Comparative Risk Analysis of Spar-Based FPSO's

by
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Based on M.S. Engineering Thesis
"Risk Analysis of Oil Storage on Spars in the Gulf of Mexico,"

by
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Final Project Report Prepared for the Minerals Management Service
Under the MMS/OTRC Cooperative Research Agreement
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August 2003

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INTRODUCTION

The objective of this research was to conduct a Comparative Risk Analysis for a Spar-based Floating Production Storage and Offloading (FPSO) facility in the Gulf of Mexico. This work represents an extension of a previously completed project where the oil spill and fatality risks were analyzed for a tanker-based FPSO in the Gulf of Mexico (Gilbert et al. 2001). In the earlier work, the risks for the tanker-based FPSO were compared with three types of deepwater production systems that have already been operated successfully in the Gulf of Mexico: a Spar and a Tension Leg Platform (TLP) with oil pipelines; and a shallow-water jacket serving as a hub and host to deepwater production. The results from the original project guided the current research in the following ways:

- Oil spills due to transportation from the facility to the shore terminal was the main discriminator between the various systems. Therefore, this risk was the focus of the current project and a comparison was developed for the spar-based FPSO (oil transport through storage on the facility and offloading to shuttle tankers) with a conventional spar (oil transport through a pipeline).
- An important factor in the oil-spill risk was how the distributions of the largest spill sizes were modeled. Developing a practical method to accommodate different assumptions for these distributions and to incorporate the uncertainty in these distributions was addressed in the current project.
- The measure of risk for oil spills in the original work was the average volume spilled in the operational lifetime of a facility, where the average represents the average for a large fleet of similar facilities operating in the Gulf of Mexico. In order to gain additional insight into the risk, the variability in performance between individual facilities was also addressed in the current project.
- There was significant uncertainty in the estimated value for the average volume spilled for each facility type, such that it was very difficult to distinguish the estimated performance of one type of facility from another. In the current project, an approach was developed to use operational data from these facilities to update the estimated performance so that the risk could be periodically re-assessed in the future as more data become available.

This report provides a brief description of the methodology and a summary of the major results and conclusions. A more detailed description of this work is provided in Chemadurov (2002).

METHODOLOGY

The methodology used herein was an extension of that developed for the original Comparative Risk Analysis for Deepwater Production Facilities (Gilbert et al. 2001 and 2002). In summary, this methodology consisted of the following steps.

Conceptual Study System

A conceptual model of the study system was developed. The intent was for this model to be representative of what a typical spar-based FPSO might look like in the Gulf of Mexico. Input was obtained from industry representatives to develop this model. The conceptual model was to adapt the conventional spar from the original study (Gilbert et al. 2001) and to replace ballast tanks in the hull with oil storage tanks (Figure 1 and Halkyard 1996). The design would be similar in concept to the Brent Spar (Figures 2), where “wet-oil storage” is used so that oil and sea water are placed together in each tank to maintain ballasting during storage and offloaded operations.

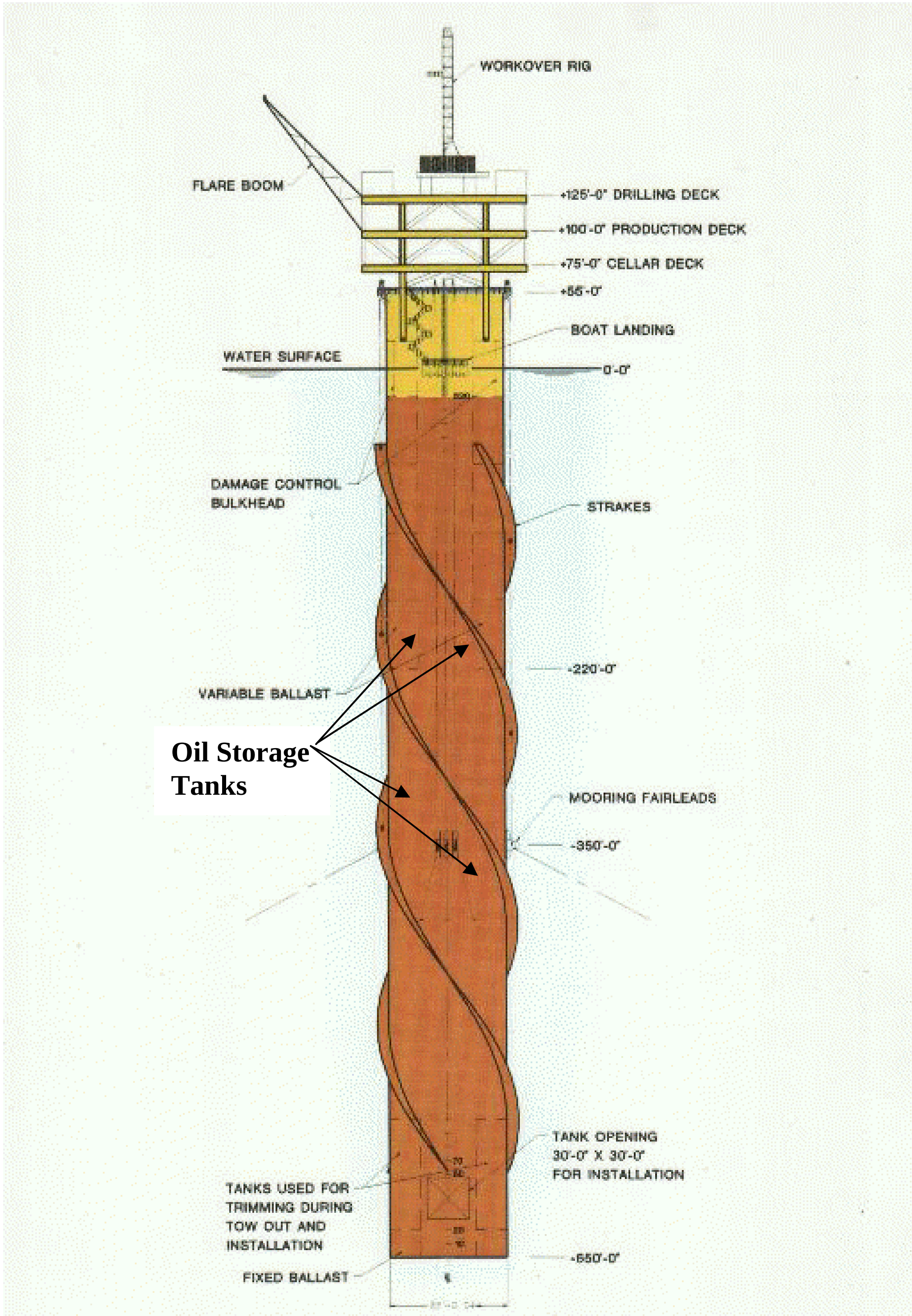

Figure 1 Schematic of Spar-based FPSO

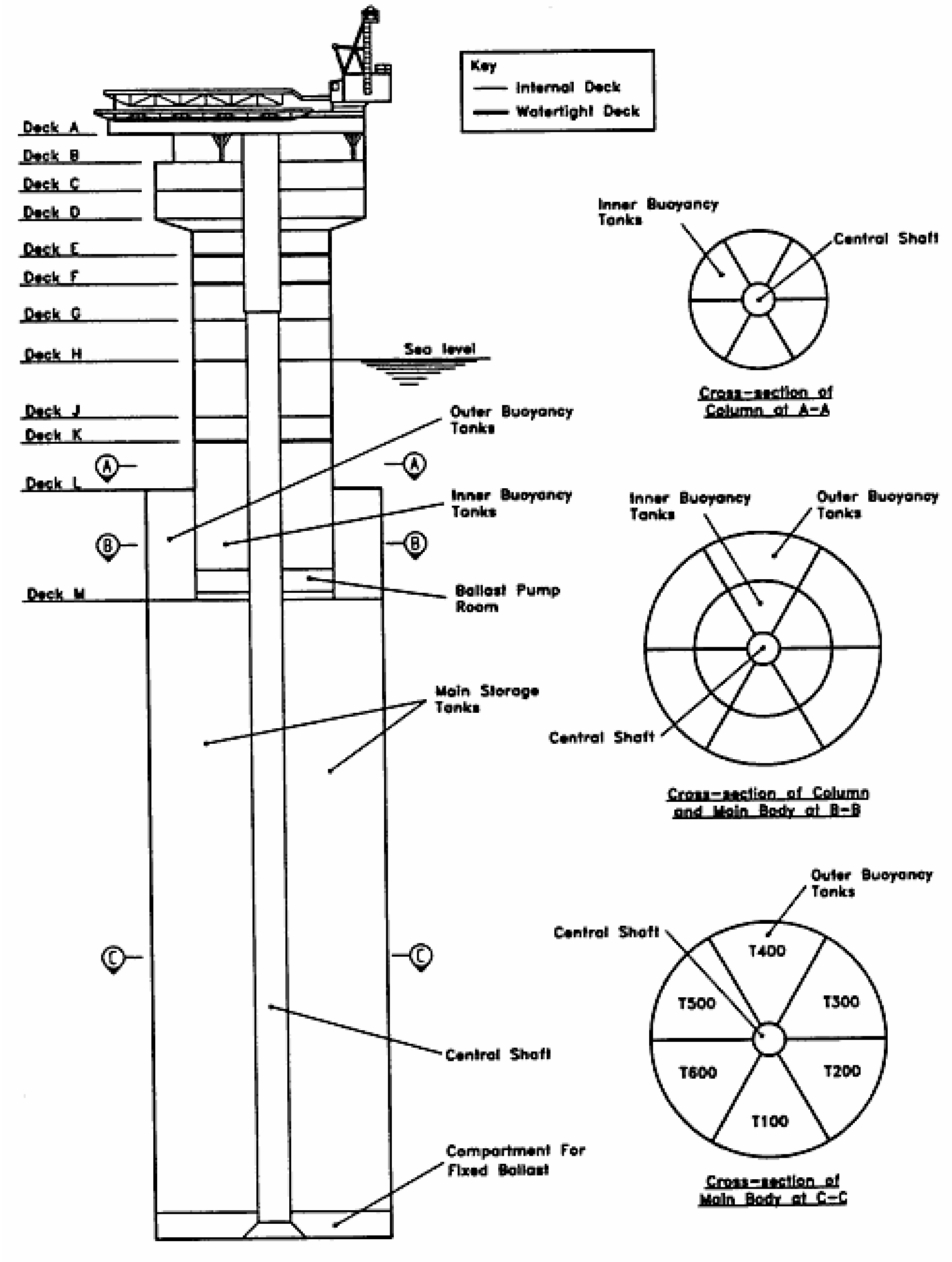

Figure 2(a) General Arrangement for Brent Spar (Shell 2002)

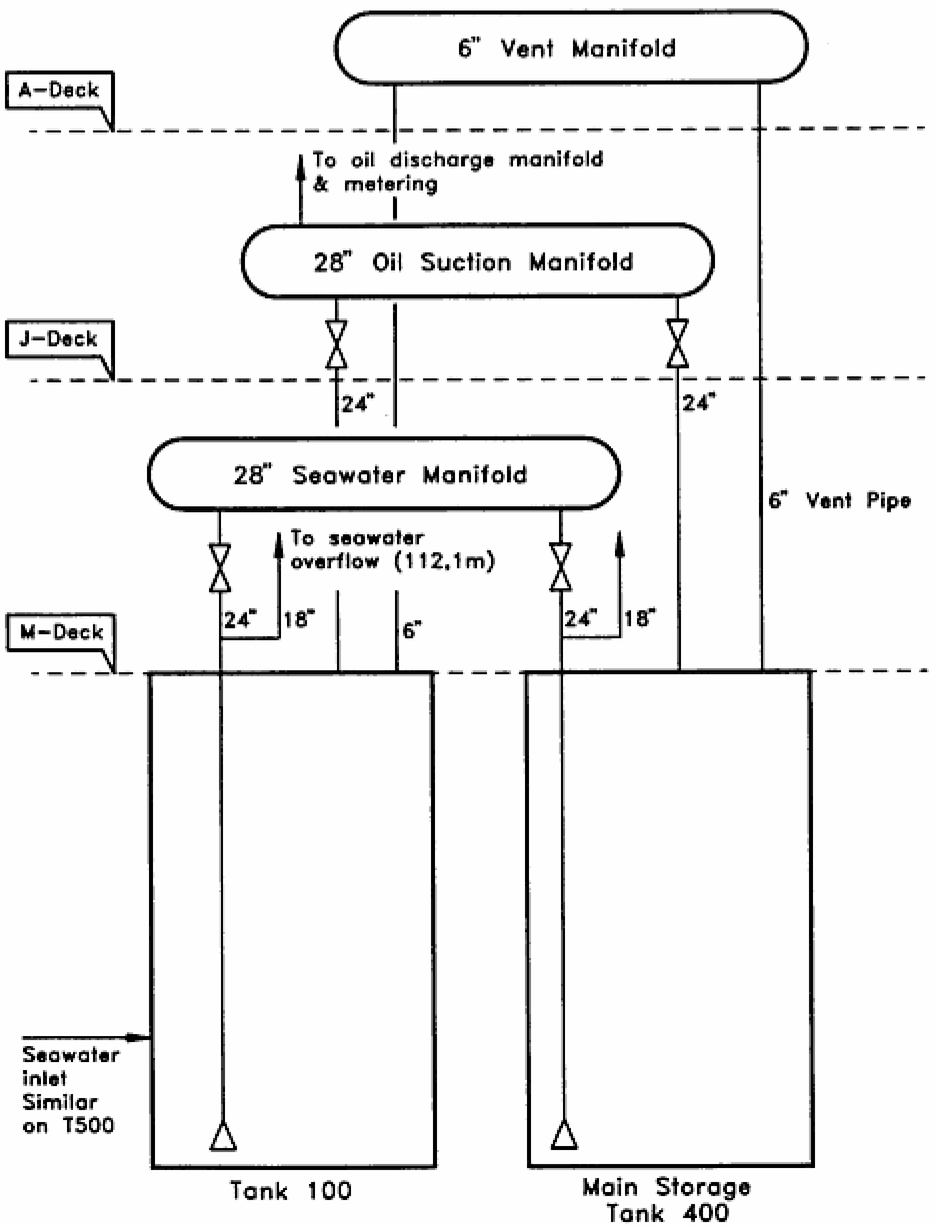

Figure 2(b) Schematic of Storage Tanks with Main Piping System for Brent Spar (Shell 2002)

Sub-Systems

Each facility was divided into a series of sub-systems so that data and expertise, which are typically specific to individual sub-systems, could be incorporated rationally and conveniently. For the purposes of the current project, the focus was on the oil transportation system which was divided into storage and shuttle tanker transport for the spar-based FPSO and export riser and oil pipeline for the conventional spar.

Spill Size Distribution

The distribution of possible spill sizes was modeled for each sub-system. The range of possible spill sizes was divided into a series of categories and then the annual frequency of occurrence in each category was estimated. This approach essentially models the distribution of spill sizes as series of steps or uniform distributions in each spill-size range (Figure 3).

The distribution of the largest-spill size, which depends on the sub-system, is not necessarily uniform because smaller spills in this category are generally more likely than larger spills due to physical constraints on the possible volume spilled in an incident. In the original study, this distribution was modeled as a triangular distribution. However, there was significant uncertainty in the actual shape of this distribution because there are few to no directly relevant data points upon which to base it.

In order to account for uncertainty in the largest-spill size distribution in the current project, the distribution was represented by a general form that could take on variety of shapes. Specifically, a Beta distribution model was used and the shape of the distribution is controlled by a parameter in this model that is denoted r (Figure 4). For r equal to one, the distribution is a uniform distribution like that used in the smaller spill-size categories; for r equal to two, the distribution is triangular like that used in the largest spill-size category in the original study; and as r increases, the smaller spill sizes in the category become more likely relative to the larger spill sizes (Figure 4). For the current project, the uncertainty in the true value of r for the largest-spill size in the shuttle tanker and pipeline sub-systems was modeled as a uniform distribution between one and five.

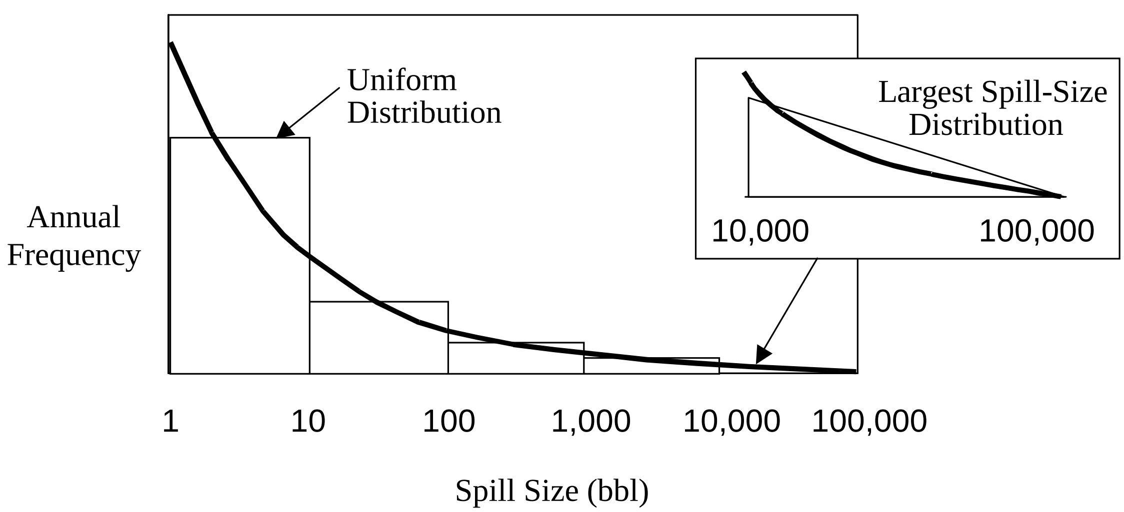

Figure 3 Model for Spill-Size Distribution

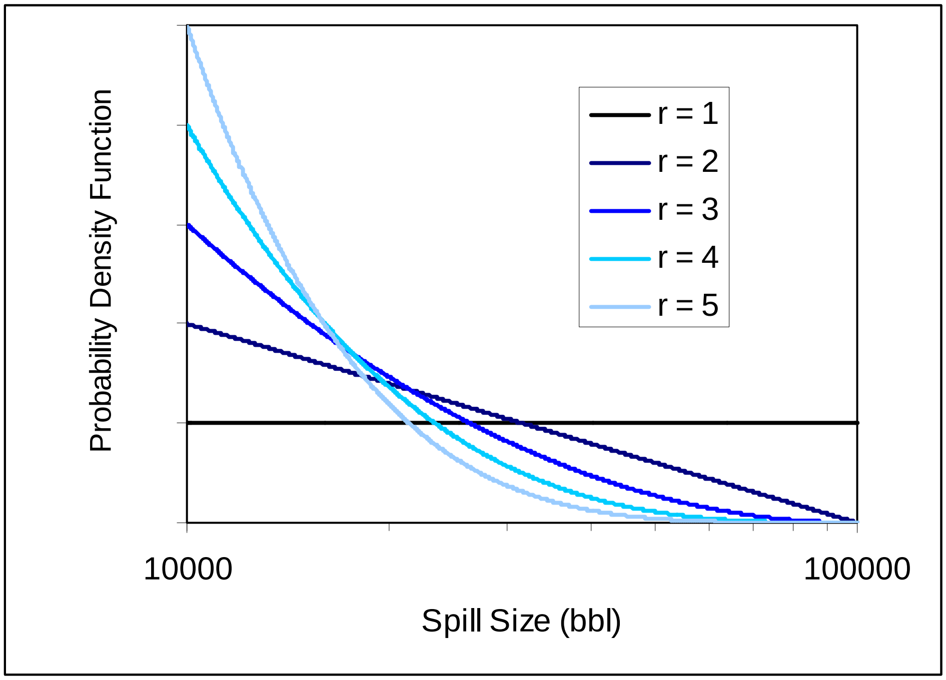

Figure 4 Alternative Models for the Largest Spill-Size Distribution

Spill Frequencies

The frequency of spills in each spill-size category for each sub-system (Figure 3) was estimated based on available data and expert input. For the conventional spar, the same information from the original study for the export riser and pipeline sub-systems was used here (Table 1).

Table 1 Estimated Spill Frequencies for Oil Transport with Conventional Spar

Conventional Spar - Export Pipeline Riser

Exposure (riser-years)	20

Spill Size Range (bbl)	Distribution in Range (see Fig. 4)	Estimated Frequency	
		Expected Value (per riser-year)	Coefficient of Uncertainty
1-10	Uniform (r = 1)	2.0E-03	1.15
10-100	Uniform (r = 1)	1.8E-03	1.15
100-1,000	Uniform (r = 1)	6.8E-03	1.15
1,000-10,000	Uniform (r = 1)	6.8E-04	1.15
10,000-100,000	Uncertain (1 < r < 5)	1.4E-04	1
100,000-500,000	Not Applicable	0	Not Applicable
500,000-1,000,000	Not Applicable	0	Not Applicable

Conventional Spar - Pipeline

Exposure (mile-years)	2900

Spill Size Range (bbl)	Distribution in Range (see Fig. 4)	Estimated Frequency	
		Expected Value (per mile-year)	Coefficient of Uncertainty
1-10	Uniform (r = 1)	3.7E-04	0.52
10-100	Uniform (r = 1)	3.2E-04	0.53
100-1,000	Uniform (r = 1)	1.2E-04	0.64
1,000-10,000	Uniform (r = 1)	1.2E-04	0.64
10,000-100,000	Uncertain (1 < r < 5)	2.5E-05	1.13
100,000-500,000	Not Applicable	0	Not Applicable
500,000-1,000,000	Not Applicable	0	Not Applicable

For the storage sub-system on the spar-based FPSO, there was very little information from the original study that could be applied directly. Therefore, a compilation and review was performed of data from the North Sea, where large gravity-based structures with wet-oil storage have been used since the 1970's (Figure 5). While these facilities are not spars, they provide a reasonable analog with oil storage where there is operational history. Based on a study of these facilities (Vinnem and Vinnem 1998), the main contributors to spills from the storage system have been identified as puncture of a tank due to dropped objects, structural failure of the hull, and operational errors. In addition, the frequencies of large spills from these storage systems were estimated based on historical data at $1x10^{-3}$ per year per facility for spill sizes between 6,000 and 60,000 bbls and $1x10^{-4}$ per year per facility for spill sizes between 60,000 and 600,000 bbls. Since there have been no occurrences of large spills from these facilities to date, these estimates are considered conservative. In addition, the technology and operating standards have improved significantly since the 1970's.

The estimated spill frequencies in the original study for the storage system in the tanker-based FPSO are about one order of magnitude smaller than those from the North Sea data for Gravity Based Structures. This difference seems reasonable given the age and source of the data. Therefore, the estimated frequencies for the tanker-based FPSO storage system were applied to the spar-based FPSO without adjustment (Table 2). One consideration in applying these estimates directly is the capacity of the storage system on the spar-based FPSO. It will likely be less than 1,000,000 bbl and maybe even as small as 500,000 bbl, so the frequency for the 500,000 to 1,000,000 bbl spill-size category on the spar-based FPSO could arguably be as small as zero. Since this category does not contribute much to the total risk, it was kept the same as for the tanker-based FPSO for simplicity. Another consideration with the spar-based FPSO is the potential effect of drilling operations on the risk of oil spills from storage. Since the data from North Sea Gravity Based Structures includes this effect and since these data support the estimates in Table 2, no adjustment was made to explicitly account for drilling operations.

