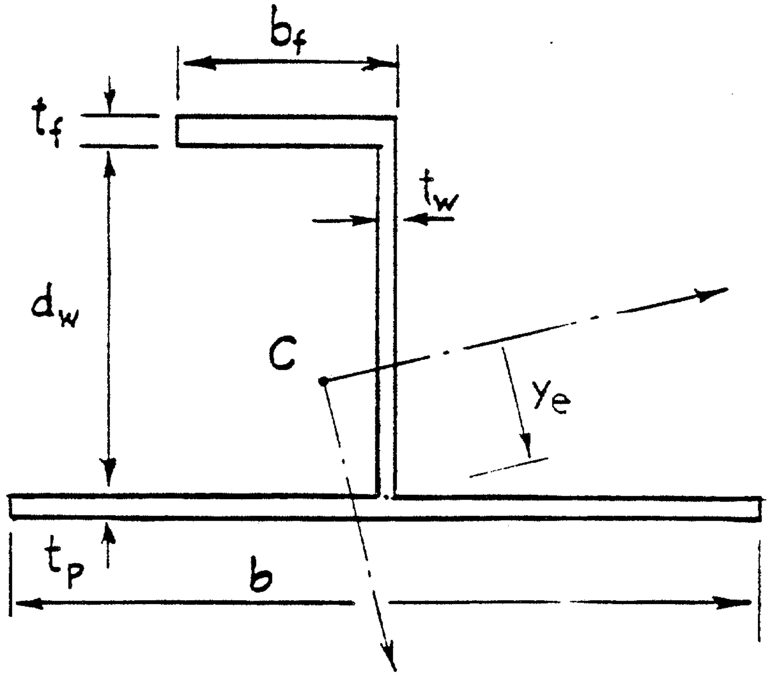



**Figure 14:** Definitions for Angle Stiffener

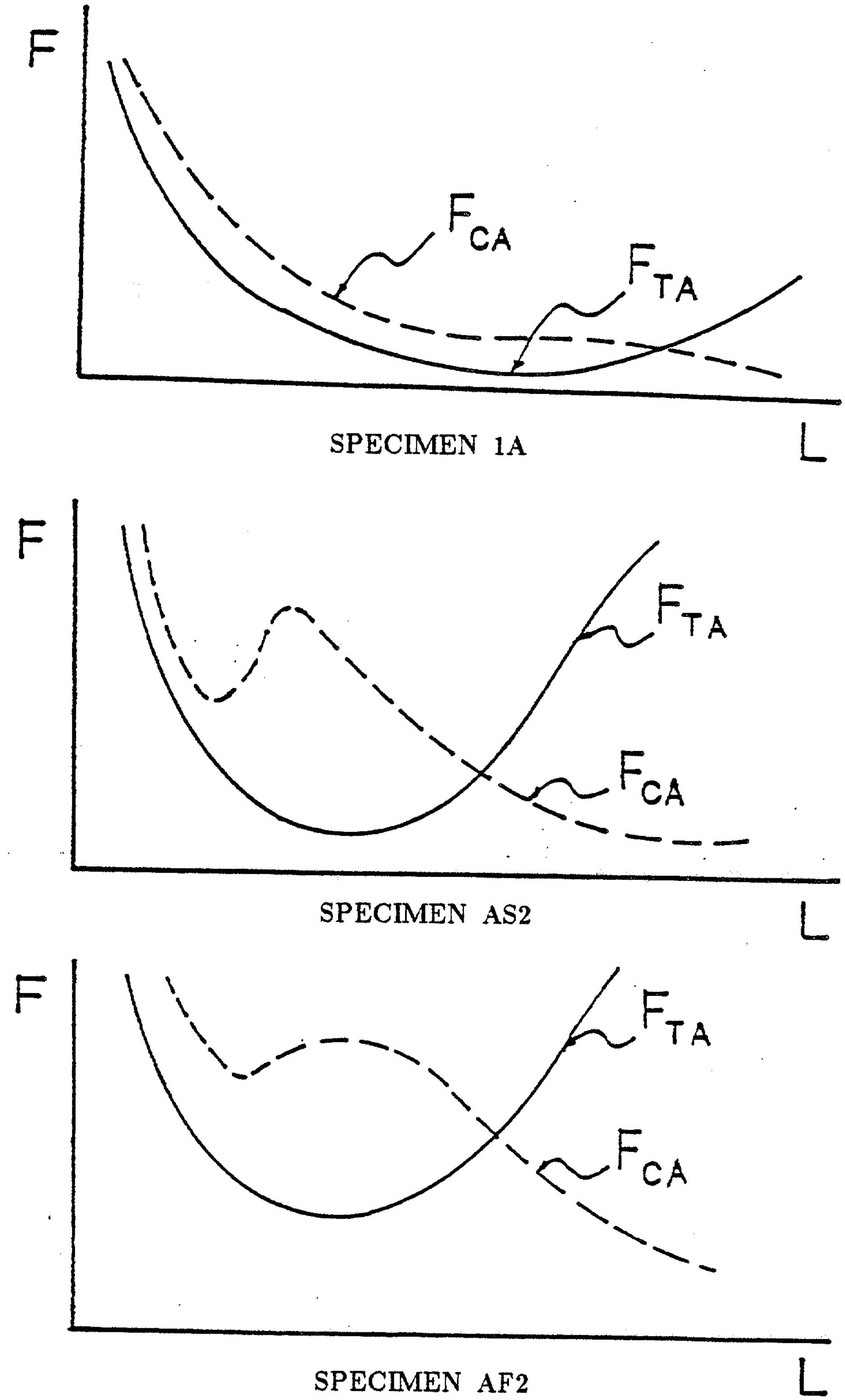



**Figure 15:** Comparison of $F_{CA}$ and $F_{TA}$ with Respect to Length for Specimens 1A, AS2 and AF2[6, 24]

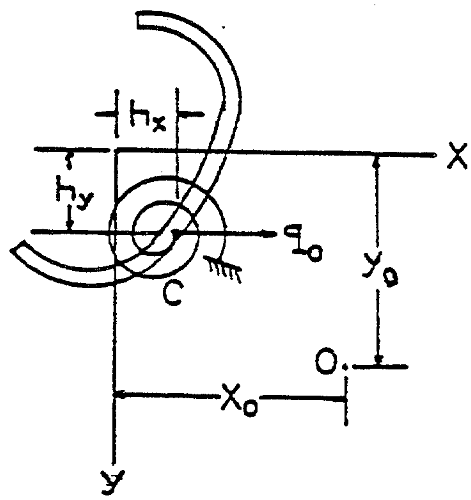