Figure 5 Maureen Gravity Based Structure with Oil Storage Tanks (Phillips 66 2002)

For the shuttle tanker sub-system on the spar-based FPSO, the frequency estimates from the original study for the tanker-based FPSO shuttle tanker were applied directly (Table 2). The main differences between the tanker-based and spar-based FPSO's are the motions of the storage facility during offloading and the effect of dry (tanker-based FPSO) versus wet (spar-based FPSO) oil storage on offloading operations. Given the significant uncertainty in the estimated frequencies and given that large shuttle tanker spills are most likely to result from problems on the shuttle tanker versus the storage facility (such as a collision during transport), these differences were not considered significant enough to distinguish the estimated frequencies between the two types of facilities.

Table 2 Estimated Spill Frequencies for Oil Transport with Spar-based FPSO

Spar-based FPSO - Storage

Exposure (years)	20

Spill Size Range (bbl)	Distribution in Range (see Fig. 4)	Estimated Frequency	
		Expected Value (per year)	Coefficient of Uncertainty
1-10	Not Applicable	0	Not Applicable
10-100	Not Applicable	0	Not Applicable
100-1,000	Not Applicable	0	Not Applicable
1,000-10,000	Uniform (r = 1)	9.0E-05	1.41
10,000-100,000	Uniform (r = 1)	9.0E-05	1.41
100,000-500,000	Uniform (r = 1)	1.0E-05	1.41
500,000-1,000,000	Uniform (r = 1)	1.0E-05	1.41

Spar-based FPSO - Shuttle Tanker

Exposure (docking calls)	3049

Spill Size Range (bbl)	Distribution in Range (see Fig. 4)	Estimated Frequency	
		Expected Value (per year)	Coefficient of Uncertainty
1-10	Uniform (r = 1)	5.4E-04	0.51
10-100	Uniform (r = 1)	2.0E-04	0.62
100-1,000	Uniform (r = 1)	1.4E-04	0.68
1,000-10,000	Uniform (r = 1)	3.5E-05	1.13
10,000-100,000	Uniform (r = 1)	4.7E-06	1.16
100,000-500,000	Uncertain (1 < r < 5)	3.1E-06	1.2
500,000-1,000,000	Not Applicable	0	Not Applicable

RESULTS

The results of the comparative risk analysis are shown on Figures 6 through 9. Figure 6 shows that the total volume spilled for either the conventional spar or the spar-based FPSO is expected to be about the same. In both cases, the expected value for an average facility is approximately 3,000 bbl. The physical meaning of this result is that if a large number of similar facilities were operated in the Gulf of Mexico for 20-years each, then the average volume of oil spilled per facility over 20 years is expected be approximately 3,000 bbl. The large confidence bounds on Figure 6 reflect the significant uncertainty in the performance of an average facility since there has not yet been a single spar operating in the Gulf of Mexico for its lifetime. The bounds for the conventional spar are larger compared to the original study (larger by 500 to 1,000 bbl on either side) since uncertainty in the distribution for the largest-spill size is now incorporated into the calculations. The confidence bounds for the spar-based FPSO are larger than for the conventional spar since less historical information is available.

A break-down of the total risk by spill-size category is shown on Figures 7 and 8. The risk for the conventional spar with a pipeline is dominated by the possibility of spills between 1,000 bbl and 100,000 bbl (Figure 8a). However, the risk for the spar-based FPSO arises from the possibility of spills over a much larger range from 1,000 bbl up to 1,000,000 bbls, with spills greater than 100,000 bbl contributing the majority of the total risk (Figure 8b). This result means that while the risk in terms of the total volume spilled for the average facility is comparable between the two types of spars, the risk for the conventional spar is due to more frequent but smaller spills than that for the spar-based FPSO.

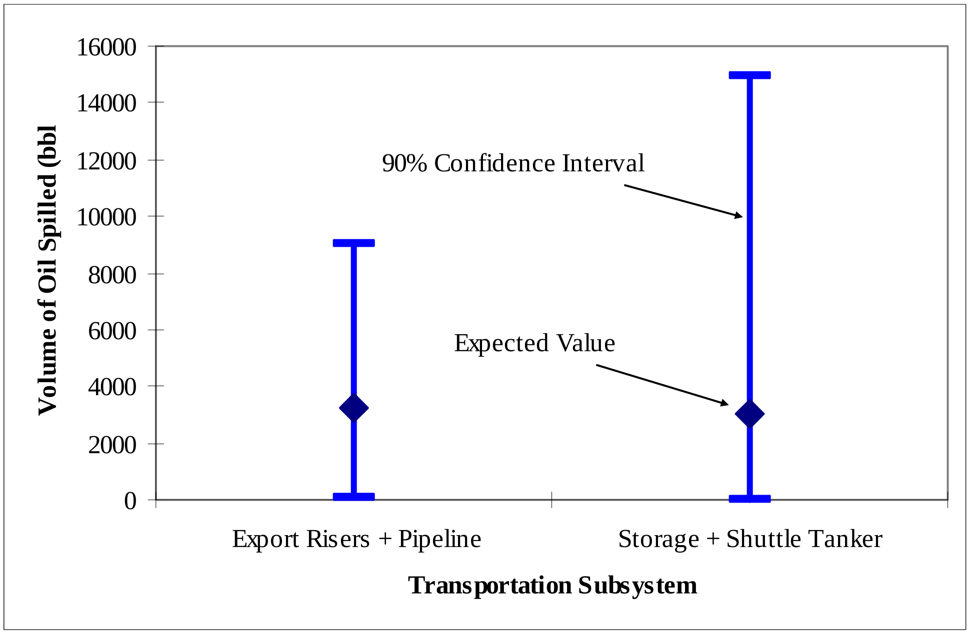

Figure 6 Volume Spilled in Lifetime for an Average Facility – Comparison of Conventional Spar with Pipeline and Spar-based FPSO with Shuttle Tankers

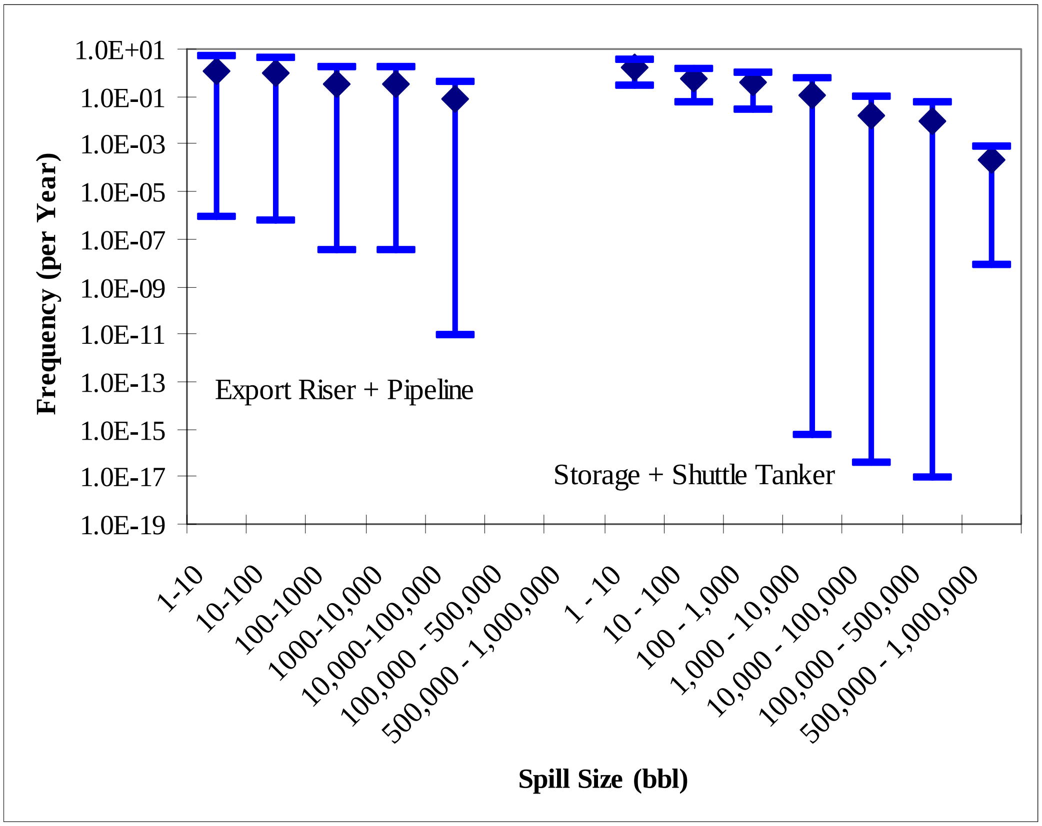

Figure 7 Frequency Breakdown by Spill Size for an Average Facility – Comparison of Conventional Spar with Pipeline and Spar-based FPSO with Shuttle Tankers

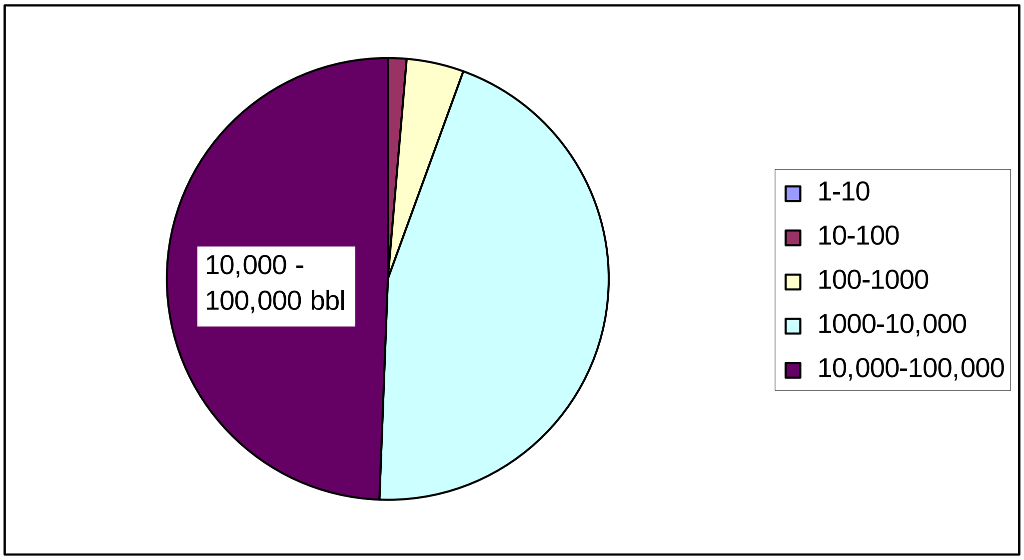

Figure 8(a) Contribution of Spill-Size Category to Expected Value of Total Volume Spilled for a Conventional Spar with Pipeline

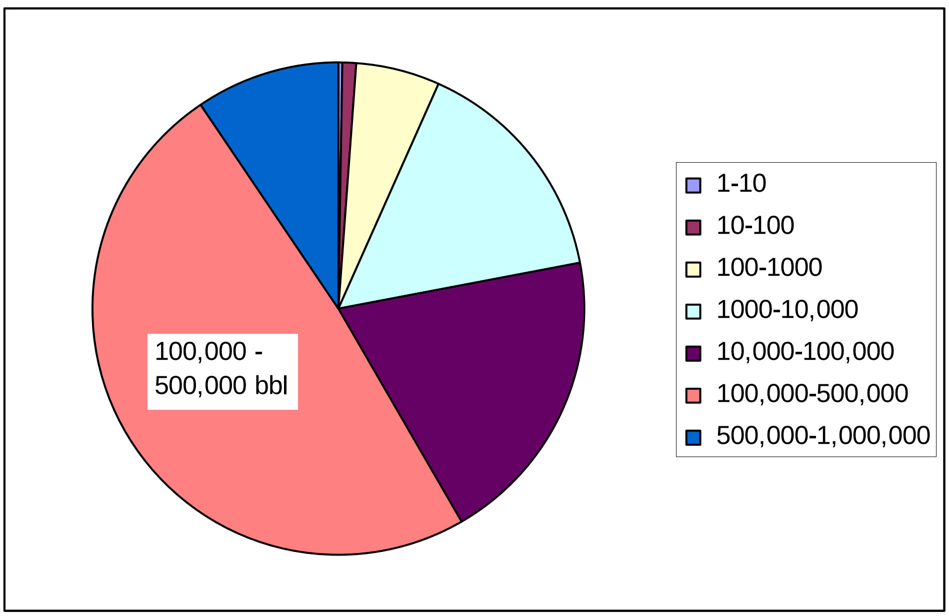

Figure 8(b) Contribution of Spill-Size Category to Expected Value of Total Volume Spilled for a Spar-based FPSO with Shuttle Tankers

In order to provide greater understanding into the risk for each type of spar, the performance for individual spars is shown on Figure 9 in terms of the frequency for different total volumes spilled in the lifetime. Several conclusions can be drawn from this figure. First, the majority of facilities in both cases will have total volumes spilled that are less than 100 bbl (Figure 9), even though the expected value for the average is greater than 1,000 bbl (Figure 6). Second, about 10 percent of the conventional spar facilities will have a total volume spilled in a 20-year operational lifetime that is greater than 10,000 bbl while about 2 percent of the spar-based FPSO facilities will have spills this large. Finally, while only a very small percentage of spar-based FPSO facilities will have a total volume spilled in excess of 100,000 bbl, this possibility has a significant contribution to the oil-spill risk for this type of facility (Figure 8b).

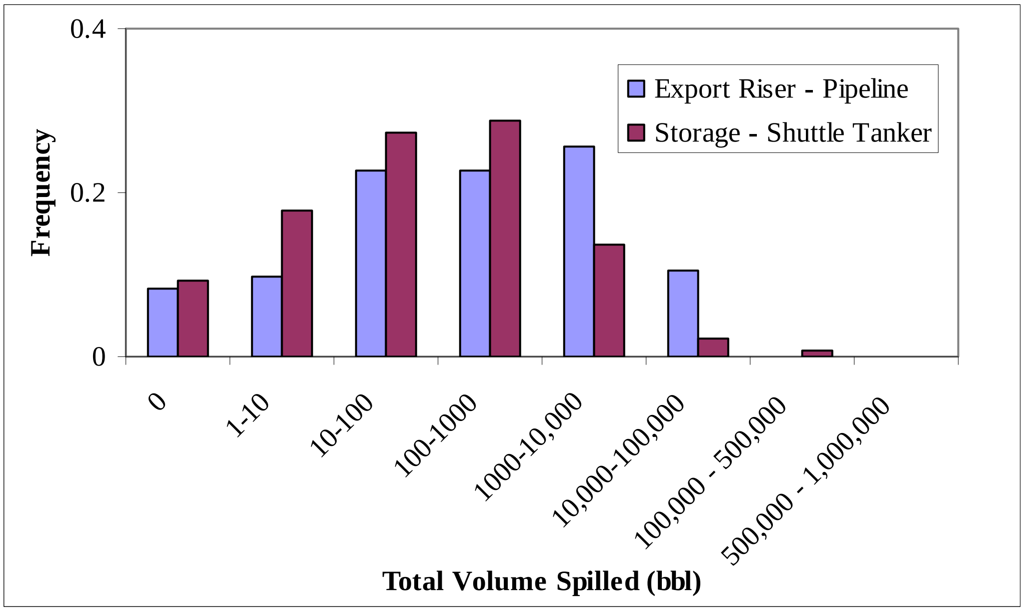

Figure 9 Frequency of Total Volume Spilled in Lifetime for an Individual Spar – Comparison of Conventional Spar with Pipeline with Spar-based FPSO with Shuttle Tankers

A relevant question that we attempted to answer with this work is the following: Given that it is not really possible to distinguish between the different types of facilities due to the considerable uncertainty (Figure 6), how many spar-years of operation in the Gulf of Mexico would be required to being to distinguish them? The mathematical details of how this question was addressed are presented in Chemadurov (2002) and the results are shown on Figure 10. This figure shows how the confidence bounds in Figure 6 could be reduced in the future as more historical data become available. The width of the confidence bounds are approximately proportional to the standard deviation; so a 50-percent reduction in the standard deviation roughly means a 50-percent reduction in the width of the confidence bounds.

The results on Figure 10 indicate that even if 10,000 spar-years of data were available (e.g., 500 spars each operated for 20 years in the Gulf of Mexico), the reduction in the width of the confidence bounds would be negligible (a reduction of less than 20 percent for the conventional spar and less than a 5 percent reduction for the spar-based FPSO). The expected values on Figure 6 are very close together so reductions in the confidence bounds of more than 90 percent would be required to begin distinguishing between these two types of facilities. Therefore, significantly more than 10,000 spar-years of data would be required to meaningfully update these risks with actuarial information.

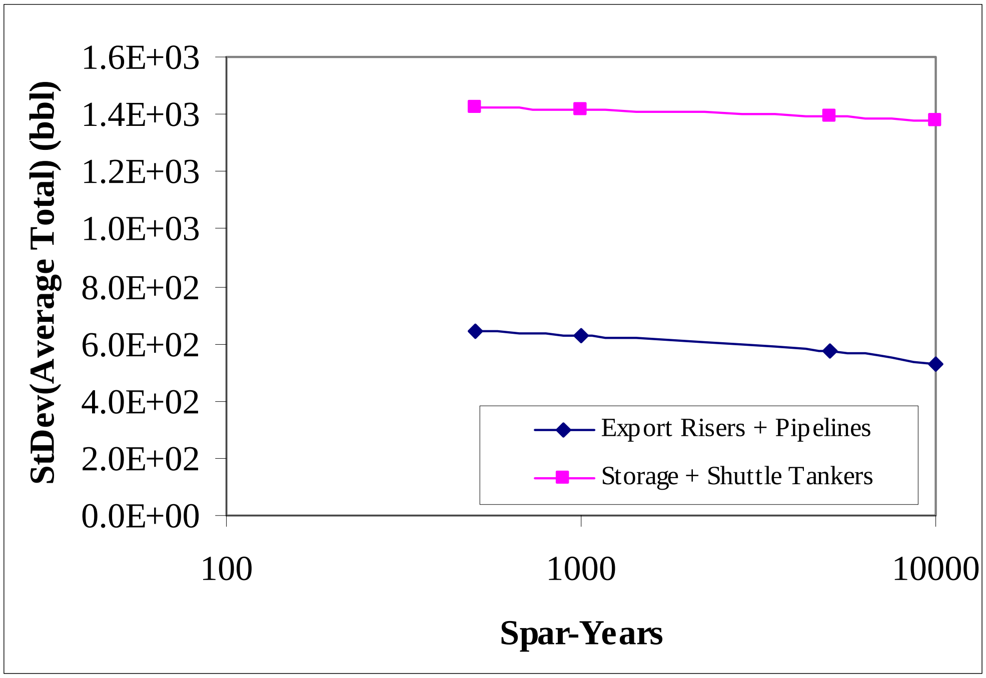

Figure 10 Estimated Effect of Updating Risk based on Operational Data

CONCLUSIONS

The main conclusions from this work are:

1. The oil-spill risk for a spar-based FPSO with oil shuttle tankers is comparable to that for a conventional spar with an oil pipeline.
2. The oil-spill risks for both types of spars are governed by the possibility of rare but large oil spills greater than 1,000 bbl in size.
3. It would require much more than 10,000 spar-years of data for each type of facility to be able to distinguish the oil spill risks based on actuarial information alone.

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Risk Analysis of Oil Storage on Spars in the Gulf of Mexico

By

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Thesis

Presented to the Faculty of the Graduate School

of the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science in Engineering

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August 2002

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Chapter 1: Introduction

A quantitative risk analysis was completed to analyze the risk to the environment associated with proposed oil storage on spars in the Gulf of Mexico.

1.0 Objectives

This report is an extension of the Comparative Risk Assessment (CRA) model developed by Gilbert et al (2001a). The first objective is to develop a general and realistic model to quantify the risk for oil spills. The second objective is to eliminate restrictions within the model and evaluate their uncertainty, in regards to the rate of spill occurrences and the spill's magnitude. Lastly, the mean rate of occurrence, N, and the statistical parameter describing the spill size distribution, R, will be updated in terms of their variance in order to judge the amount of information that can be learned from prolonged spar operations in the Gulf of Mexico.

This report will only examine the risk posed from transporting oil to the shore from a spar production facility. Two transportation systems will be looked at: the conventional system of export risers and a pipeline to the shore and an alternative system of storage in the spar with offloading and transport by shuttle tankers (also known as floating drilling, production, storage and offloading system or FDPSO and as a spar-based FPSO).

1.1 Background

As larger hydrocarbon reservoirs are discovered in deeper water (greater than 3,000 feet of water), new methods for developing these fields are being explored. However, as the water depth increases, so does the distance from the offshore facility to the shore, which poses problems in the transfer of the produced hydrocarbons.

The current method for offloading and transporting the produced oil from a spar involves the use of pipelines along the ocean floor. The use of pipelines is appealing if the reservoir is located near existing pipelines or is near the shore. However, the deepwater reservoirs are not near the existing infrastructure and extension of the network can be long and expensive.

One potential alternative is the use of current spar technology with the addition of oil storage. The spar, Figure 1.1, is a deep-draft floating caisson, similar to a buoy, which has been adapted for drilling and production. The hull allows for its potential use for oil storage. The oil would then be unloaded and transferred via shuttle tankers.

Spars are currently in use in the Gulf of Mexico for drilling and production, but not storage. In the North Sea, the Brent Spar was used for oil storage, but not drilling and production.

Before the spar can be implemented with integrated oil storage in the Gulf of Mexico, a risk analysis is helpful to evaluate its potential threat to the environment. This thesis describes a model developed for evaluating the environmental performance of a spar. This model included the variations in performance from spar to spar as well as the uncertainty in the performance of an average spar due to a lack of historical data. It allows for a comparison of the two different types of oil transportations systems with a spar in deepwater.

1.2 Organization

This thesis is divided into seven chapters. Following this introduction, Chapter 2 will present a background of the problem, along with information on past studies and data collected for evaluation in this report. The model used to evaluate the environmental risk due to oil storage on spars will be discussed in

Chapter 3. Uncertainties in the model will be considered in Chapter 4. Chapter 5 will present a method for updating the data. A comparison of the two methods for transporting oil with and without oil storage on a spar will be conducted in Chapter 6. Finally, Chapter 7 will present the conclusions of the analysis and recommendations for future work. Appendix A contains the Visual Basic® code written for the Monte Carlo simulations and tables used for updating are presented in Appendix B.

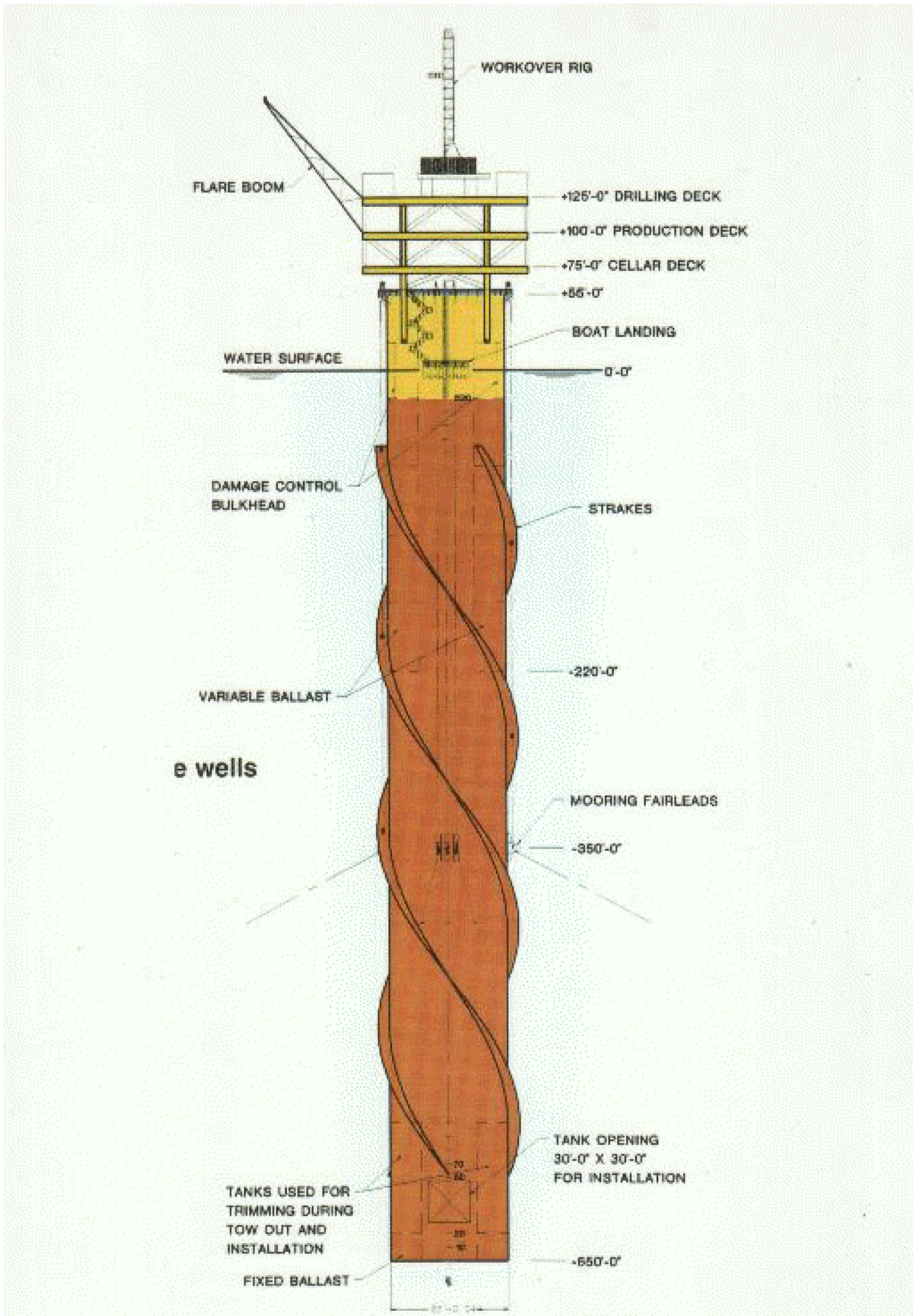


Figure 1.1. Schematic of a Spar (Centre for Oil and Gas Engineering 2002)

Chapter 2: Literature Review

This chapter discusses the use of spars as they are currently used in the Gulf of Mexico, as well as their adaptation for oil storage. Gravity based structures in the North Sea will also be examined given their similarity to current and future spar use, in regards to oil storage. Data collected on the North Sea's past experience with oil storage offshore will also be presented, as well as oil spill data for the Gulf of Mexico. Lastly, the methods of analysis chosen to quantify the relevant risk will be discussed.

2.0 Spar History

The spar is a hollow, cylindrical, floating platform, similar to a buoy, which utilizes a mooring system to help maintain its position. In the past, spars have been used for many purposes including storing oil and for gathering oceanographic data.

In the North Sea, the Brent Spar had been used successfully for oil storage, but was not utilized for drilling or oil production. Oil storage on the Brent Spar provided an efficient means of collecting produced oil from multiple offshore installations and holding it until it could be offloaded via shuttle tankers at an appropriate time. Since the Brent Spar was located in deeper water further off the Norwegian coast, this method helped to eliminate the need for additional miles of pipelines.

Currently in the Gulf of Mexico, there are three spars (Neptune (1996), Genesis (1998), and Hoover/Diana (1998)) that are designed for drilling and oil production, but not storage and offloading. All of these spars use pipelines for offloading oil to shore. At present, there are no known spars in the world's waters that both produce and store oil.

The advantage of using a spar design for drilling and production is its ability to function in deep water (greater than 3000 ft). As oil and gas production continues, reserves closer to shore are being depleted, thus forcing development of oil fields into deeper waters. This poses a problem for oil transport with pipelines.

Current methods in the Gulf of Mexico for offloading the produced oil involve the use of pipelines along the ocean floor. As production fields move farther away from shore, the further they move from existing pipelines able to transport the oil. In addition, it becomes more expensive to extend the pipelines. Therefore, spars with oil storage are being looked upon as an alternative due to their potential ability to produce and store oil. The oil would then be offloaded and transported via shuttle tankers.

The spar design lends itself to oil storage because of its hull. The hull is typically divided into compartments, which are flooded with seawater to provide ballast for the structure. These compartments could also be designed to accommodate oil storage. Approximately 500,000 bbls of oil could be stored on the spar without affecting current designs of the hull size (Halkyard, 1996). In addition, soft tanks, not part of the original design, could be added to the hull for oil storage (Glanville, 1991).

2.1 Offshore Oil Storage Structures

The following discusses past and current offshore structures which have/are used for oil storage in the field: the Brent Spar and Gravity Based Structures, GBS.

2.1.1 Brent Spar

An example of oil storage on a spar is the Brent Spar. In 1974, the Brent Spar was installed in the North Sea with the main goal of providing a means to maintain an optimum level of crude oil production where pipelines were not present or feasible (Bax, 1974). This would also reduce the number of shutdowns and lost production time due to the transfer of oil to shuttle tankers.

Wet storage, storing oil with seawater, was chosen as the method to store the crude oil in the six storage tanks in the lowest section of the hull. The storage tanks were designed to handle a net storage capacity of 300,000 bbls. The processed crude oil from adjacent production platforms in the Brent field was then transferred to the spar for storage until offloaded by shuttle tankers.

Experience from the Brent Spar with integrated oil storage has proven to be successful. However, in January 1977, the Brent Spar realized a structural limitation in the design of its storage tanks when there was an accidental build up of differential pressures, which caused two tanks to rupture. These two tanks were repaired, but were not used again to store oil. Instead, they functioned as settling/emergency tanks (Shell 2002). A general arrangement of the Brent Spars configuration is shown in Figure 2.1 and a schematic of the storage tanks with the main piping system shown in Figure 2.2.

In 1991, the Brent Spar was decommissioned after 15 years of successful service.

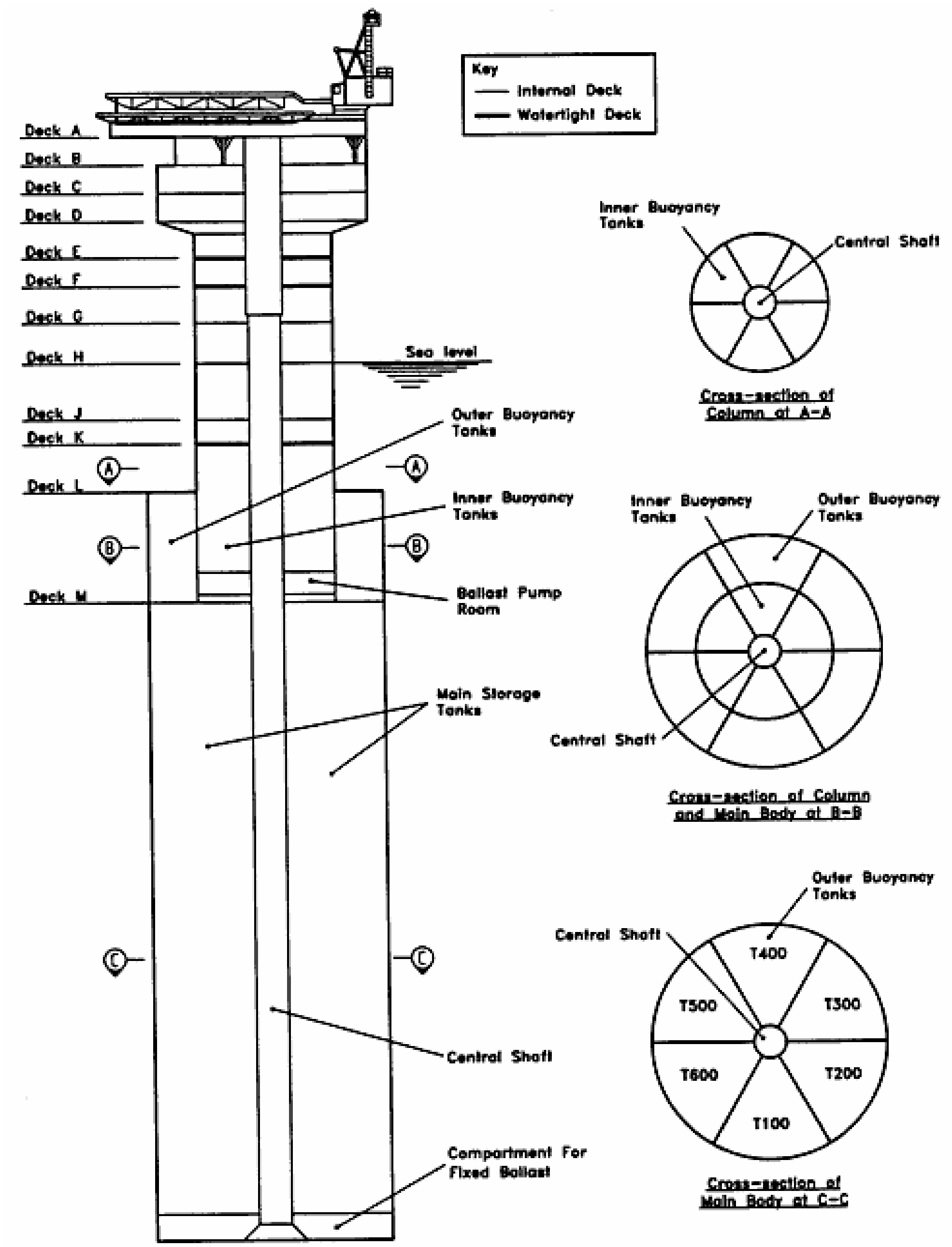


Figure 2.1. General Arrangement of the Brent Spar (Shell 2002)

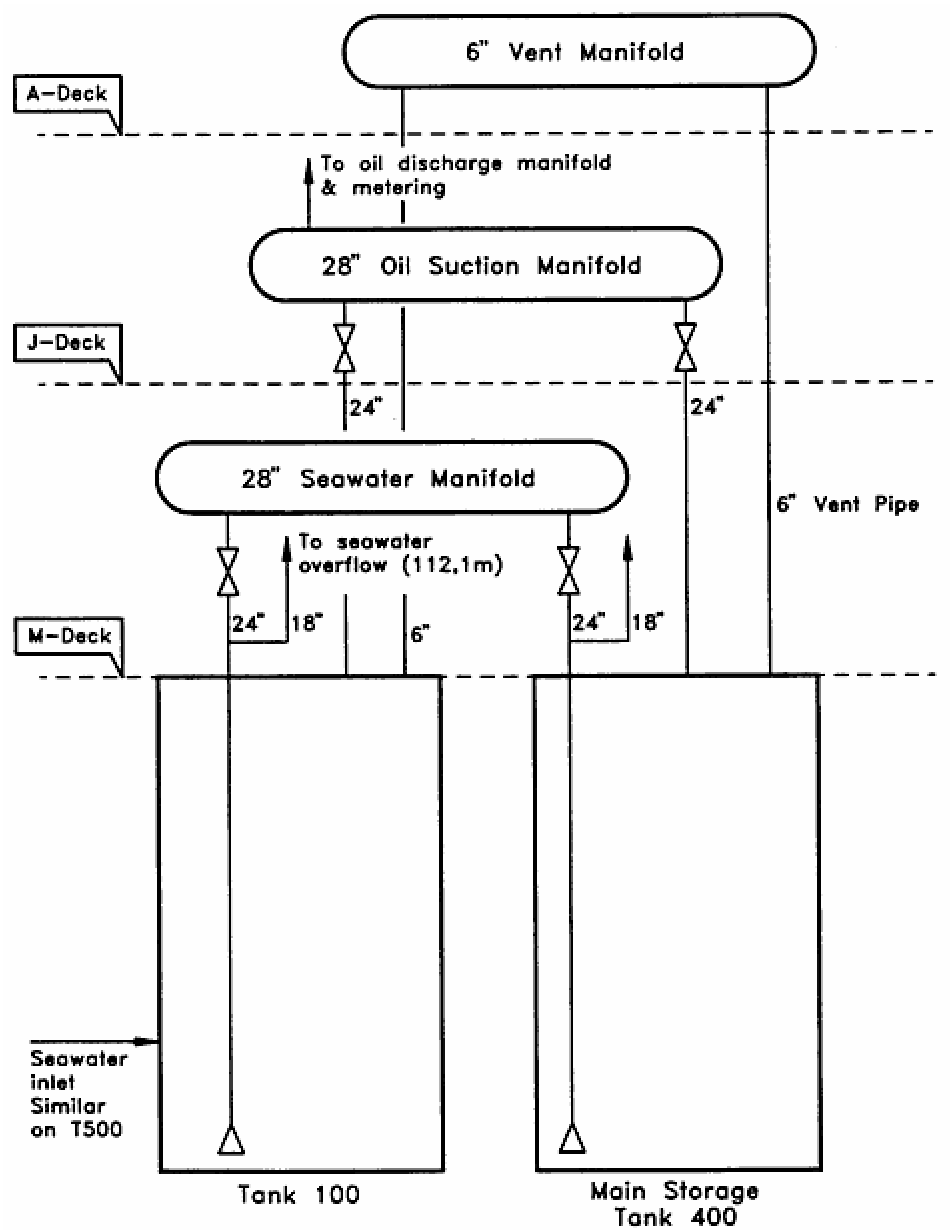


Figure 2.2. Schematic of Brent Spar's Storage Tanks with Main Piping System (Shell 2002)

2.1.2 Gravity Based Structures

Since there are only a few historical cases of oil storage on spars, data from gravity based structures (GBS) was also used. Gravity based structures have been used throughout the world as platforms for drilling and production and/or storage.

An example of oil storage on a GBS is the Maureen, depicted in Figure 2.3, which was a steel gravity platform installed in 1983 and operated in the North Sea. This structure functioned in a similar manner as the Brent Spar for oil storage, except it rested on the sea floor. Maureen had a storage capacity of 650,000 bbls and also used wet storage. It also incorporated the use of gas pressure to keep the oil at the desired level for ballast (Agostoni, 1985). In 2001, the Maureen GBS was decommissioned after 18 years of successful service.

Figure 2.3. Maureen GBS with Three Oil Storage Tanks (Phillips 66 2002)

2.2 Oil Storage

There are different methods used to store oil. One method is “dry” storage, which consists of storing the oil in a dry tank within the hull. This method is not typically used for a spar since the storage tanks also act as ballast for the structure. Therefore, the stability of the spar is affected as the tanks are continually loaded and offloaded with crude oil. This is the current method used on tanker-based FPSO systems.

The other method is “wet” storage, which stores the oil with seawater. During production, the crude oil displaces the water, whereas during offloading, the storage compartments are refilled with seawater (Halkyard, 2000). This method, also used for gravity based structures in the North Sea, allows for greater control over the stability of the structure by incorporating the oil storage into its ballasting procedure. Consideration must also be made for the difference in specific gravity between the oil and seawater, which affects the spar’s buoyancy.

A problem with the wet storage method is the inability to visually inspect the storage tanks for corrosion. Nevertheless, experience from the Brent Spar and gravity based structures storing oil indicates that there is little or no corrosion on the inside of the tanks exposed to the oil. Areas exposed to the seawater that are susceptible to corrosion can be cathodically protected (Bax, 1974).

2.3 Available Experience and Data

The following presents data collected on past risk analyses conducted on offshore operations in the North Sea and the Gulf of Mexico. Additional data gathered for this study’s risk analysis is also presented.

2.3.1 North Sea

According to the Worldwide Offshore Accident Databank Statistical Report for 1998, for years 1980-1997 and the entire North Sea, there have been 627 reported spills from fixed units, including gravity based structures, GBS (DNV, 1999). This results in an average spill occurrence of 8×10^{-3} per unit per year for fixed units. Note, a spill includes crude oil, gas, chemicals, etc.

2.3.1.1 Norwegian Sector

Information on the Norwegian sector will be presented in terms of past risk analysis studies conducted by outside sources and then information collected for this study.

2.3.1.1.1 Past Studies

Norway has a considerable amount of past experience in the North Sea with gravity based structures. A study conducted by Vinnem (1998) on possible risks from offshore operations on the Norwegian Continental Shelf in the North Sea showed that the main contributors to the risk of oil spills are from shuttle tankers, GBS storage, and blowouts. Spar storage was not specified since Norway does not currently have any spars with storage operating in the North Sea. However, this type of oil storage could also be grouped with GBS storage. This study also proposed that the following are the main contributors to leaks from GBS storage cells:

- Puncture of one of the cells, probably most typically due to falling items
- Limited structural failure of a cell wall
- Operational errors causing some kind of overflow.

The latter contributor to oil spills may be more pronounced for wet storage than dry oil storage.

Only one major spill has occurred due to GSB storage. The spill occurred in 1977 in the UK sector of the North Sea resulting in a release of 4000 bbl (Health and Safety Executive, 1995). Since then standards have been implemented to increase the safety and to reduce the events producing/leading to pollution in the offshore community.

According to the study conducted by Vinnem (1998) on the Norwegian Continental Shelf, it has been estimated that the total number of GBS structure years, calculated as the sum of the operational years for a given time period for all installations, ranges from 200-300 for the entire North Sea between 1975 and 1999. This figure excludes the then recently installed Hibernia. Based on the fact that only one major spill has occurred between 1975 and 1999, an oil spill frequency around $4x10^{-3}$ per GBS structure year is estimated. However, this may slightly over-predict the rate of occurrence, thus the frequency in the report was reduced to $1x10^{-3}$ per GBS structure year to represent the current/updated offshore technology. This implies a spill for every 1000 GBS structure years. The study notes that for a GBS structure, if a spill were to occur, it would most likely be in the range of 6,000-60,000 bbl or 60,000-600,000 bbl. Furthermore, it was assumed that a spill occurring between 6,000-60,000 bbl is 9 times more likely than a spill between 60,000-600,000 bbl.

2.3.1.1.2 Gathered Data

A database on acute (accidental) offshore crude oil spills for 1990-September 2001 from the Norwegian Pollution Control Authority was analyzed for this report. The database contained the name of the offshore installation, date, amount spilled, and the possible cause. Possible causes for the spill were not given for all spills listed, thus they could not be differentiated into the various areas of the offshore platform's operations (i.e. due to production system or due to transportation system).

This database was further subdivided based on structure type. All spills resulting from gravity based structures were analyzed and grouped according to year. Each year was then further subdivided into spill size ranges, where the total number of spills, total volume spilled, and the average volume spilled were recorded (Table 2.1).

The data on oil spills from GBS could not be further subdivided based on activity. Therefore, all spills listed might not have occurred solely due to GBS storage, but could have been due to loading/offloading of storage operations or other activities, such as drilling or production.

Knowing the number of oil spills, the corresponding exposure can be determined in terms of oil produced, offloading lifts, or oil stored. Information regarding crude oil production and storage in the North Sea for Norway was received from the Norwegian Petroleum Directorate. Crude oil production, shown in Table 2.2, was listed according to structure name and per month for years 2000-2001. Only information from gravity-based structures was compiled from the given database.

Table 2.1. Crude Oil Spills from Gravity Based Structures in the North Sea for the Norwegian Sector

(Source: Norwegian Pollution Control Authority)																
			1 - 10 bbl			10 - 100 bbl			100 - 1,000 bbl			1,000 - 10,000 bbl			10,000 - 100,000 bbl		
Year	Total Number of Spills	Total Volume Spilled	Number of Spills	Total Volume Spilled (bbl)	Average Volume Spilled (bbl)	Number of Spills	Total Volume Spilled (bbl)	Average Volume Spilled (bbl)	Number of Spills	Total Volume Spilled (bbl)	Average Volume Spilled (bbl)	Number of Spills	Total Volume Spilled (bbl)	Average Volume Spilled (bbl)	Number of Spills	Total Volume Spilled (bbl)	Average Volume Spilled (bbl)
1990	23	58	22	43	2	1	16	16	0	0	0	0	0	0	0	0	0
1991	12	219	8	31	4	3	51	17	1	138	0	0	0	0	0	0	0
1992	10	5740	5	8	2	4	106	27	0	0	0	1	5625	0	0	0	0
1993	34	137	29	66	2	5	71	14	0	0	0	0	0	0	0	0	0
1994	13	66	12	53	4	1	13	13	0	0	0	0	0	0	0	0	0
1995	18	123	15	39	3	3	84	28	0	0	0	0	0	0	0	0	0
1996	9	41	8	29	4	1	12	12	0	0	0	0	0	0	0	0	0
1997	9	59	7	22	3	2	37	18	0	0	0	0	0	0	0	0	0
1998	6	59	5	22	4	1	38	38	0	0	0	0	0	0	0	0	0
1999	2	8	2	8	4	0	0	0	0	0	0	0	0	0	0	0	0
2000	7	27	7	27	4	0	0	0	0	0	0	0	0	0	0	0	0
TOTAL	**143**	**6536**	**120**	**346**	**3**	**21**	**427**	**20**	**1**	**138**	**138**	**1**	**5625**	**5625**	**0**	**0**	**0**

Table 2.2. Crude Oil Production from Gravity Based Structures in the North Sea for the Norwegian Sector

Crude Oil Production (bbl) 2000-2001 (Source: Norwegian Petroleum Directorate)								
Month	**DRAUGEN A**	**GULLFAKS A**	**GULLFAKS B**	**GULLFAKS C**	**OSEBERG A**	**STATFJORD A**	**STATFJORD B**	**STATFJORD C**
1/1/2000	5,438,200	2,644,588	2,595,150	3,004,325	6,782,006	1,603,394	2,671,294	2,850,063
2/1/2000	5,843,731	2,609,256	2,514,781	2,813,513	6,300,875	1,843,275	2,530,956	2,971,600
3/1/2000	6,165,538	2,351,519	2,200,544	2,930,563	7,207,638	1,608,200	2,314,975	2,825,681
4/1/2000	6,589,425	2,533,206	2,310,538	974,606	6,456,425	1,626,325	2,501,419	2,301,825
5/1/2000	6,652,225	2,665,919	2,352,688	2,804,006	6,372,600	1,431,425	2,171,719	2,803,881
6/1/2000	3,667,075	2,548,231	2,272,738	2,980,506	7,090,313	1,472,606	2,461,113	567,681
7/1/2000	6,691,531	2,511,994	2,245,156	3,024,819	7,944,450	1,732,125	2,541,069	2,622,263
8/1/2000	6,815,531	815,975	803,131	2,947,069	7,180,175	1,686,625	2,945,319	2,261,069
9/1/2000	6,175,175	2,039,213	2,111,394	2,707,931	7,688,300	849,200	2,882,481	2,243,031
10/1/2000	6,699,819	1,849,375	2,231,594	1,253,875	8,193,125	1,457,619	2,747,344	2,294,363
11/1/2000	6,541,650	1,784,656	2,251,319	2,557,038	8,267,281	1,525,519	2,490,456	2,291,113
12/1/2000	6,121,144	1,775,550	2,170,163	2,766,944	8,897,869	1,593,588	1,916,775	2,394,619
Sum - Year 2000	**73,401,044**	**26,129,481**	**26,059,194**	**30,765,194**	**88,381,056**	**18,429,900**	**30,174,919**	**28,427,188**
Average - Year 2000	**6,116,754**	**2,177,457**	**2,171,599**	**2,563,766**	**7,365,088**	**1,535,825**	**2,514,577**	**2,368,932**
1/1/2001	6,845,919	3,613,744	1,884,744	2,641,813	9,201,738	1,596,831	2,417,375	2,273,575
2/1/2001	4,000,881	3,497,131	1,707,913	2,313,531	7,396,613	1,377,825	1,742,625	1,968,456
3/1/2001	6,753,356	3,519,288	1,807,881	2,514,069	7,842,900	1,364,081	2,172,663	2,284,150
4/1/2001	6,554,494	3,662,550	1,823,706	2,548,831	8,232,150	1,492,350	2,333,469	2,208,313
5/1/2001	3,482,488	3,561,081	1,838,031	792,475	8,383,738	1,325,925	2,269,013	2,329,588
6/1/2001	6,612,150	3,611,688	1,785,494	2,189,294	5,450,150	1,496,831	489,838	2,362,344
7/1/2001	6,808,406	3,810,869	1,723,256	2,089,744	8,197,575	1,510,975	2,234,744	2,495,144
8/1/2001	6,823,969	2,567,638	1,486,894	2,321,644	8,932,338	1,632,600	2,211,863	2,471,069
9/1/2001	6,450,581	3,958,150	1,609,825	2,084,281	9,224,094	1,437,650	2,118,400	2,223,350
10/1/2001	6,808,194	3,975,000	1,669,669	2,418,963	8,886,638	1,247,800	2,274,063	2,456,706
11/1/2001	6,143,819	2,860,856	1,054,300	2,561,894	9,148,756	1,290,188	2,223,181	2,198,706
12/1/2001	6,835,669	4,065,244	1,580,063	2,669,044	9,039,875	1,305,569	2,346,456	2,149,906
Sum - Year 2001	**74,119,925**	**42,703,238**	**19,971,775**	**27,145,581**	**99,936,563**	**17,078,625**	**24,833,688**	**27,421,306**
Average - Year 2001	**6,176,660**	**3,558,603**	**1,664,315**	**2,262,132**	**8,328,047**	**1,423,219**	**2,069,474**	**2,285,109**

The database for crude oil storage, shown in Table 2.3, was listed according to field name and per month for years 2000-2001. However, the values reported were based on the amount in storage at the end of the month. This does not represent the total amount of crude oil that passes through the storage tanks per month. Also, the database reports the volume by field, instead of by structure. Therefore, this value encompasses all structures producing oil for that field.