**Figure 16:** Arbitrary Cross Section of Compression Member

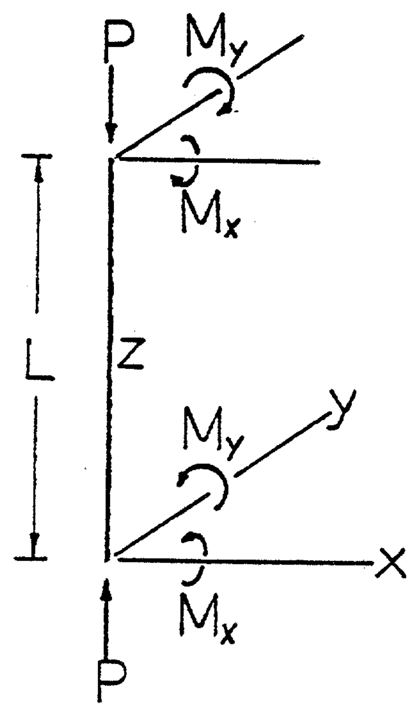



**Figure 17:** External Applied Bending Moments and Axial Forces

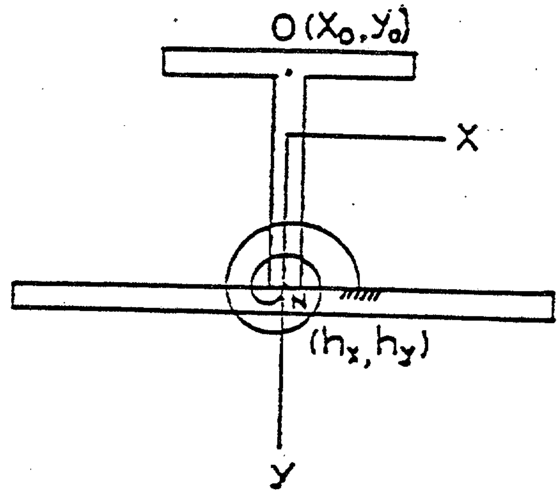



**Figure 18:** Idealized Cross Section of Tee Stiffener

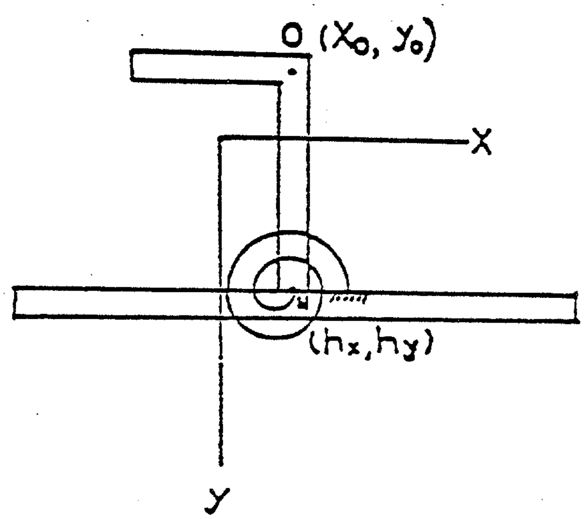



**Figure 19:** Idealized Cross Section of Angle Stiffener