Table 2.3. Crude Oil Storage from Gravity Based Structures in the North Sea for the Norwegian Sector

Crude Oil in Tank Stock at End of Month (bbl) (Source: Norwegian Petroleum Directorate)			
Month	**DRAUGEN**	**GULLFAKS**	**STATFJORD**
1/1/2000	868,056	2,371,838	2,994,575
2/1/2000	1,376,875	3,085,506	3,941,156
3/1/2000	95,794	559,900	2,407,644
4/1/2000	211,469	1,335,063	2,013,281
5/1/2000	199,175	1,038,675	3,062,244
6/1/2000	533,519	1,452,913	2,428,394
7/1/2000	519,206	1,740,413	2,067,106
8/1/2000	861,006	1,068,813	2,331,631
9/1/2000	433,356	1,714,656	2,495,450
10/1/2000	563,644	1,897,088	1,800,519
11/1/2000	321,406	2,510,706	3,516,038
12/1/2000	610,300	2,460,138	2,766,538
Average	**549,484**	**1,769,642**	**2,652,048**
1/1/2001	775,150	2,878,331	2,441,819
2/1/2001	727,594	1,077,325	2,078,050
3/1/2001	431,544	1,355,563	2,689,238
4/1/2001	657,056	1,864,838	2,240,863
5/1/2001	414,813	1,330,881	2,184,788
6/1/2001	694,138	2,876,519	2,785,581
7/1/2001	816,063	1,100,506	2,053,750
8/1/2001	100,831	1,914,094	2,122,781
9/1/2001	605,063	3,165,506	2,669,694
10/1/2001	815,663	1,400,844	3,207,475
11/1/2001	717,113	2,796,238	3,139,600
12/1/2001	825,938	1,198,231	2,066,963
Average	**631,747**	**1,913,240**	**2,473,383**

Information on the amount of oil stored was difficult to obtain. The following GBS structures were determined from Norwegian operations in the North Sea: the Draugen A; Gullfaks A, B, and C; Oseberg A; and Statfjord A, B, and C. Information on crude oil storage was only available for the Draugen, Gullfaks, and Statfjord fields.

The total amount offloaded for the month would then be equal to the volume produced from all platforms in the field for the entire month minus the volume stored at the end of the month. Therefore, if the average size of the storage tank of the shuttle tanker is known, then the number of times the structure was offloaded in a month can be determined.

If the maximum volume of oil storage for the GBS is known, then the minimum number of offloadings can be determined based on oil production. For example, the Draugen and Gullfaks have a storage capability of 1,250,000 and 3,750,000 bbl, respectively; and produce approximately 6,600,000 and 7,000,000 bbl/month, respectively. This would result in six and two offloadings/month for the Draugen and Gullfaks, respectively. If an average shuttle tanker of 600,000 bbl were assumed, then there would be approximately 24 docking calls per month per structure, including offloading at the structure offshore and offloading on shore, for each GBS structure. This implies an approximate spill frequency of 1×10^{-3} per GBS per year for spills greater than 1,000 bbl.

2.3.1.2 United Kingdom Sector

Information on the United Kingdom sector will also be presented in terms of past risk analysis studies conducted by outside sources and then information collected for this study.

2.3.1.2.1 Past Studies

A study performed on the possible risks from offshore operations on the United Kingdom's Continental Shelf (UKCS) in the North Sea reported that large spills are mainly attributed to spills from oil-based drilling mud, and spills from pipelines and oil storage (Health and Safety Executive (HSE), 1995). The HSE study reports a spill of 3300 bbls resulted from a flange that parted during loading in 1977, as well as the 4000 bbl spill from a leak in a storage cell. Oil spills resulting from crude loading and crude storage are shown in Table 2.4 as a partial listing of all oil spills on the UKCS originally presented in the HSE report. Based on this compilation of oil spill data, there were a total of 1250 spills from all activities between 1975 and 1989. Therefore, the frequency of oil spills from crude loading and crude storage are 0.07 and 0.04 per year, respectively.

Table 2.4. Oil Spills on the UKCS for 1975-1989 (HSE, 1995)

Spill Size Range (bbl)	Crude Loading	Crude Storage
< 0.7	0	0
0.7 - 2.1	15	3
2.1 - 7.0	10	10
7.0 - 21.0	23	14
21.0 - 70.0	16	3
70.0 - 210.0	3	2
210.0 - 700.0	2	0
700.0 - 2100.0	0	0
2100.0 - 7000.0	1	1
> 7000.0	0	1
Unknown	3	5
Total Number	73	39

Therefore, from 1975-1989, there have been three significant events on the UKCS involving loading/storage facilities. This implies a frequency around 0.3 per year for a significant spill, greater than 2100 bbl. Note the UK study used years and not platform-years as the measure of exposure. If this data were extrapolated for the next ten years, then the frequency should be adjusted to incorporate for improved technology since two of the significant oil spills were known to occur in 1977.

Overall, the study on the UKCS predicts a spill over 68 bbl to occur approximately twice a year and a spill over 6800 bbl to occur about once every 5 years within the North Sea.

2.3.1.2.2 Gathered Data

Information on crude oil spills for the United Kingdom in the North Sea was received from the United Kingdom's Maritime and Coastguard Agency. The data was presented in the Advisory Committee on Protection of the Sea (ACOPS) Annual Reports. These reports were only available for the years 1995-2000. From this data, only year 2000 had a list of all spills reported for that year. Evaluation of this list showed that all spills from gravity based structures were less than 4 bbl. From 1995-1999, there did not appear to be any spills greater than 13 bbl from gravity based structures or spars. This data does not include any information of oil spills from the Brent Spar since it was decommissioned in 1991.

2.3.2 Gulf of Mexico

Information on the frequency of oil spills from operations related to transportation of oil from spars was gathered from the report "Comparative Risk Analysis for Deepwater Production Systems", by Gilbert et al (2001a). Frequencies for spar-export riser and spar-pipeline used the same values and the frequencies for spar-storage and spar-shuttle tanker utilized those for FPSO-storage and FPSO-shuttle tanker. Tables of these values are presented in Tables 2.5, 2.6, 2.7, and 2.8 for spar-export pipeline risers, spar-pipelines, spar-shuttle tanker, and spar- storage, respectively.

These frequencies from the Gulf of Mexico were compared to those from the North Sea and were determined to be similar. In terms of storage, the data from the North Sea determined a spill frequency of 1.0 X 10^{-3} and 1.0 X 10^{-4} per GBS structure year to represent the range of 6000 – 60,000 bbl and 60,000 – 600,000 bbl, respectively. For the Gulf of Mexico, spill frequencies of 1 x 10^{-4} per year for 1,000 – 10,000 bbl and 10,000 – 100,000 bbl and 1 x 10^{-5} per year for 100,000 – 500,000 bbl and 500,000 – 1,000,000 bbl were determined. These differ by an order of magnitude, reflected the limited amount of data on spills from oil storing structures offshore in the Gulf of Mexico. Therefore, the Gulf of Mexico frequencies were used since they represent the Gulf of Mexico and the results from this study can be compared in the future to those from that report.

Table 2.5. Analysis Input Spar-Export Pipeline Riser

Spar - Export Pipeline Risers

Exposure (riser-years)	**20**

		Combined (Expert+Data)		Expert-Based		Data-Based Estimate	
Spill Size Range (bbl)	**E(Consequence) (bbl)**	**Expected Frequency (per riser-year)**	**Coefficient of Uncertainty for Frequency**	**Expected Frequency (per riser-year)**	**Coefficient of Uncertainty for Frequency**	**Expected Frequency (per riser-year)**	**Coefficient of Uncertainty for Frequency**
1-10	3.9	2.0E-03	1.15	1.0E+00	1.00	2.0E-03	0.58
10-100	39	1.8E-03	1.15	1.0E+00	1.00	1.8E-03	0.58
100-1000	391	6.8E-04	1.15	1.0E+00	1.00	6.8E-04	0.58
1000-10,000	3,909	6.8E-04	1.15	1.0E+00	1.00	6.8E-04	0.58
10,000-100,000	21,968	1.4E-04	1.00	1.0E+00	1.00	1.4E-04	0.00
100,000 - 500,000	N/A	0.0E+00	0.00	0.0E+00	0.00	0.0E+00	0.00
500,000 - 1,000,000	N/A	0.0E+00	0.00	0.0E+00	0.00	0.0E+00	0.00

Table 2.6. Analysis Input Spar-Pipeline

Spar - Pipeline

Exposure (mile-years)	**2900**

		Combined (Expert+Data) Estimate		Expert-Based Extrapolation Bias		Data-Based Estimate	
Spill Size Range (bbl)	**E(Consequence) (bbl)**	**Expected Frequency (per mile-year)**	**Coefficient of Uncertainty for Frequency**	**Expected Frequency (per mile-year)**	**Coefficient of Uncertainty for Frequency**	**Expected Frequency (per mile-year)**	**Coefficient of Uncertainty for Frequency**
1-10	3.9	3.7E-04	0.52	1.0E+00	0.33	3.7E-04	0.40
10-100	39	3.2E-04	0.53	1.0E+00	0.33	3.2E-04	0.41
100-1000	391	1.2E-04	0.64	1.0E+00	0.33	1.2E-04	0.55
1000-10,000	3,909	1.2E-04	0.64	1.0E+00	0.33	1.2E-04	0.55
10,000-100,000	21,968	2.5E-05	1.13	1.0E+00	0.33	2.5E-05	1.08
100,000 - 500,000	N/A	0.0E+00	0.00	1.0E+00	0.00	0.0E+00	0.00
500,000 - 1,000,000	N/A	0.0E+00	0.00	1.0E+00	0.00	0.0E+00	0.00

Table 2.7. Analysis Input Spar-Shuttle Tanker

Spar - Shuttle Tanker

Exposure (docking calls)	**3049**

		Combined (Expert+Data) Estimate		Expert-Based Extrapolation Bias		Data-Based Estimate	
Spill Size Range (bbl)	**E(Consequence) (bbl)**	**Expected Frequency (per docking call)**	**Coefficient of Uncertainty for Frequency**	**Expected Frequency (per docking call)**	**Coefficient of Uncertainty for Frequency**	**Expected Frequency (per docking call)**	**Coefficient of Uncertainty for Frequency**
1-10	3.9	5.4E-04	0.51	1.0E+00	0.33	5.4E-04	0.39
10-100	39	2.0E-04	0.62	1.0E+00	0.33	2.0E-04	0.52
100-1000	391	1.4E-04	0.68	1.0E+00	0.33	1.4E-04	0.60
1000-10,000	3,909	3.5E-05	1.13	1.0E+00	0.33	3.5E-05	1.08
10,000-100,000	39,087	4.7E-06	1.16	1.0E+00	0.33	4.7E-06	1.11
100,000 - 500,000	167,288	3.1E-06	1.20	1.0E+00	0.33	3.1E-06	1.15
500,000 - 1,000,000	N/A	0.0E+00	0.00	0.0E+00	0.00	0.0E+00	0.00

Table 2.8. Analysis Input Spar-Storage

Spar - Storage

Exposure (years)	20

Spill Size Range (bbl)	E(Consequence) (bbl)	Combined (Expert+Data) Estimate: Expected Frequency (per year)	Combined (Expert+Data) Estimate: Coefficient of Uncertainty for Frequency	Expert-Based Extrapolation Bias: Expected Frequency (per year)	Expert-Based Extrapolation Bias: Coefficient of Uncertainty for Frequency	Data-Based Estimate: Expected Frequency (per year)	Data-Based Estimate: Coefficient of Uncertainty for Frequency
1-10	N/A	0.0E+00	0.00	1.0E+00	0.00	0.0E+00	0.00
10-100	N/A	0.0E+00	0.00	1.0E+00	0.00	0.0E+00	0.00
100-1000	N/A	0.0E+00	0.00	1.0E+00	0.00	0.0E+00	0.00
1000-10,000	3,909	9.0E-05	1.41	4.5E-02	1.00	2.0E-03	1.00
10,000-100,000	39,087	9.0E-05	1.41	4.5E-02	1.00	2.0E-03	1.00
100,000 - 500,000	248,534	1.0E-05	1.41	5.0E-03	1.00	2.0E-03	1.00
500,000 - 1,000,000	721,348	1.0E-05	1.41	5.0E-03	1.00	2.0E-03	1.00

2.4 Methods of Analysis

The method of analysis for the evaluation of the risk involved with oil storage on a spar is similar to that undertaken by Gilbert (2001a) and Jaber (2000) involving oil storage on Floating Production Storage and Offloading (FPSO) structures in the Gulf of Mexico. In their study, they utilized historical data and expert opinion based data. The expert opinion (based on expertise from industry and related governmental agencies) was used to refine the historical data collected. Their knowledge was also used to judge the historical data in regards to current practices.

The historical data and expert opinion were then combined to determine the expected rate of oil spill occurrences for four different offshore structures. From past studies, they decided that a systems risk would be studied by subsystem (risk from riser, risk from pipelines, etc.) instead of the entire system. In order to quantify the risk, relevant exposure factors were determined for each subsystem. Table 2.9 shows the exposure factors used for each subsystem to evaluate and quantify the risk to the environment.

Table 2.9. Exposure Factors Used to Evaluate the Risk to the Environment (Jaber, 2000)

Subsystem	Measure of Exposure
Well Systems	Volume produced (bbl)
Risers	Number of riser-years
Pipelines	Mile-years
Topsides	Volume produced (bbl)
Shuttle Tanker and Offshore Support Vessels	Number of docking calls per year

Each subsystem used a separate normalizing factor or measure of exposure to ensure an accurate representation of the system and to provide the ability to compare the different subsystems. For example, the magnitude of a spill has a direct relationship to the amount of oil that is possible to be spilled from that subsystem. Each subsystem has its own separate means in which oil could be spilled. For example, the subsystem pipeline has many miles of pipelines that could be damaged or punctured, resulting in a release of oil. Whereas every time a shuttle tanker offloads oil from a spar, the chance of a spill increases. Spar storage and export risers are normalized by the operational lifetime in years. Table 2.10 summarizes the exposure factors applicable for this study.

Table 2.10. Exposure Factors for Spar Subsystems

Subsystem	Measure of Exposure
Export Risers	Years
Pipeline	Mile-Years
Shuttle Tanker	Number of Docking Calls per Year
Storage	Years

A Poisson distribution was assumed to be representative of the frequency of oil spill occurrences. Based on this distribution, analytical values were determined for the expected number of oil spills. Once the number of oil spills was determined, the average volume of oil spilled was calculated based on an assumed volume of oil spilled. In their study, the results proved to be reasonable. However, they did not allow for the randomness of a spill size and the distribution representing the spill size for the largest spill size range was constrained by assuming it to be triangular.

2.5 Conclusions

There has been successful storage of oil in spars and gravity based structures in the North Sea (other areas were not examined). Currently in the Gulf of Mexico, there are three spars, which drill and produce oil. However, they do not currently store oil although the capability to do so is there.

Data bases obtained for offshore operations in the North Sea gave an initial estimate for the number and size of spills from the different types of offshore structures. However, the data was not detailed enough to allow for the spills to be subdivided based on offshore operations or activities (e.g. storage, transportation via pipeline or shuttle tanker, etc.). In addition, it was difficult to locate information for various measures of exposure, such as the amount of oil stored per structure or the number of offloadings per month from a spar of GBS structure. Based on the North Sea data from spar and GBS structures with oil storage, there have been no significant spills in the past 10 years, indicating a spill frequency of 0.12 and 0.013 per structure year for 6,000-60,000 bbl or 60,000-600,000 bbl, respectively.

The process and frequencies used by Jaber (2000) and Gilbert et al (2001a) to evaluate the risk to the environment from an offshore structure will be implemented for analysis of oil storage on a spar. This method is reasonable since it examines a system by subsystems or operations, therefore providing reasonable measures of exposure to evaluate the number and size of oil spills.

Chapter 3: Model for Environmental Performance

This section will describe the development of a general model, based on the CRA model (Gilbert et al 2001a), for quantifying the environmental risk due to oil spills. Illustration of the model will be demonstrated using a spar with oil storage in regard to operations involved in the transportation of crude oil from the offshore facility. For a spar with oil storage, this includes the operations involving storage and shuttle tankers. This chapter considers a spar whose mean rate of spill occurrence and spill size distribution are assumed known based on limited historical data and expert opinion.

The model will be used to evaluate variations in the performance between individual spars and fleets of spars, i.e. the "average" spar. This will demonstrate that the typical spill volume for a spar or fleet of spars is significantly less than the average total volume spilled for an individual spar.

3.0 Development of Model

The quantitative risk to the environment is based on the volume of oil spilled or released accidentally into the environment within its lifetime. The volume spilled is divided into two measures of the risk: 1) the expected total volume of oil spilled, representing the chronic environmental risk and 2) the expected maximum volume of oil spilled, representing the acute environmental risk (Gilbert et al 2001a). These expected volumes of oil spilled are determined from a fleet of spars operating in the Gulf of Mexico that are producing oil for a known lifetime. Within this lifetime, the average total volume of oil spilled and the average maximum spill size can be computed. For this analysis, only the chronic environmental risk will be evaluated; the acute environmental risk for a spar with oil storage will not be assessed.

The following will describe the process used to evaluate the chronic environmental risk, or the total volume spilled. According to Gilbert et al (2001b), the total volume spilled for a single spar can be expressed mathematically as:

$$\mathrm{TOTAL_C} = \sum_{i=0}^{X_{occur}} \mathrm{Consequence_i} \tag{3.1}$$

where $\mathrm{TOTAL_C}$ is the total volume of oil spilled over the lifetime of a spar, X_{occur} is the number of oil spill occurrences in the lifetime of the spar, and $\mathrm{Consequence_i}$ is the individual volume of oil spilled in each occurrence.

Therefore, if a single spar operating in the Gulf of Mexico for 20 years (lifetime) has ten spills (X_{occur}) with magnitudes (Consequence) of 415, 4, 62, 572, 18570, 1, 10, 13, 2791, and 56 bbl, then $\mathrm{TOTAL_C}$ is 22,494 bbl. Note, this is only a hypothetical case for one spar. It is just as possible to have a spar operating in the Gulf of Mexico for 20 years with a total volume spilled greater or less than the case presented.

Since the spill size can range over several orders of magnitude from several barrels to a million barrels, it is difficult to determine the average spill size for a spar. Therefore, the spill sizes are divided into spill size ranges, with a difference of one order of magnitude on a logarithmic scale. By subdividing the range of spill sizes, the uncertainty inherent in estimating the number of occurrences and the size of spills from historical data can be reduced. Equation 3.1 is then rewritten as:

$$TOTAL_C = \sum_{j=1}^{n} \sum_{i=0}^{(X_{occur})_j} (Consequence_i)_j \tag{3.2}$$

where n is the number of spill size ranges and j represents the spill size range, i.e. j equal to one indicates a spill size range from 1 to 10 bbl. Table 3.1 presents the hypothetical case presented previously, but with the spills subdivided based on magnitude.

Table 3.1. Hypothetical Spill Occurrences for One Spar

Spill Size Range (bbl)	Random Spill Size for Lifetime (bbl)				Σ(Consequence$_i$)
1 - 10	4	1			5
10 - 100	62	10	13	56	141
100 - 1,000	415	572			987
1,000 - 10,000	2791				2791
10,000 - 100,000	18570				18570
				TOTAL$_C$ =	22494

3.0.1 Modeling the Number of Occurrences

A Poisson distribution was assumed to represent the number of spill occurrences. Jaber (2000) used this same assumption in his assessment of the environmental risk associated with an FPSO (Floating, Production, Storage, and Offloading structure) in the Gulf of Mexico, which proved to be reasonable. The Poisson distribution is reasonable for this situation because of the distribution's assumptions:

a. The occurrence of a spill can occur at any time,
b. The occurrence of a spill in a given time interval is independent of another spill occurring in a non-overlapping time interval, and
c. The probability of a spill occurring in a given time interval is proportional to the length of the time interval, given the time interval is small (Ang and Tang, 1975).

Based on these assumptions, the probability that the number of occurrences is x is written mathematically as:

$$P_X(X = x) = \frac{(\nu\, t)^x}{x!} e^{-\nu\, t} \tag{3.3}$$

where X is a random variable for the number of occurrences, x is the actual number of occurrences, ν is the mean rate of occurrence, and t is the exposure or time interval.

The physical representation of this distribution also seems reasonable since the probability of occurrence is greater for a smaller number of occurrences and decreases as the number of occurrences increases. Figure 3.1 illustrates this relationship.

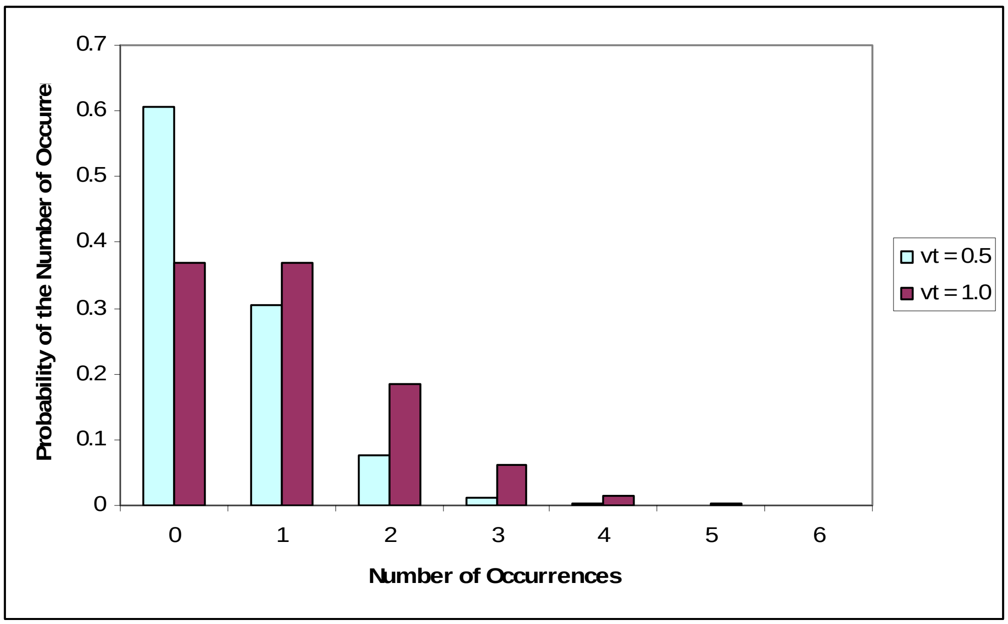


Figure 3.1. Illustration of the Poisson Distribution

The expected number of occurrences is computed analytically as:

$$E(X_{occur})_i = \nu_i t \tag{3.4}$$

where ν_i is the mean rate of occurrence or frequency of spills for each spill size range determined by multiplying the expected frequency by the exposure factor for the subsystem (Tables 2.5 thru 2.8), and t is the length of exposure (Gilbert et al, 2001b). The length of exposure, t, is the number of spars operating for an average 20 year lifetime. Therefore, t equal one represents one spar operating for 20 years and t equal two represents two spars operating for 20 years.

The variance in the number of spill occurrences in spill size range i, is expressed as:

$$Var(X_{occur\ i}) = \nu_i t \tag{3.5}$$

Table 3.2 displays a sample calculation for the subsystem of spar-shuttle tanker.

Table 3.2. Sample Calculations for the Mean and Variance of the Number of Occurrences for Spar-Shuttle Tanker

Spill Size Range (bbl)	ν	Exposure, t (per 20 year lifetime)	E(Xoccur)	Var(Xoccur)
1 - 10	1.65E+00	1	1.65E+00	1.65E+00
10 - 100	6.10E-01	1	6.10E-01	6.10E-01
100 - 1,000	4.27E-01	1	4.27E-01	4.27E-01
1,000 - 10,000	1.07E-01	1	1.07E-01	1.07E-01
10,000 - 100,000	1.43E-02	1	1.43E-02	1.43E-02
100,000 - 500,000	9.45E-03	1	9.45E-03	9.45E-03
500,000 - 1,000,000	0.00E+00	1	0.00E+00	0.00E+00

3.0.2 Modeling the Spill Size

By assuming a uniform distribution on a logarithmic scale (Figure 3.2), the actual spill size, x, has the same probability of occurring, whether its magnitude is small or large. Take for example a spill within the spill size range of 1 to 10 bbl, where on a logarithmic scale log(1) = 0 and log(10) = 1. A uniform distribution assumes that a spill of 2 bbl has the same chance of occurring as a spill of 9 bbl. As long as the spill size range is small enough, a uniform distribution is valid.

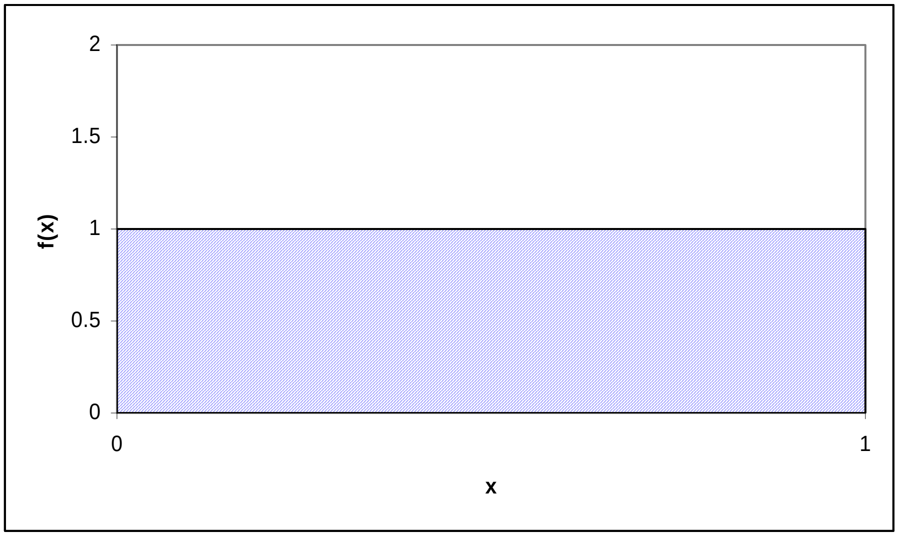
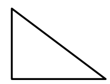
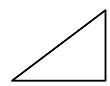


Figure 3.2. Standard Uniform Distribution

However, a uniform distribution does not accurately represent the largest spill size range. Therefore, all spill size ranges were modeled by a beta distribution, which allows for any distribution to exist based on the chosen statistical parameters, q and r. Three special cases of the beta distribution are the uniform and the two triangular distributions described in Table 3.3. When r is greater than q, the distribution is skewed to the left, and when r is less than q, the distribution is skewed to the right.

Table 3.3. Special Cases of the Beta Distribution

q	r	Resulting Distribution	
1	1	Uniform	
1	2	Triangular	
2	1	Triangular	

Spill size ranges that are uniform (generally all ranges except the largest) can be modeled by a beta distribution with both q and r equal to one. For the largest spill size range, expert opinion (Gilbert 2001a) and historical data agree that a smaller spill size (closer to the lower bound) has a greater probability of occurring; therefore, an expected spill size much less than that determined from a uniform distribution is expected. However, there still exists the chance of a larger spill (closer to the upper bound) occurring. In the CRA report, the largest spill size ranges for the subsystems of pipelines and shuttle tankers were both modeled to represent spills that tended toward the lower bound. For the case of storage industry agreed that all spill size ranges should be modeled by a uniform distribution. Examination of the distributions produced from changes in q and r with x representing the modeled parameter (i.e., the consequence or spill size), shown in Figure 3.3, indicates that for the analysis of the largest spill size range for all subsystems, excluding spar-storage, q should remain equal to one and r should be equal to 3.

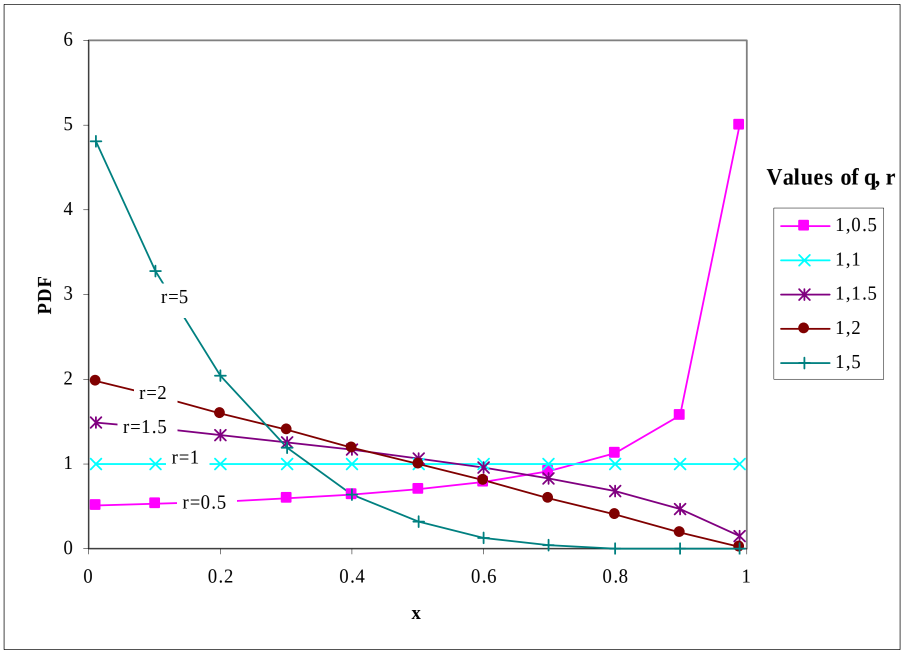


Figure 3.3. Standard Beta Distribution with q = 1 and r Varying

The expected total volume spilled for each spill size range is expressed analytically by the following:

$$E(Consequence) = \int_a^b \frac{\Gamma(q+r)}{\Gamma(q)\Gamma(r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q-r-1}} e^x dx \qquad (3.6)$$

where a = ln(lower bound) and b = ln(upper bound) of the spill size range and x = ln(spill size).

This equation reduces to the following for q equal to one:

$$E(Consequence) = \int_a^b \frac{\Gamma(1+r)}{\Gamma(1)\Gamma(r)} \frac{(b-x)^{r-1}}{(b-a)^r} e^x dx \qquad (3.7)$$

The variance of the distribution for each spill size range i is described as:

$$Var(Consequence)_n = \int_a^b \frac{\Gamma(1+r)}{\Gamma(1)\Gamma(r)} \frac{(b-x)^{r-1}}{(b-a)^r} (e^{x)})^2 dx - \left[\int_a^b \frac{\Gamma(1+r)}{\Gamma(1)\Gamma(r)} \frac{(b-x)^{r-1}}{(b-a)^r} e^x dx \right]^2 \tag{3.8}$$

Table 3.4 summarizes the expected spill sizes and their respective variances for each spill size range. Note, the column for q =1 and r =3 is only used for the largest spill size range in some subsystems.

Table 3.4. Summary of E(Consequence) and Var (Consequence)

	q = r = 1		q = 1, r = 3	
Spill Size Range (bbl)	**E(Consequence) (bbl)**	**Var (Consequence) (bbl^2)**	**E(Consequence) (bbl)**	**Var (Consequence) (bbl^2)**
1 - 10	3.9	6.22.E+00	N/A	N/A
10 - 100	39	6.22.E+02	N/A	N/A
100 - 1,000	391	6.22.E+04	N/A	N/A
1,000 - 10,000	3,909	6.22.E+06	N/A	N/A
10,000 - 100,000	39,087	6.22.E+08	21,968	2.16.E+08
100,000 - 500,000	248,534	1.28.E+10	167,288	5.23.E+09
500,000 - 1,000,000	721,348	2.07.E+10	612,562	1.05.E+10

3.1 Analytical Solution for an Individual Spar

The following are the equations representing the analytical approximation for the expected total volume spilled and the variance in the total volume spilled for an individual spar. This is an extension of the CRA model and will now allow for evaluation of the amount of variability between individual spars so that the range of possible performances can be evaluated that correspond to the "average" spar.

3.1.1 Expected Total Volume Spilled

The total volume spilled can be determined analytically for an individual spar assuming that the rate of spill occurrences and the spill size distribution are known by rewriting Equation 3.2 as the sum of the product of the expected number of occurrences multiplied by the expected spill size for all spill size ranges:

$$E(TOTAL_C) = \sum_{i=1}^{n} \left[E(X_{occur})_i * E(Consequence)_i \right] \qquad (3.9)$$

where $E(X_{occur})_i$ is the expected number of occurrences in spill size range i and $E(Consequence)_i$ is the expected spill volume in spill size range i.

Analytical calculations determining the expected total volume spilled for a spar with oil storage for the subsystem shuttle tanker are displayed in Table 3.5. Based on historical data and expert opinion for spar-shuttle tanker, the largest spill size range was determined to be 100,000 – 500,000 bbls.

Table 3.5. Analytical Calculations of $E(TOTAL_C)$ for Spar-Shuttle Tanker

Spill Size Range (bbl)	E(Xoccur)	E(Consequence) (bbl)	$E(TOTAL_C)$ (bbl)
1 - 10	1.65E+00	3.9	6
10 - 100	6.10E-01	39	24
100 - 1,000	4.27E-01	391	167
1,000 - 10,000	1.07E-01	3,909	417
10,000 - 100,000	1.43E-02	39,087	560
100,000 - 500,000	9.45E-03	167,288	1581
500,000 - 1,000,000	0.00E+00	0	0
		Sum	**2755**

3.1.2 Variance in the Total Volume Spilled

The uncertainty in the total volume spilled can be accounted for in terms of the variance (square of the standard deviation) of the total volume spilled. For the analytical solution, the following equation is used for an individual spar where the number of occurrences and the consequence are statistically independent:

$$Var(TOTAL_C) \cong \sum_{j=1}^{n}[E(X_{occur})_j Var(Consequence)_j + Var(X_{occur})_j E(Consequence)_j^2] \tag{3.10}$$

where $E(X_{occur\ i})$ and $Var(X_{occur\ i})$ are the expected number and the variance of the number of spill occurrences, respectively, and $E(Consequence_i)$ and $Var(Consequence_i)$ are the expected value and variance in the volume spilled due to the spill size distribution, respectively.

A summary of the analytical calculations of the variance is shown in Table 3.6 for a spar with oil storage for the subsystem shuttle tanker.

Table 3.6. Analytical Calculations of Var($TOTAL_C$) for Spar-Shuttle Tanker

Spill Size Range (bbl)	ν	E(Xoccur)	Var(Xoccur)	E(Consequence) (bbl)	Var (Consequence) (bbl^2)	Var($TOTAL_C$) (bbl^2)
1 - 10	1.65E+00	1.65E+00	1.65E+00	3.9	6.22.E+00	3.54E+01
10 - 100	6.10E-01	6.10E-01	6.10E-01	39	6.22.E+02	1.31E+03
100 - 1,000	4.27E-01	4.27E-01	4.27E-01	391	6.22.E+04	9.18E+04
1,000 - 10,000	1.07E-01	1.07E-01	1.07E-01	3,909	6.22.E+06	2.29E+06
10,000 - 100,000	1.43E-02	1.43E-02	1.43E-02	39,087	6.22.E+08	3.08E+07
100,000 - 500,000	9.45E-03	9.45E-03	9.45E-03	167,288	5.23.E+09	3.14E+08
500,000 - 1,000,000	0.00E+00	0.00E+00	0.00E+00	0	0.00.E+00	0.00E+00
					Sum	**3.47E+08**

3.1.3 Total Risk from Transportation of Oil

The total risk is comprised of all subsystems involved in the transportation of oil. As mentioned in Chapter 2, for a spar with oil storage, the method of transportation would also include the use of shuttle tankers. Originally the two subsystems were analyzed separately for ease in computations. The results from spar-storage and spar-shuttle tanker were then combined, simply by adding the results from the analytical equations for the total expected volume of oil spilled and the corresponding variance, as displayed in Table 3.7.

Table 3.7. Summary of Analytical Calculations per Spill Size Range for Spar with Oil Storage and Shuttle Tanker Transport

	Storage		Shuttle Tanker		Storage + Shuttle Tanker	
Spill Size Range (bbl)	**$TOTAL_C$ (bbl)**	**$Var(TOTAL_C)$ (bbl^2)**	**$TOTAL_C$ (bbl)**	**$Var(TOTAL_C)$ (bbl^2)**	**$TOTAL_C$ (bbl)**	**$Var(TOTAL_C)$ (bbl^2)**
1 - 10	0	0.00E+00	6.4	3.54E+01	6.4	3.54E+01
10 - 100	0	0.00E+00	24	1.31E+03	23.8	1.31E+03
100 - 1,000	0	0.00E+00	167	9.18E+04	166.8	9.18E+04
1,000 - 10,000	7	3.87E+04	417	2.29E+06	424.1	2.33E+06
10,000 - 100,000	70	3.87E+06	560	3.08E+07	630.5	3.47E+07
100,000 - 500,000	50	1.49E+07	1581	3.14E+08	1630.8	3.29E+08
500,000 - 1,000,000	144	1.08E+08	0	0.00E+00	144.3	1.08E+08
Total	**271**	**1.27E+08**	**2755**	**3.47.E+08**	**3027**	**4.74.E+08**

The variances due to the uncertainty in the spill size distribution or consequence and the number of occurrences are shown separately in Table 3.8 for an individual spar. This shows the variance due to the uncertainty in the consequence only contributes to 14% of the total variance of the total volume spilled, whereas the variance in the number of occurrences contributes 86% to the total variance. Therefore, this demonstrates that the uncertainty in the rate of spill occurrences plays a greater role in the total volume spilled than the actual spill size within a spill-size category. In addition, this also confirms that the subdivision of spill size ranges was small enough to be represented by separate distributions.

Table 3.8. Analytical Variance Terms Involved in the Variance of the Total Volume Spilled for an Individual Spar

	Storage		Shuttle Tanker		Storage + Shuttle Tanker	
Spill Size Range (bbl)	**Variance due to Consequence**	**Variance due to Occurrences of Spills**	**Variance due to Consequence**	**Variance due to Occurrences of Spills**	**Variance due to Consequence**	**Variance due to Occurrences of Spills**
1 - 10	0.00E+00	0.00E+00	1.02E+01	2.52E+01	1.02E+01	2.52E+01
10 - 100	0.00E+00	0.00E+00	3.79E+02	9.32E+02	3.79E+02	9.32E+02
100 - 1,000	0.00E+00	0.00E+00	2.65E+04	6.52E+04	2.65E+04	6.52E+04
1,000 - 10,000	1.12E+04	2.75E+04	6.64E+05	1.63E+06	6.75E+05	1.66E+06
10,000 - 100,000	1.12E+06	2.75E+06	8.91E+06	2.19E+07	1.00E+07	2.46E+07
100,000 - 500,000	2.56E+06	1.24E+07	4.94E+07	2.65E+08	5.20E+07	2.77E+08
500,000 - 1,000,000	4.13E+06	1.04E+08	0.00E+00	0.00E+00	4.13E+06	1.04E+08
Total	**7.82E+06**	**1.19E+08**	**5.90E+07**	**2.88E+08**	**6.69E+07**	**4.07E+08**

3.2 Analytical Solution for an Average Spar

The following equations are based on those for an individual spar, but are now averaged to represent an average spar operating in the Gulf of Mexico.

3.2.1 Expected Average Total Volume Spilled

The expected average total volume spilled can be expressed analytical by the following expression:

$$E(\overline{TOTAL_C}) = \frac{\sum_{k=1}^{n_{spar}} \sum_{i=1}^{n} \left[E(X_{occur})_i * E(Consequence)_i \right]_k}{n_{spar}} \qquad (3.11)$$

where n_{spar} is the number of spars operating in the fleet.

3.2.2 Variance in the Average Total Volume Spilled

The uncertainty in an average spar or for a fleet of spars is given by the following equation:

$$Var(\overline{TOTAL_C}) \cong \sum_{j=1}^{n} [E(X_{occur})_j Var(\overline{Consequence})_j + Var(\overline{X_{occur}})_j E(Consequence)_j^2]$$

(3.12)

where, the variance in the average consequence and the variance in the average number of occurrences is the variance in the consequence and the variance in the number of occurrences divided by the number of spars, n_{spar}, respectively:

$$Var(\overline{Consequence}) = \frac{Var(Consequence)}{n_{spar}} \tag{3.13}$$

$$Var(\overline{X_{occur\ i}}) = \frac{Var(X_{occur})}{n_{spar}} \tag{3.14}$$

Table 3.9 displays a sample calculation of the variance in the average total volume spilled for n_{spar} = 10. A plot of the effect of averaging spar performances in terms of the uncertainty is demonstrated in Figure 3.4. This reveals that as the number of spars within a fleet goes to infinity, then the variance in the average total volume spilled goes toward zero. This is also expressed by Equation 3.12. Therefore, if the mean rate of occurrences and the spill size distribution are known for certain, then there does not exist any uncertainty in the average total volume spilled.

Table 3.9. Analytical Calculations of $Var(\overline{TOTAL_C})$ with n_{spar} = 10 for Spar-Shuttle Tanker

Spill Size Range (bbl)	E(Xoccur)	Var(Average Xoccur)	E(Consequence) (bbl)	Var (Average Consequence) (bbl^2)	Var(Average $TOTAL_C$) (bbl^2)
1 - 10	1.65E+00	1.65E-01	3.9	6.22E-01	3.54E+00
10 - 100	6.10E-01	6.10E-02	39	6.22E+01	1.31E+02
100 - 1,000	4.27E-01	4.27E-02	391	6.22E+03	9.18E+03
1,000 - 10,000	1.07E-01	1.07E-02	3,909	6.22E+05	2.29E+05
10,000 - 100,000	1.43E-02	1.43E-03	39,087	6.22E+07	3.08E+06
100,000 - 500,000	9.45E-03	9.45E-04	167,288	5.23E+08	3.14E+07
500,000 - 1,000,000	0.00E+00	0.00E+00	0	0.00E+00	0.00E+00
				Sum	**3.47E+07**

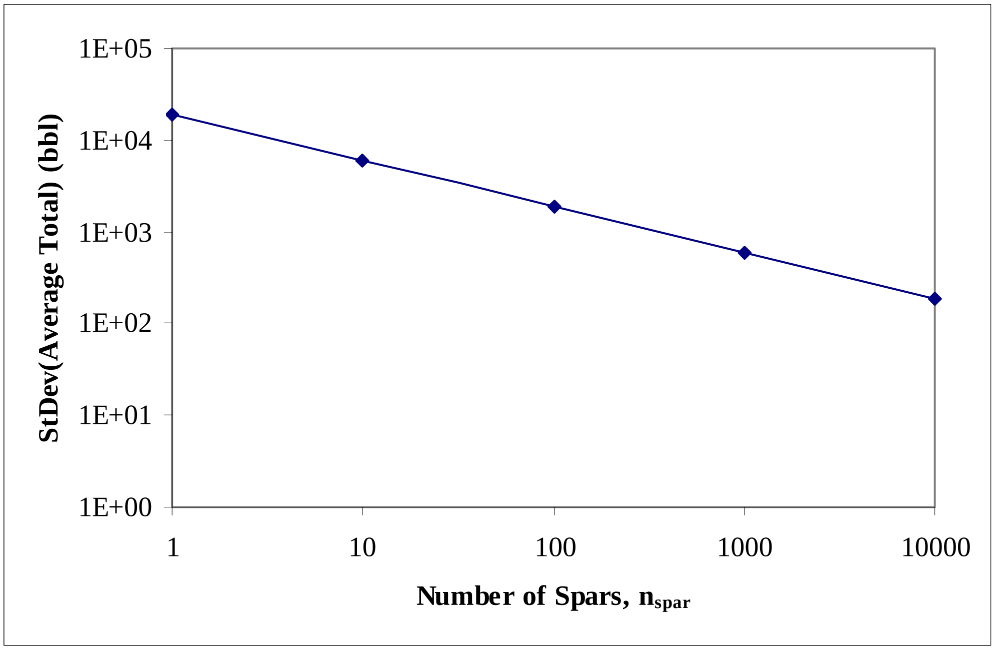


Figure 3.4. Effect of Averaging Spar Performances in Terms of the Uncertainty for Spar-Shuttle Tanker

The CRA model developed by Gilbert et al (2001a) represented the "average" spar since the mean rate of occurrence, ν, and the spill size distribution, q and r, were assumed known to represent an average spar. Thus, the variance in the total volume spilled was zero since the two parameters were known with certainty. The only uncertainty the CRA model presented was in the mean rate of occurrence, which will be discussed in Chapter 4. The initial model being presented here was for an individual spar and then adjusted to describe an average spar. The adjustment to the model for an average spar is the same as the CRA model since with a large enough fleet of spars, the variance in the total volume spilled will tend toward zero.

3.3 Numerical Solution – Monte Carlo Simulation

A numerical Monte Carlo simulation was performed to validate the accuracy of the analytical approximations and to illustrate what the results mean. Development of the Monte Carlo simulation was achieved by writing a Visual Basic® macro in Microsoft Excel®. This program simulated multiple spars operating in the Gulf of Mexico using the historical data compiled for the mean rate of occurrence, ν, and the spill size distribution. The number of occurrences and the spill sizes were simulated based on this data. A total of 100,000 realizations were conducted in order to verify the analytical results and for increased precision. A flowchart of the process used is presented in Figure 3.5 and the Visual Basic® code written for the simulations is contained in Appendix A.

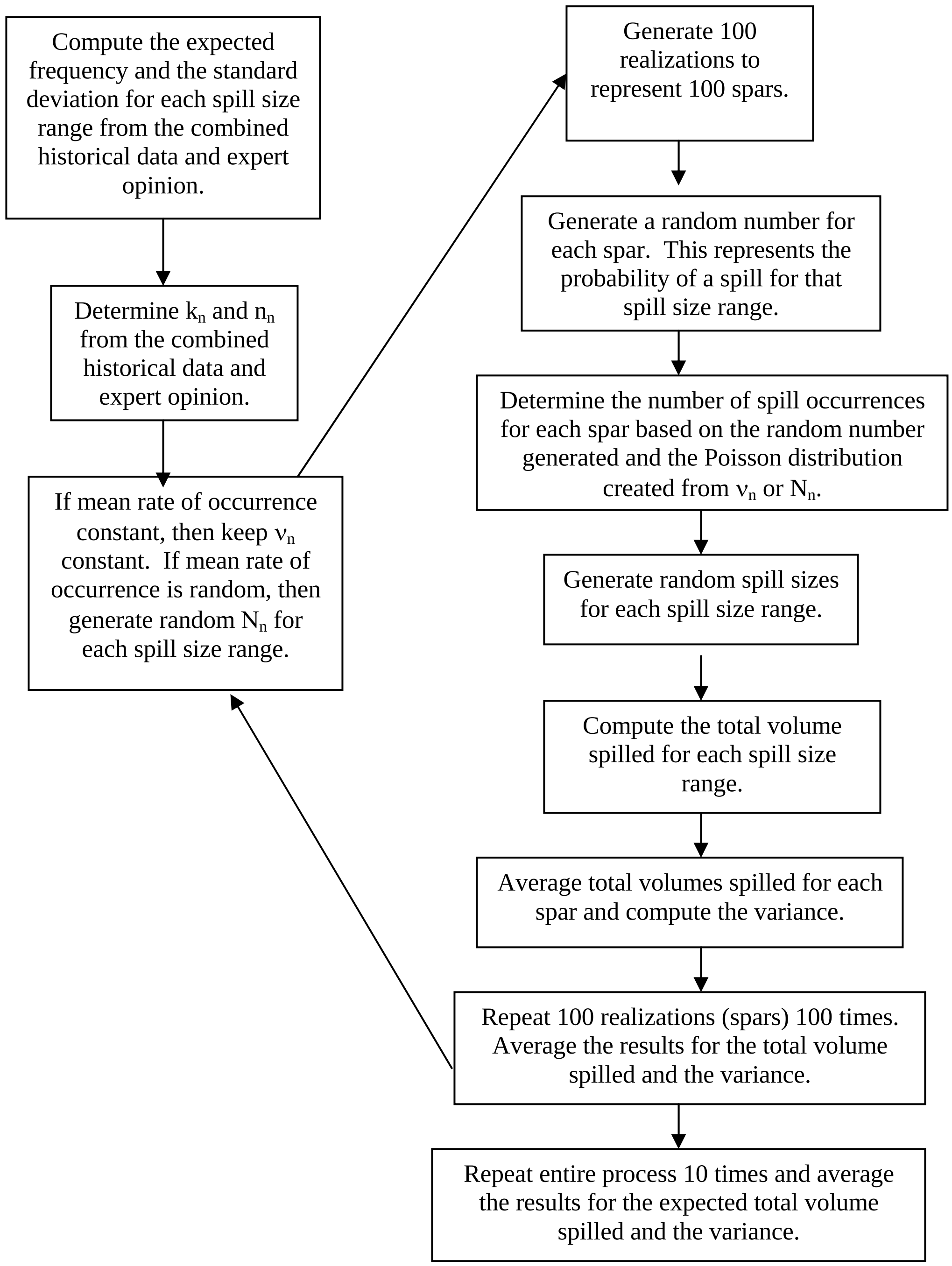


Figure 3.5. Flowchart of Monte Carlo Process

3.3.1 Simulating the Number of Occurrences

Using Monte Carlo simulation, the number of spill occurrences for a spar can be generated based on the Poisson distribution by the following procedure:

1) Generate a single realization, representing a single spar operating in the Gulf of Mexico for a lifetime.
2) Using Microsoft Excel®, create a random number between 0 and 1; this represents the probability that x number of spills will occur.
3) Create a lookup table to determine the number of spill occurrences based on the probability (random number). The lookup table is the cumulative probability density function of the Poisson distribution based on ν, the mean rate of occurrence and t, the length of exposure for the system.

For example, consider a spar with oil storage for the shuttle tanker subsystem. If random numbers of 0.164 and 0.472, representing the probability of x number of spills, are generated for the spill size range of 1 – 10 bbl, then the number of occurrences expected are one and two, respectively. The lookup table used to determine the number of occurrences is shown in Table 3.10 and the graphical representation in Figure 3.6. The number of occurrences is determined as the number corresponding to the cumulative density function, CDF, value that is less than the random number or probability of occurrence.

Table 3.10. Lookup Table for Spill Size Range 1 – 10 bbl using ν for Spar-Shuttle Tanker

CDF for x	Number of Occurrences, x
0	0
0.192731	1
0.510055	2
0.771285	3
0.914654	4
0.973666	5
0.993099	6
0.998431	7
0.999686	8
0.999944	9
0.999991	10
0.999999	11
1	12

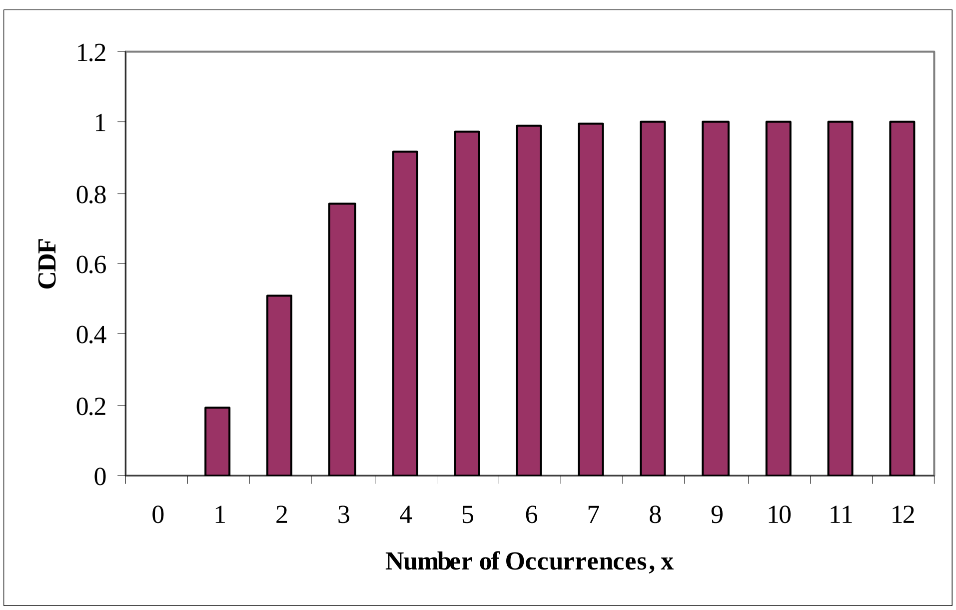


Figure 3.6. Graphical Representation of the Lookup Table for Spill Size Range 1 – 10 bbl for Spar-Shuttle Tanker

3.3.2 Simulating the Spill Sizes

The equation used to model the spill sizes for a uniform distribution (i.e., q = r = 1) with the Monte Carlo simulation is given as:

$$(Consequence_{\ i})_{j} = e^{[(rand \times (b-a))+a]} \tag{3.15}$$

where rand is a random number between 0 and 1 and a = ln(lower bound) and b = ln(upper bound). Therefore, if random numbers of 0.080 and 0.742 are generated for the spill size range of 10,000 – 100,000 bbl for one spar, then the magnitude of the spill sizes are 12,022 and 55,207 bbl, respectively. Thus, for one and two spill occurrences, the total volume spilled is 12,022 bbl and 67,230 bbl, respectively.

For spill size ranges where q = 1 and r = 3, then the consequence was modeled using the inverse beta function in Excel, based on a randomly generated number between 0 and 1. Using the same random numbers, 0.080 and 0.742, then the resulting spill sizes are 10,652 and 23,088 bbl, respectively. Note, these spill sizes are less than those using a uniform distribution. Table 3.11 displays an example of one realization for spar-shuttle tanker for all applicable spill size ranges. The spill sizes for 100,000 – 500,000 were generated based on beta distribution with q =1 and r = 3.

Table 3.11. Summary of Simulation for a Single Realization

Spill Size Range (bbl)	**ν**	**Xoccur**	**Spill Size (bbl)**			**$TOTAL_C$ (bbl)**
1 - 10	1.65E+00	3	2	1	7	10
10 - 100	6.10E-01	2	34	79	56	113
100 - 1,000	4.27E-01	0	523	174	811	0
1,000 - 10,000	1.07E-01	1	5,783	6,815	1,300	5,783
10,000 - 100,000	1.43E-02	0	48,410	62,693	26,739	0
100,000 - 500,000	9.45E-03	0	136,798	101,855	123,567	0
500,000 - 1,000,000	0.00E+00	N/A	N/A	N/A	N/A	0
					Sum	**5,906**

3.4 Comparison of Analytical and Numerical Solutions

A comparison of the analytical and numerical Monte Carlo results is shown in Table 3.12 for the expected total volume spilled and the standard deviation in the total volume spilled.

Table 3.12. Comparison of Analytical and Numerical Solutions for Spar-Shuttle Tanker

	$E(TOTAL_C)$ (bbl)	$StDev(TOTAL_C)$ (bbl)
Analytical	2755	18,631
Monte Carlo	2600	16,866

Results from the two methods of determining the expected total volume spilled and the standard deviation in the total volume spilled shows that they are similar. Therefore, the analytical approximations are reasonable.

In addition, the Monte Carlo simulation also provides a means of learning more about the performance of an individual spar that just its expected total volume spilled and the standard deviation. By plotting a histogram of the frequencies for the total volume spilled, shown in Figure 3.7, it is demonstrated that for spar-shuttle tanker a total volume spilled between 100 - 1,000 bbl is most prevalent. Note, this is an order of magnitude less than the expected total volume spilled for this subsystem. Also, the smaller total volumes and even zero total volumes spilled are more likely to be observed than a very large total volume spilled.

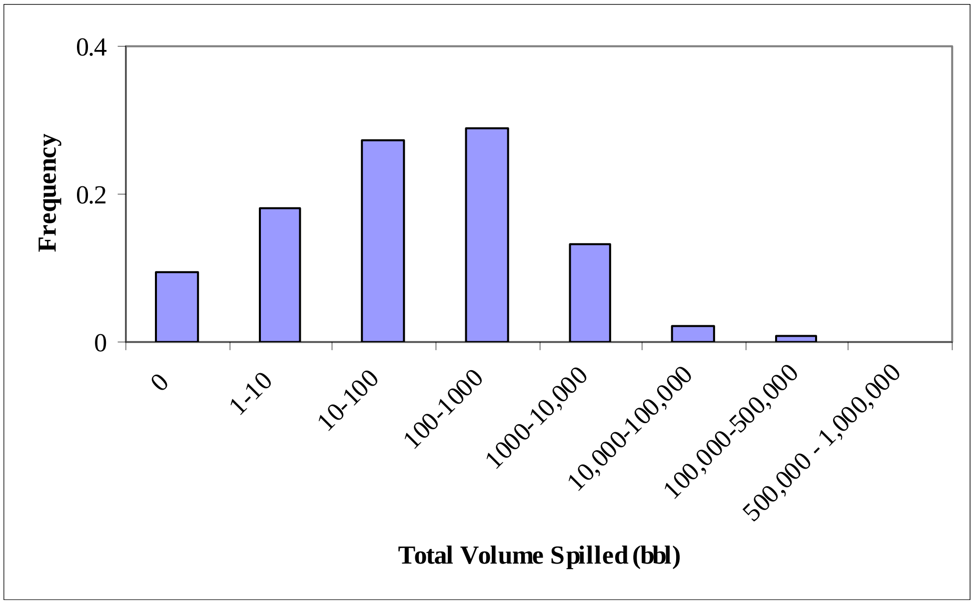


Figure 3.7. Frequency of Total Volume Spilled for an Individual Spar-Shuttle Tanker

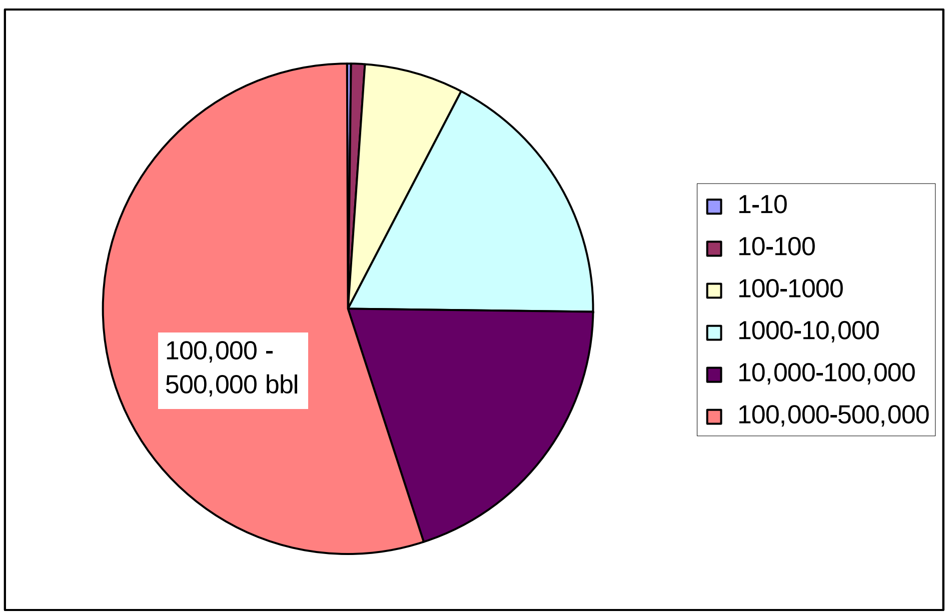


Figure 3.8. Contribution of Spill Size Range to Total Volume Spilled for an Individual Spar-Shuttle Tanker

The contribution of each spill size range to the total volume spilled is demonstrated in Figure 3.8. This demonstrates that the largest spill size range contributes the most to the expected total volume spilled, although it has the smallest rate of spill occurrences. On the contrary, the ranges which contain the most reliable information, the smallest spill size ranges, contribute the least. This reveals the need to acquire more knowledge on the rate of occurrences and size of spills for the largest spill size range, since it is the one that contributes the most and contains the greatest amount of uncertainty.

The median or 50th percentile value for the total volume spilled is approximately 60 bbl. This value is significantly less than the expected total volume spilled for spar-shuttle tanker, indicating that most spars will have relatively small volumes spilled. Typical percentile values are summarized in Table 3.13. Notice that 90-percent of the spills are still less than the expected total volume spilled. Only approximately 10-percent of the spills will be about or more than the expected value for this subsystem.

Table 3.13. Percentile Values for Total Volume Spilled for an Individual Spar-Shuttle Tanker

	Total Volume Spilled (bbl)
50%	61
90%	1,821
95%	5,632
99%	64,732

3.5 Conclusion

The equations presented in this chapter are used to determine the expected total volume spilled for an individual spar. It is assumed that the expected mean rate of occurrence, ν, is known as well as the distribution of the spill size range. The variance (square of the standard deviation) in the total volume spilled for an individual spar then represents the uncertainty in the number of occurrences and the consequence or spill size. The analytical values were verified with the use of a Monte Carlo simulation. The Monte Carlo simulation also demonstrated that spill volumes less than the expected total volume spilled are more likely.

Results from a spar with storage and transportation via shuttle tankers demonstrated that the total variance is dominated by the uncertainty in the number of occurrences and that the uncertainty in the spill size distribution only plays a minor role. The lesser role of the uncertainty in the spill size distribution also conveys that the designated spill size ranges were adequate enough to be represented effectively by a given distribution based on historical data.

Chapter 4: Uncertainty in Environmental Performance Model

Uncertainties are introduced into the analysis due to the lack of actual data and confidence in the model parameters when estimating the future performance of a spar. This chapter will discuss the sources of uncertainty and their effect on the expected total volume spilled and the resulting measure of uncertainty, the variance. Equations presented in Chapter 3 will be extended to incorporate this uncertainty. The CRA model included uncertainty in the mean rate of occurrence, N, but not uncertainty in the spill size distribution, R.

4.0 Sources of Uncertainty

The uncertainties stem from 1) the mean occurrence rate for spills, ν in Chapter 3, and 2) the spill size distribution, q and r in Chapter 3. Subdividing the spill sizes into ranges reduces part of the uncertainty in these two values. It also reveals that more is generally known about the smaller spill size ranges, since spills in these ranges are more frequent. Therefore, the distribution and the average spill size are better known for the smaller spill size ranges.

4.0.1 Uncertainty in Spill Occurrences

In Chapter 3, the frequency of spill occurrences, ν, used an assumed value based on limited historical data and expert opinion. However, the actual number of occurrences can vary from spar to spar, therefore creating an uncertainty in the number of spills. In order to account for this realistic variation and uncertainty, the frequency of spill occurrences will be modeled by a gamma function, given that a Poisson distribution modeled the number of occurrences, as described in Chapter 3 (Ang and Tang, 1984). This is accomplished by modeling the mean rate of occurrence, N_n, as a continuous random variable for each spill size range, n.

The expected value of the number of occurrences is still expressed by Equation 3.4. To now incorporate the uncertainty from the number of occurrences and the frequency of occurrences, both which are unknown, the variance due to the number of occurrences, X, is now expressed as:

$$Var(X_{occur}) = Var(X_{occur} | E(N)) + Var(\mathrm{N}) \qquad (4.1)$$

where the variance given the mean value of N is:

$$Var(X_{occur} | E(N)) = \nu t \qquad (4.2)$$

and the variance due to the mean rate of occurrence, N is:

$$Var(\mathrm{N}) = t^2 StDev(\nu)^2 \qquad (4.3)$$

where

$$StdDev(\nu) = E(\nu) * c.o.v.(\nu) \qquad (4.4)$$

Table 4.1 displays a sample calculation for the subsystem of spar-shuttle tanker.

Table 4.1. Sample Calculations for the Mean and Variance of the Number of Occurrences for Spar-Shuttle Tanker

Spill Size Range (bbl)	ν	StDev(ν)	Exposure, t (per 20 year lifetime)	E(Xoccur)	Var(Xoccur given E(N))	Var(N)	Var(Xoccur)
1 - 10	1.65E+00	8.41E-01	1	1.65E+00	1.65E+00	7.07E-01	2.35E+00
10 - 100	6.10E-01	3.76E-01	1	6.10E-01	6.10E-01	1.41E-01	7.51E-01
100 - 1,000	4.27E-01	2.92E-01	1	4.27E-01	4.27E-01	8.54E-02	5.12E-01
1,000 - 10,000	1.07E-01	1.21E-01	1	1.07E-01	1.07E-01	1.45E-02	1.21E-01
10,000 - 100,000	1.43E-02	1.66E-02	1	1.43E-02	1.43E-02	2.75E-04	1.46E-02
100,000 - 500,000	9.45E-03	1.13E-02	1	9.45E-03	9.45E-03	1.28E-04	9.58E-03
500,000 - 1,000,000	0.00E+00	0.00E+00	1	0.00E+00	0.00E+00	0.00E+00	0.00E+00

4.0.2 Uncertainty in the Spill Size Distribution

In Chapter 3, the distribution for the largest spill size was considered to be represented by a beta distribution with statistical parameters q = 1 and r = 3, for all cases except spar-storage, which was described by a uniform distribution. Figure 4.1 shows the physical representation of these two distributions in terms of the beta distribution.

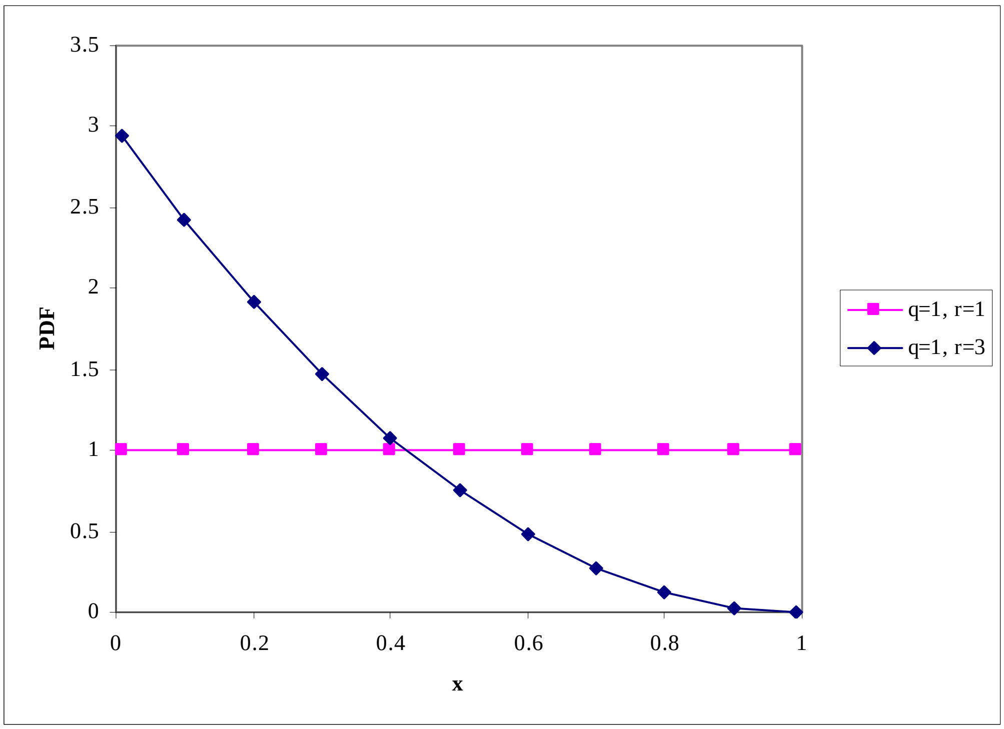


Figure 4.1. Standard Beta Distribution

The largest spill size range is difficult to define in terms of a representative distribution and may not be as easily described with a beta distribution using q =1 and r = 3. This is due to the limited amount of data available resulting from the fact that spills of this magnitude seldom occur. This is demonstrated in Table 2.1, which shows that more spills occur with smaller magnitudes than do spills with larger magnitudes. Although it is not desired to have large releases of crude oil, the lack of data creates a greater amount of uncertainty when trying to model its behavior.

In an effort to better represent the spill size distribution and ease previous assumptions, the distribution for the largest spill size range in terms of the statistical parameter r will be allowed to be random, thus represented as R. Modeling this parameter and its uncertainty are important since it is the largest spill size range which tends to dominate the risk. Previously, r was equal to 3; therefore, R will be modeled with the same mean by $1 \le R \le 5$.

The applicability of using $1 \le R \le 5$ will now be evaluated to ensure that it will produce spill sizes that are representative of the spill size range. A plot of the expected value of x, $E(e^x)$, (Figure 4.2) for q equal to one and r increasing shows that the expected value of x decreases with increasing r. A largest spill size range of 10,000-100,000 bbl was used to represent an actual case. From several workshops conducted (Gilbert et al, 2001a), an expert opinion was expressed that, for this range, there would not be a spill size greater than approximately 30,000 bbl due to safety measures in place. If this were true, then for an expected spill size of 30,000 bbl the statistical parameter r would be about 1.5. This results in a distribution between a uniform and triangular (skewed to the left) distribution. However, other expert opinion expressed a belief that the expected spill size would be even smaller. The graph also demonstrates that as r increases there is a diminishing effect on the expected value of x.

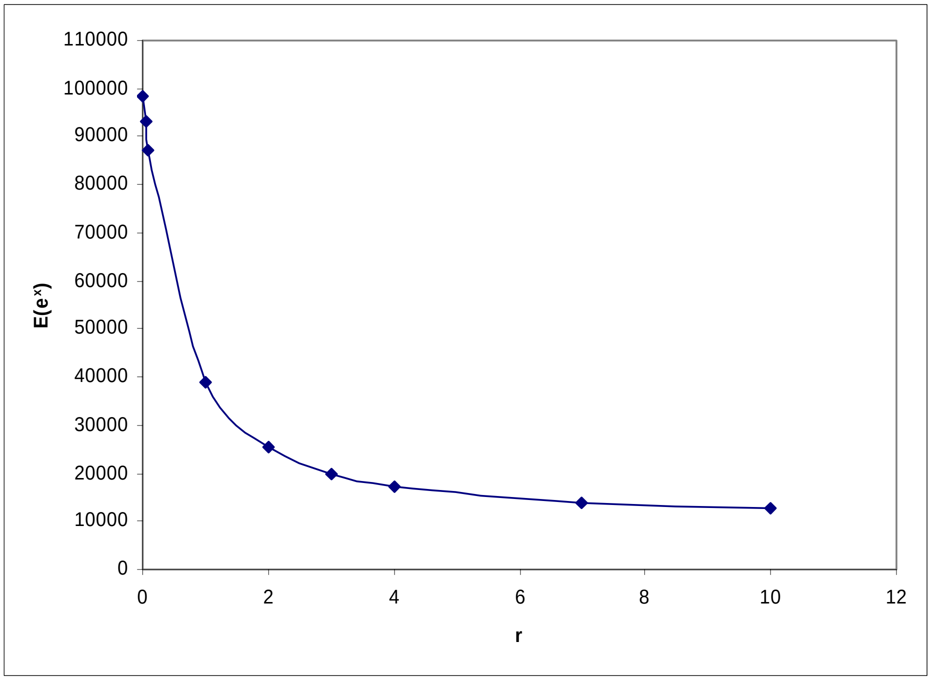


Figure 4.2. Expected Value of x for a Largest Spill Size Range of 10,000-100,000 bbl

Since expert opinion tended to agree that the likelihood of a spill greater than about 1/3 of the spill size range was rare, the probability that the expected value of x was greater than this value was examined with respect to r, as shown in Figure 4.3 for a spill size range of 10,000 to 100,000 bbl. This resulted in values for r of 4.5 and 7 with a 1% and 5% probability that the spill would be greater than 30,000 bbl, respectively.

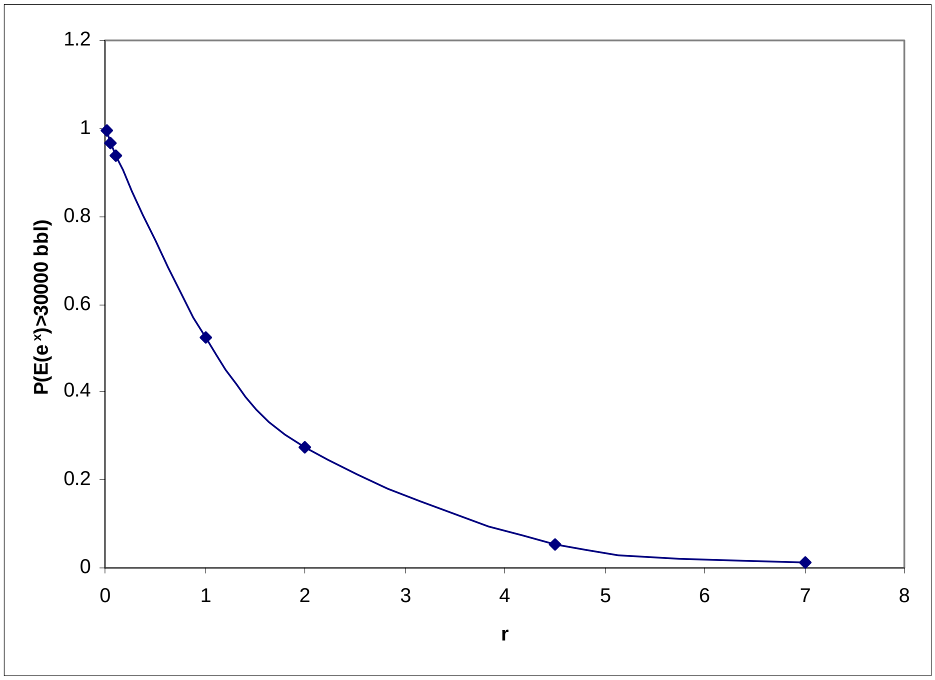


Figure 4.3. Probability $E(e^x) > 30{,}000$ bbl

Gilbert et al (2001a) had assumed that the largest spill size range was best represented by a triangular distribution, a beta distribution with q = 1 and r = 2. To further evaluate the range for which R could vary, two other distributions with $0 \leq R \leq 4$ and $1 \leq R \leq 3$ and the triangular distribution of q = 1 and r = 2 were analyzed in terms of their frequency of spill sizes within a spill size range of 10,000 – 100,000 bbls. The two other ranges have a mean of 2, thus allowing the triangular distribution to be the most prevalent.

Results of the simulations (Figure 4.4) show that the four distributions are similar in shape, as expected, except for the tail end of the distribution with $0 \leq R \leq 4$ and the upper end for $1 \leq R \leq 5$, where the frequencies increased. By allowing R to vary between 0 and 4, R is allowed to be less than q, thus creating a distribution that is skewed to the right. Therefore, this distribution has a greater expected spill size than the other two distributions. Although the distribution for $0 \leq R \leq 4$ permits a greater number of large spill sizes, it mainly affects the tail end of the range and not the occurrences in the middle. According to expert opinion, a large spill within this category is relatively unlikely compared to a small spill within this category. Thus, this distribution does not seem to be representative of the largest spill size range.

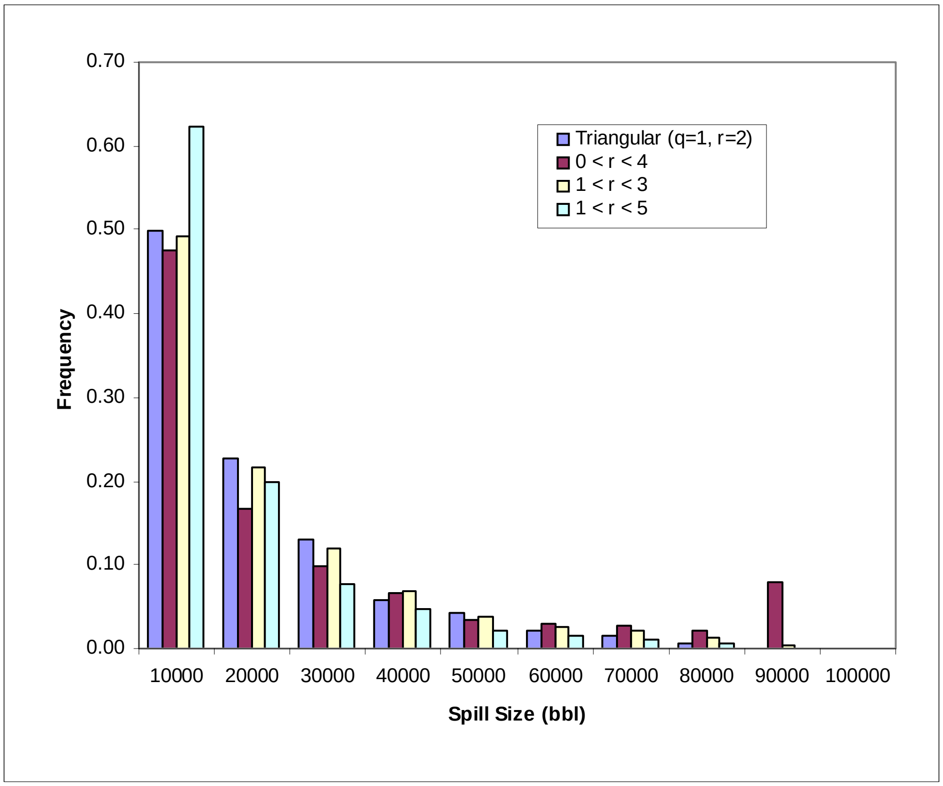


Figure 4.4. Frequency of Spill Occurrence for Four Beta Distributions

The distribution with $1 \leq R \leq 3$ also resembles a triangular distribution, but allows for a few more occurrences to take place for the spills in the upper region of the bounds. The expected spill size is then slightly greater than that from a triangular distribution, but less than for the distribution with $0 \leq R \leq 4$. Therefore, allowing R to vary between 1 and 3 might be more reasonable than between 0 and 4.

However, the distribution with $1 \leq R \leq 5$, again similar to a triangular distribution, allows more spill size occurrences which fall into the smallest category. This is because larger values of R create a distribution that creates a greater number of smaller spills to occur than large spills.

A summary of the expected spill sizes is presented in Table 4.2. This shows that more small spills are likely to occur for $1 \le R \le 5$, than the other three distributions.

Table 4.2. Summary of Expected Spill Sizes for the Largest Spill Size Range Using the Beta Distribution

Spill Size Range (bbl)	Expected Spill Size (bbl)			
	q = 1; r = 2 (Triangular)	q = 1; r = 0 to 4	q = 1; r = 1 to 3	q = 1; r = 1 to 5
1 - 10	N/A	N/A	N/A	N/A
10 - 100	N/A	N/A	N/A	N/A
100 - 1,000	N/A	N/A	N/A	N/A
1,000 - 10,000	N/A	N/A	N/A	N/A
10,000 - 100,000	25,264	32,731	26,577	21,968
100,000 - 500,000	184,579	216,072	190,375	167,288
500,000 - 1,000,000	638,674	671,622	645,652	612,562

The expected total volume spilled for each spill size range is expressed analytically by the following:

$$E(Consequence|E(R)) = \int_b^z \int_c^d \frac{\Gamma(1+R)}{\Gamma(1)\Gamma(R)} \frac{(x-a)^{q-1}(b-x)^{R-1}}{(b-a)^{q-R-1}} \frac{1}{d-c} e^x \, dR\,dx \tag{4.5}$$

where a = ln(lower bound) and b = ln(upper bound) of the spill size range, x = ln(spill size), and c and d are the range in R.

This equation reduces to the following for q equal to one:

$$E(Consequence|E(R)) = \int_b^z \int_c^d \frac{\Gamma(1+R)}{\Gamma(1)\Gamma(R)} \frac{(b-x)^{R-1}}{(b-a)^R} \frac{1}{d-c} e^x \, dRdx \tag{4.6}$$

The variance of the distribution for each spill size range i is described as:

$$Var(Consequence|E(R))_n = \int_a^b \int_c^d \frac{\Gamma(1+R)}{\Gamma(1)\Gamma(R)} \left[\frac{(b-x)^{R-1}}{(b-a)^R}\right] \frac{1}{d-c} (e^x)^2 \, dRdx$$
$$- \left[\int_a^b \int_c^d \frac{\Gamma(1+R)}{\Gamma(1)\Gamma(R)} \left(\frac{(b-x)^{R-1}}{(b-a)^R}\right) \frac{1}{d-c} e^x \, dRdx\right]^2 \tag{4.7}$$

If r is constant, then the integration with respect to R drops out of the equation, as well as all terms representing R.

Since it was determined that the largest spill size range is best described by a uniform distribution with $1 \leq R \leq 5$, the mean value is 3. The computed analytical expected values and variances in the consequence at the mean value of R for all spill size ranges are the same as those tabulated in Table 3.4.

To incorporate the uncertainty from the consequence for each spill size range and the statistical parameter, R, both which are unknown, the variance due to the consequence is now expressed as:

$$Var(Consequence)_j \cong Var(Consequence|E(R))_j + \left(\frac{d\mu_{Consequence}}{dR}\right)_j^2 Var(R)_j \tag{4.8}$$

where the variance in the consequence given the mean value of R is as given in Equation 4.7. The expected value and the variance in R are represented by the following equation, respectively:

$$E(R) = \frac{d+c}{2} \tag{4.9}$$

$$Var(R) = \frac{(d-c)^2}{12} \qquad c \leq R \leq d \tag{4.10}$$

where c and d represent the lower and upper bounds of R, respectively.

The change in the expected spill size with respect to R, $\left(\frac{d(\mu_{Consequence})}{dR}\right)$, is evaluated by plotting the change in spill size against the change in r and computing the slope or change at the expected value of r for the distribution. Figure 4.2 shows this plot for a spill size range of 10,000 – 100,000 bbl. Table 4.3 contains the three parameters which contribute to the variance in the consequence.

Note, since R is only varied for the largest spill size range, the uncertainty in R only contributes to the variance for that range. For a spill size range with a beta distribution represented by statistical parameters of q = r = 1 or where r is constant, then the variance in r is zero.

Table 4.3. Parameters Contributing to Var(Consequence)

	Var(Consequence) due to Spill Size Distribution (bbl^2)		Parameters of Uncertainty due to R	
Spill Size Range (bbl)	**q = r = 1**	**q = 1, 1<R<5**	**d$\mu_{Consequence}$/dR (bbl)**	**Var (R)**
1 - 10	6.22.E+00	N/A	N/A	0
10 - 100	6.22.E+02	N/A	N/A	0
100 - 1,000	6.22.E+04	N/A	N/A	0
1,000 - 10,000	6.22.E+06	N/A	N/A	0
10,000 - 100,000	6.22.E+08	2.16.E+08	6.63E+03	1.33
100,000 - 500,000	1.28.E+10	5.23.E+09	5.26E+04	1.33
500,000 - 1,000,000	2.07.E+10	1.05.E+10	2.00E+05	1.33

Sample calculations for the variance in the consequence for a spar-shuttle tanker are presented in Table 4.4.

Table 4.4. Sample Calculations for the Variance in the Consequence for a Spar-Shuttle Tanker

Spill Size Range (bbl)	Var (Consequence given E(R)) (bbl^2)	$d\mu_{Consequence}/dR$	Var(R)	Var (Consequence) (bbl^2)
1 - 10	6.22E+00	0	0	6.22E+00
10 - 100	6.22E+02	0	0	6.22E+02
100 - 1,000	6.22E+04	0	0	6.22E+04
1,000 - 10,000	6.22E+06	0	0	6.22E+06
10,000 - 100,000	6.22E+08	0	0	6.22E+08
100,000 - 500,000	5.23E+09	5.26E+04	1.33	8.90E+09
500,000 - 1,000,000	0.00E+00	0	0	0.00E+00
			Sum	**9.53.E+09**

4.1 Effect of Uncertainty

The effect of the uncertainty in the two parameters, N and R, will be analyzed using analytical approximations and numerical Monte Carlo solutions.

4.1.1 Analytical Solution for an Individual Spar

The following are the equations representing the analytical approximation for the expected total volume spilled and the variance in the total volume spilled for an individual spar.

4.1.1.1 Expected Total Volume Spilled

The uncertainty in the total volume spilled is now a function of the uncertainty in the frequency of spill occurrences, N, and the spill size distribution, which is a function of R. Previously, it was assumed that the performance of an average spar was known, i.e., the frequency of spills and their respective sizes.

Now, the analytical solutions from Chapter 3 can be extended to include these changes. In terms of the expected total volume spilled, the equation is the same:

$$E(TOTAL) = \sum_{i=1}^{n} \left[E(X_{occur})_i * E(Consequence)_i \right] \quad (4.11)$$

Thus, the results for a spar with oil storage are the same as those presented in Chapter 3, shown again in Table 4.5.

Table 4.5. Analytical Calculations of E(TOTAL$_C$) for Spar-Shuttle Tanker

Spill Size Range (bbl)	E(Xoccur)	E(Consequence) (bbl)	E(TOTAL$_C$) (bbl)
1 - 10	1.65E+00	3.9	6.4
10 - 100	6.10E-01	39	24
100 - 1,000	4.27E-01	391	167
1,000 - 10,000	1.07E-01	3,909	417
10,000 - 100,000	1.43E-02	39,087	560
100,000 - 500,000	9.45E-03	167,288	1581
500,000 - 1,000,000	0.00E+00	0	0
		Sum	**2755**

4.1.1.2 Variance in the Total Volume Spilled

The variance of the total volume spilled is approximated by the same equation as in Chapter 3:

$$Var(TOTAL_C) \cong \sum_{j=1}^{n} [E(X_{occur})_j Var(Consequence)_j + Var(X_{occur})_j E(Consequence)_j^2] \quad (4.12)$$

However, the two variance terms, Equations 4.1 and 4.8, are updated to include the uncertainty introduced by incorporating random variables into the analysis. Table 4.6 shows the analytical calculations for the variance in the total volume spilled for spar-shuttle tanker.

Table 4.6. Summary of Analytical Calculations for Var(TOTAL$_C$) for Spar-Shuttle Tanker

Spill Size Range (bbl)	E(Xoccur)	Var(Xoccur)	E(Consequence) (bbl)	Var (Consequence) (bbl^2)	Var($TOTAL_C$) (bbl^2)
1 - 10	1.65E+00	2.35E+00	3.9	6.22E+00	4.62E+01
10 - 100	6.10E-01	7.51E-01	39	6.22E+02	1.53E+03
100 - 1,000	4.27E-01	5.12E-01	391	6.22E+04	1.05E+05
1,000 - 10,000	1.07E-01	1.21E-01	3,909	6.22E+06	2.52E+06
10,000 - 100,000	1.43E-02	1.46E-02	39,087	6.22E+08	3.12E+07
100,000 - 500,000	9.45E-03	9.58E-03	167,288	8.90E+09	3.52E+08
500,000 - 1,000,000	0.00E+00	0.00E+00	0	0.00E+00	0.00E+00
				Sum	**3.86E+08**

4.1.2 Analytical Solution for an Average Spar

The following equations are based on those for an individual spar, but are now averaged to represent an average spar.

4.1.2.1 Expected Average Total Volume Spilled

The expected average total volume spilled is identical to Equation 3.11. The difference is due to the uncertainty in the mean rate of occurrence and the spill size distribution, demonstrated in the variance of the average total volume spilled.

4.1.2.2 Variance in the Average Total Volume Spilled

The uncertainty in an average spar or for a fleet of spars is given by the following equation, which is identical to Equation 3.12:

$$Var(\overline{TOTAL_C}) \cong \sum_{j=1}^{n}[E(X_{occur})_j Var(\overline{Consequence})_j + Var(\overline{X_{occur}})_j E(Consequence)_j^2] \tag{4.13}$$

where the variance in the average consequence and number of occurrences is expressed by the following equations:

$$Var(\overline{Consequence})_j \cong \frac{Var(Consequence|E(R))_j}{n_{spar}} + \left(\frac{d\mu_{Consequence}}{dR}\right)_j^2 Var(R)_j \tag{4.14}$$

$$Var(\overline{X_{occur}}) = \frac{Var(X_{occur}|E(N))}{n_{spar}} + Var(\text{N}) \tag{4.15}$$

where n_{spar} is the number of spars operating in the fleet.

Table 4.7 displays a sample calculation of the variance in the average total volume spilled for n_{spars} = 10. For this example of a spar-shuttle tanker, the variance in the total volume spilled only includes the uncertainty due to N and R.

Table 4.7. Analytical Calculations of $Var(\overline{TOTAL_C})$ with n_{spar} = 10 for Spar-Shuttle Tanker

Spill Size Range (bbl)	E(Xoccur)	Var(Average Xoccur)	E(Consequence) (bbl)	Var (Average Consequence) (bbl^2)	Var(Average $TOTAL_C$) (bbl^2)
1 - 10	1.65E+00	8.72E-01	3.9	6.22E-01	1.43E+01
10 - 100	6.10E-01	2.02E-01	39	6.22E+01	3.47E+02
100 - 1,000	4.27E-01	1.28E-01	391	6.22E+03	2.22E+04
1,000 - 10,000	1.07E-01	2.52E-02	3,909	6.22E+05	4.51E+05
10,000 - 100,000	1.43E-02	1.71E-03	39,087	6.22E+07	3.50E+06
100,000 - 500,000	9.45E-03	1.07E-03	167,288	4.20E+09	6.97E+07
500,000 - 1,000,000	0.00E+00	0.00E+00	0	0.00E+00	0.00E+00
				Sum	**7.37E+07**

An illustration of the change in the variance of the average total volume spilled is presented in Figure 4.5. As the number of spars or the fleet size increases, the variance in the average total volume spilled tends toward a constant value. This value is the uncertainty in the total volume spilled only due to the uncertainty in N and R. Any uncertainty from the number of occurrences, X, and the spill size or consequence is eliminated, thus representing an average spar. This model now represents the CRA model's average spar; however, that model did not consider the uncertainty in the distribution of the largest spill size range due to R. The change in variance presented in Chapter 3 is also plotted to demonstrate the difference in the variances between the two models.

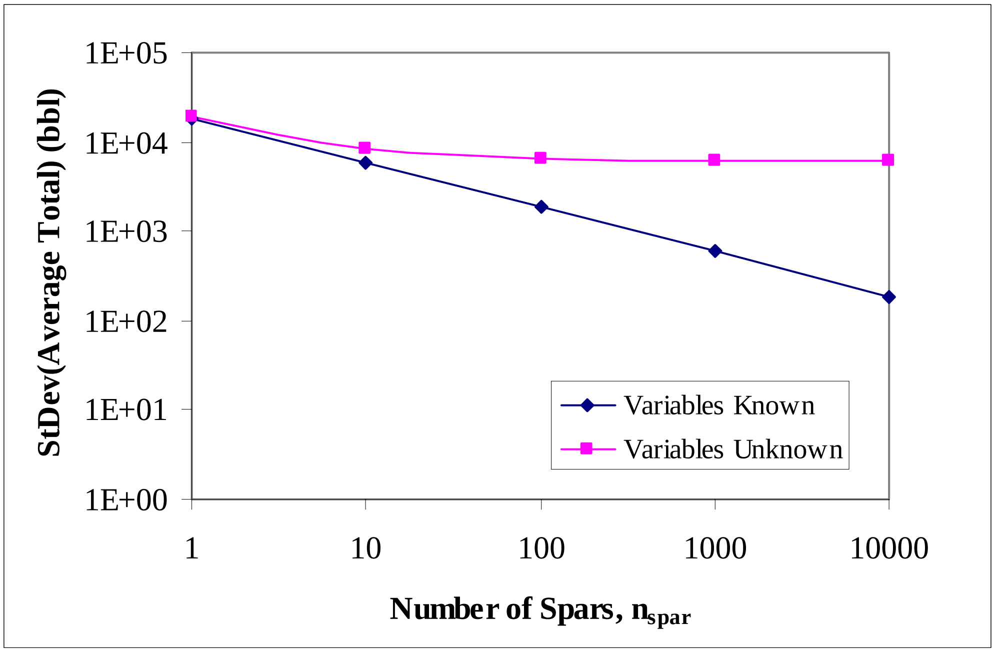


Figure 4.5. Effect of Averaging Spar Performances in Terms of the Uncertainty for Spar-Shuttle Tanker

Table 4.8 shows the amount that the variance in N and variance in R contribute to the total variance of an average spar for each spill size range. This demonstrates that the contribution of N to the total variance is greater for the larger spill size ranges. This is as expected since these spills are less frequent, thus representing the lack of actual data. For the largest spill size range, the main uncertainty is due to the variance in R, which contributes 89% to the total variance compared to the variance due to N, which contributes only 9.2%. Overall, the uncertainty due to R contributes most to the total variance with 89%, shown in Figure 4.6, although its uncertainty is only present in one spill size range. Only 11% of the uncertainty in the total variance is due to the mean rate of occurrence, N. Thus, the uncertainty in the spill size distribution for the largest spill size range is a dominant factor; reflecting the lack of data. The significance in the amount of uncertainty due to the largest spill size range, especially due to R, confirms that the extensions made to the CRA model to incorporate the uncertainty in N and R provides a more realistic model of an actual spar.

Table 4.8. Contribution of Parameters to the Total Variance of An Average Spar

Spill Size Range (bbl)	Variance due to N	Variance due to R	Contribution of Uncertainty in N to Total Variance (%)	Contribution of Uncertainty in R to Total Variance (%)
1 - 10	1.08E+01	0.00E+00	2.77E-05	0.00E+00
10 - 100	2.15E+02	0.00E+00	5.52E-04	0.00E+00
100 - 1,000	1.31E+04	0.00E+00	3.35E-02	0.00E+00
1,000 - 10,000	2.22E+05	0.00E+00	5.69E-01	0.00E+00
10,000 - 100,000	4.21E+05	0.00E+00	1.08E+00	0.00E+00
100,000 - 500,000	3.58E+06	3.47E+07	9.18E+00	8.90E+01
500,000 - 1,000,000	0.00E+00	0.00E+00	0.00E+00	0.00E+00
		Sum	**1.09E+01**	**8.90E+01**

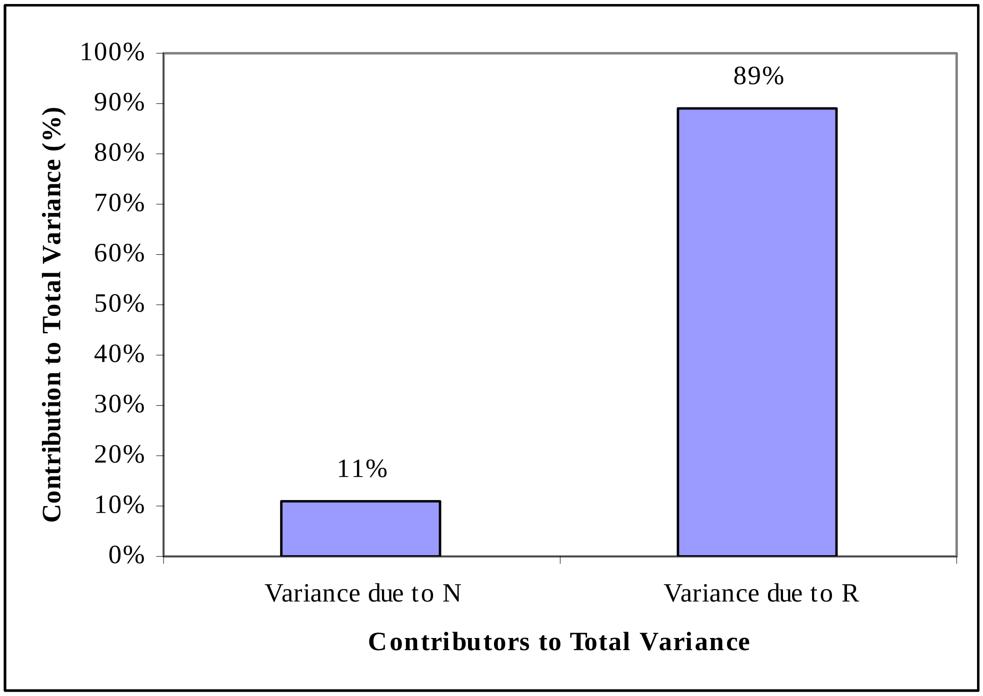


Figure 4.6. Contribution of Parameters to Total Variance of an Average Spar

4.1.3 Total Risk from Transportation of Oil

The analytical results from both spar-storage and spar-shuttle tanker, along with the total from both subsystems, for the total expected volume of oil spilled and the corresponding variance are presented in Table 4.9.

Table 4.9. Summary of Analytical Calculations per Spill Size Range for an Individual Spar with Oil Storage and Shuttle Tanker Transport

	Storage		Shuttle Tanker		Storage + Shuttle Tanker	
Spill Size Range (bbl)	$TOTAL_C$ (bbl)	$Var(TOTAL_C)$ (bbl^2)	$TOTAL_C$ (bbl)	$Var(TOTAL_C)$ (bbl^2)	$TOTAL_C$ (bbl)	$Var(TOTAL_C)$ (bbl^2)
1 - 10	0	0.00E+00	6.4	4.62E+01	6.4	4.62E+01
10 - 100	0	0.00E+00	24	1.53E+03	23.8	1.53E+03
100 - 1,000	0	0.00E+00	167	1.05E+05	166.8	1.05E+05
1,000 - 10,000	7	3.88E+04	417	2.52E+06	424.1	2.55E+06
10,000 - 100,000	70	3.88E+06	560	3.12E+07	630.5	3.51E+07
100,000 - 500,000	50	1.49E+07	1581	3.52E+08	1630.8	3.67E+08
500,000 - 1,000,000	144	1.08E+08	0	0.00E+00	144.3	1.08E+08
Total	**271**	**1.27E+08**	**2755**	**3.86.E+08**	**3027**	**5.13.E+08**

The variances due to the uncertainty in the spill size or consequence and the number of occurrences are shown separately in Table 4.10 for an individual spar. This shows the variance due to the uncertainty in the consequence contributes to 20% of the total variance of the total volume spilled, whereas the variance in the number of occurrences only contributes 80% to the total variance. Compared to the results from Chapter 3, the variance due to the consequence increased very slightly, thus correspondingly there was a minute decrease in the variance due to the occurrences. It should be noted that most of the variance in the total volume spilled will tend to go away when analyzing an average spar, as shown in Figure 4.5. In other words, the uncertainty is greatest from spar to spar, but when these values are averaged to represent an average spar, then the variance decreases.

Table 4.10. Analytical Variance Terms Involved in the Variance of the Total Volume Spilled for an Individual Spar

	Storage		Shuttle Tanker		Storage + Shuttle Tanker	
Spill Size Range (bbl)	**Total Variance due to Consequence**	**Total Variance due to Occurrences of Spills**	**Total Variance due to Consequence**	**Total Variance due to Occurrences of Spills**	**Total Variance due to Consequence**	**Total Variance due to Occurrences of Spills**
1 - 10	0.00E+00	0.00E+00	1.02E+01	3.60E+01	1.02E+01	3.60E+01
10 - 100	0.00E+00	0.00E+00	3.79E+02	1.15E+03	3.79E+02	1.15E+03
100 - 1,000	0.00E+00	0.00E+00	2.65E+04	7.83E+04	2.65E+04	7.83E+04
1,000 - 10,000	1.12E+04	2.76E+04	6.64E+05	1.85E+06	6.75E+05	1.88E+06
10,000 - 100,000	1.12E+06	2.76E+06	8.91E+06	2.23E+07	1.00E+07	2.51E+07
100,000 - 500,000	2.56E+06	1.24E+07	8.42E+07	2.68E+08	8.68E+07	2.80E+08
500,000 - 1,000,000	4.13E+06	1.04E+08	0.00E+00	0.00E+00	4.13E+06	1.04E+08
Total	**7.82E+06**	**1.19E+08**	**9.38E+07**	**2.92E+08**	**1.02E+08**	**4.12E+08**

Comparison of the variance due to the consequence from Chapter 3, where the distribution was constant, to the value when R is allowed to vary, displayed in Figure 4.7, shows that the uncertainty in R contributed to a 34% increase in the uncertainty. The additional uncertainty from allowing the mean rate of occurrence, N, to vary only increased the variance in this term by 1%. However, the uncertainty in the occurrences is still greater than that due to the consequence for an individual spar. Considering the average spar, the uncertainty due to N decreases by an order of magnitude, whereas the uncertainty due to R remains relatively unchanged.

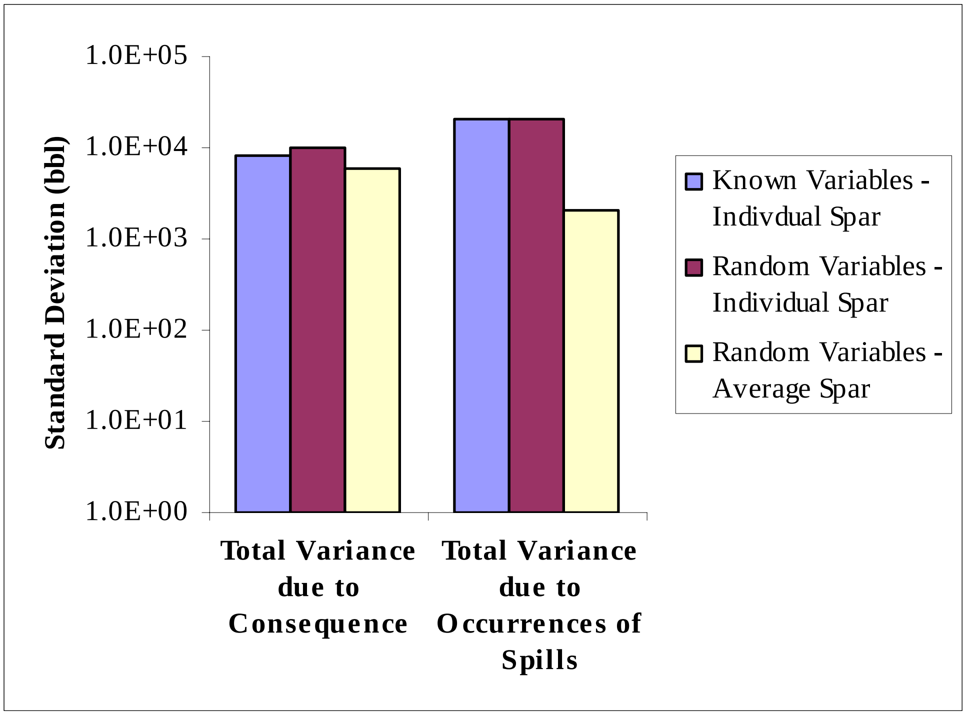


Figure 4.7. Comparison of Contributors to the Total Variance Components

4.1.4 Numerical Solution – Monte Carlo Simulation

A Monte Carlo simulation, similar to the one outlined in Chapter 3, was conducted with a total of 100,000 realizations for a spar with shuttle tanker transport. This simulation incorporated the variation in the number of occurrences as N and the spill size as R.

4.1.4.1 Simulating the Number of Occurrences

Simulation of the number of occurrences is similar to the procedure described in Chapter 3. But now the mean rate of occurrence, N, is also modeled.

This procedure is demonstrated using a spar with shuttle tanker transport. For the 1-10 bbl spill size range, ν is computed as 1.65 spills per year and the coefficient of variation as 0.51 from the historical data and expert opinion (Table

2.7). The procedure for determining these two parameters is explained in Gilbert 2001a. The statistical parameters k and n can then be calculated from the following equations:

$$k = \left(\frac{E(\nu)}{StdDev(\nu)} \right)^2 \tag{4.16}$$

$$n = \frac{E(\nu)}{(StdDev(\nu))^2} \tag{4.17}$$

The result is a k and n of 3.83 and 2.3, respectively. A random number is then generated between 0 and 1, which then uses Microsoft Excel's® inverse gamma function and the statistical parameters k_n and the inverse of n_n to obtain N. If a random number of 0.341 is generated, then N is 1.20 spills per year for this simulation. The new N is then used to determine a new Poisson distribution as shown in Table 4.11 And the graphical representation in Figure 4,8, which will then be used to determine the number of occurrences for the simulation, as explained in Chapter 3. For one simulation of all spars, N is kept constant with respect to the spill size range. Using the same probabilities for the number of occurrences as those in Chapter 3, 0.164 and 0.472, then the new number of occurrences for this N are zero and one, respectively.

Table 4.11. Lookup Table for Spill Size Range 1 – 10 bbl using N for Spar-Shuttle Tanker

CDF for x	Number of Occurrences, x
0	0
0.3006	1
0.6619	2
0.8790	3
0.9660	4
0.9922	5
0.9985	6
0.9997	7
1.0000	8

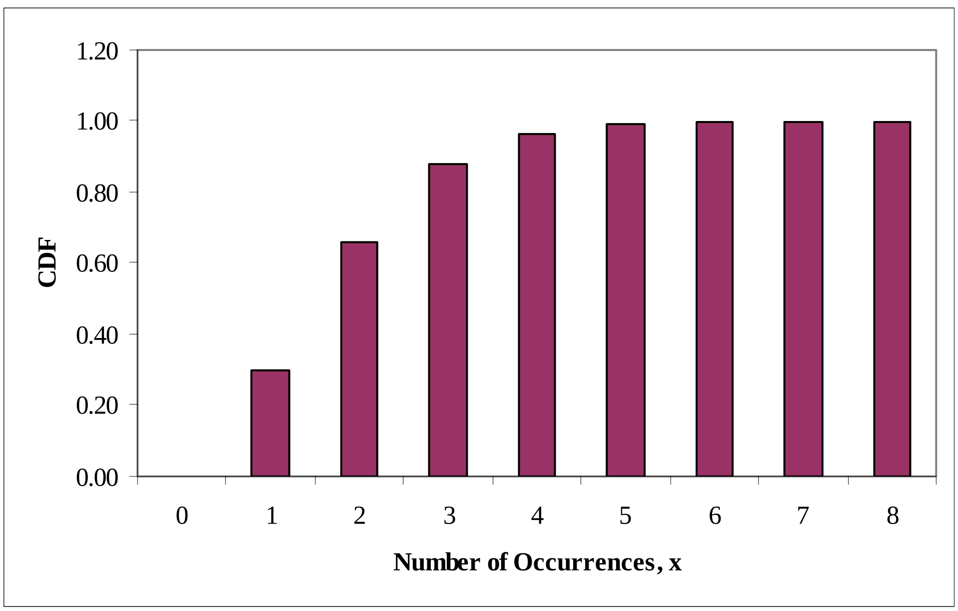


Figure 4.8. Graphical Representation of the Lookup Table for Spill Size Range 1 – 10 bbl for Spar-Shuttle Tanker

4.1.4.2 Simulating the Spill Sizes

Simulation of the spill sizes is similar to the procedure described in Chapter 3. However, when applicable, the statistical parameter R is modeled with a uniform distribution for $1 \leq R \leq 5$. The following equation is used to generate R:

$$R = rand(upper\ bound - lower\ bound) + lower\ bound \quad (4.18)$$

where rand is a random number between 0 and 1. The statistical parameters R and q = 1 are then used to determine a value of x, the spill size, using the inverse beta function in Excel, based on a randomly generated number between 0 and 1. Table 4.12 displays an example of randomly generated spill sizes based on q = r = 1 and q = 1 and $1 \leq R \leq 5$. The same random numbers were used to show the variation for the largest spill size range. Note, the spill size decreased for the spill modeled with R compared to r = 1. This will always be true since R will be greater than or equal to one.

Table 4.12. Example Calculation of Random Spill Sizes

			Spill Size (bbl)	
Spill Size Range (bbl)	**Random Number for Spill Size**	**R**	**q=1, r=1**	**q=1, 1<=R<=5**
1 - 10	0.6417	N/A	4	N/A
10 - 100	0.9507	N/A	89	N/A
100 - 1,000	0.4664	N/A	293	N/A
1,000 - 10,000	0.1451	N/A	1,397	N/A
10,000 - 100,000	0.8077	N/A	64,231	N/A
100,000 - 500,000	0.0447	N/A	107,459	N/A
500,000 - 1,000,000	0.2109	2.39	578,712	533,797

4.2 Comparison of Analytical and Numerical Solution

Results from the simulation are shown in Table 4.13. The two methods show similar results for both the total volume spilled and the standard deviation in the total. Thus, the analytical approximations are reasonable.

Table 4.13. Summary of Analytical and Monte Carlo Calculations for Spar-Shuttle Tanker

	E(TOTAL) (bbl)	StDev(TOTAL) (bbl)
Analytical	2755	19,651
Monte Carlo	2858	18,661

4.3 Conclusion

In order to more accurately represent actual conditions, the mean rate of occurrence, N, for each spill size range was allowed to vary. In addition, the spill size distribution for the largest spill size range for all subsystems, excluding spar-storage, was allowed to vary in terms of the statistical parameter, R, Thus, additional uncertainty was introduced into the analysis and is captured by the equations presented in this Chapter. The analytical values were verified with the use of a Monte Carlo simulation for spar-shuttle tanker.

In terms of the average spar, it was demonstrated that the uncertainty in the total volume spilled is due to the uncertainty in the mean rate of occurrence, N and the spill size distribution for the largest spill size range, R. Comparison of the two model parameters revealed that the uncertainty in the spill size distribution contributes the most to the overall uncertainty compared to the uncertainty in the mean rate of occurrences.

For an individual spar, the uncertainty in R had a larger affect on the variance in the consequence than the uncertainty of N in the variance of the occurrences. However, the variance due to the occurrences still contributed more to the total variance in the total volume spilled.

Chapter 5: Updating Results with Data

The model presented in Chapter 4 for an average spar included the uncertainty due to two random variables generated: the number of occurrences, X, and the spill size or consequence, C. Those equations allowed for two additional uncertainties than the original model presented in Chapter 3; the mean rate of occurrence, N, and the spill size distribution, represented by R.

Chapter 4 showed that as the number of spars tends toward infinity, the only uncertainty in the total volume spilled was due to N and R; there is no uncertainty from X and C. In order to reduce the uncertainty in N and R and to be more confident in the estimated total volume spilled, more data is needed. The Bayesian method, a technique which uses knowledge from prior events to predict future events, will be applied in order to reduce the uncertainty of the two parameters for an average spar.

5.0 Updating the Mean Rate of Occurrence, N

In Chapter 4, the mean rate of occurrence, N, was modeled as a random variable with a gamma distribution. It is desired to reduce the uncertainty in this variable by updating this parameter for each spill size range. This is accomplished by updating the statistical parameters k and n and then applying these to the expected value and variance of N. The following equations are derived based on conjugate distributions given the following assumptions (Ang And Tang 1984):

1. The number of spill occurrences is modeled by a Poisson distribution
2. The frequency of spill occurrences is modeled by a gamma distribution given the first assumption.

$$k'' = k' + x \tag{5.1}$$

$$n'' = n' + t \tag{5.2}$$

$$E(N'') = \frac{k''}{n''} \tag{5.3}$$

$$Var(N'') = \frac{k''}{(n'')^2} \tag{5.4}$$

Note N is a frequency per spar life, x is the number of spill occurrences in a spars life for the given spill size range, and t is the number of spar lives. The number of spars is based on an average operating lifetime of 20 years for one spar. In other words, t equal to one indicates 20 spar-years and t equal to two indicates 40 spar-years.

An example of the calculations for updating N is presented in Table 5.1 for a spar with shuttle tanker transport. Note k' and n' denotes the prior values, where k" and n' represent the posterior or updated values.

Table 5.1. Example Calculation for Updating N for Spar-Shuttle Tanker

Spill Size Range (bbl)	k'	n'	Var(N')	Spill Occurrences, x	Number of Spars, t	Number of Spar-Years	k"	n"	Var(N")
1-10	3.83	46.54	1.77E-03	1.65E+00	1	20	5.48	47.54	2.42E-03
10-100	2.64	86.47	3.53E-04	6.10E-01	1	20	3.25	87.47	4.24E-04
100-1000	2.13	99.92	2.14E-04	4.27E-01	1	20	2.56	100.92	2.51E-04
1000-10,000	0.78	146.96	3.63E-05	1.07E-01	1	20	0.89	147.96	4.07E-05
10,000-100,000	0.75	1040.75	6.88E-07	1.43E-02	1	20	0.76	1041.75	7.00E-07
100,000 - 500,000	0.70	1478.26	3.20E-07	9.45E-03	1	20	0.71	1479.26	3.24E-07
500,000 - 1,000,000	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

According to Equation 5.4, as t increases, the variance will decrease. In other words, with more spar-years means more data is known, thus reducing the uncertainty. The updated coefficient of variance, c.o.v., shown in Table 5.2, better represents the decline in the variance in N, since it is a non-dimensional measure of the magnitude of uncertainty for each spill-size range.

Table 5.2. Coefficient of Variance, c.o.v., for Example Calculation of Updating N

Spill Size Range (bbl)	c.o.v.'	c.o.v."
1-10	6.21E+00	3.71E+00
10-100	2.02E+01	1.50E+01
100-1000	3.21E+01	2.46E+01
1000-10,000	2.12E+02	1.76E+02
10,000-100,000	1.62E+03	1.57E+03
100,000 - 500,000	2.53E+03	2.48E+03
500,000 - 1,000,000	N/A	N/A

Table 5.3 shows the analytical updated variance in N for each spill size range from Spar-Shuttle Tanker. Spar-years represent the number of years that a group of spars may be operating within the Gulf of Mexico. This does not mean that an individual spar is operating for that time period or that all spars are operating for the same time period. Spar-years are a unit of measure, in which information on the rate of occurrence and size of spills can be accumulated for future prediction of the risk involved. For example, in the Gulf of Mexico, if there are four spars operating for nine, five, four, and two years, respectively, then the total would be 20 spar-years.

Table 5.3. Summary of Var”(N) for Different Spar-Years for Spar-Shuttle Tanker

	Var(N')	Var(N") per Spar-Year							
Spill Size Range (bbl)	0	20	100	200	500	1000	2000	5000	10,000
1-10	1.77E-03	2.42E-03	2.06E-03	1.71E-03	1.07E-03	5.88E-04	2.55E-04	6.23E-05	1.83E-05
10-100	3.53E-04	4.24E-04	3.88E-04	3.49E-04	2.61E-04	1.74E-04	9.34E-05	2.87E-05	9.44E-06
100-1000	2.14E-04	2.51E-04	2.32E-04	2.12E-04	1.64E-04	1.14E-04	6.40E-05	2.09E-05	7.11E-06
1000-10,000	3.63E-05	4.07E-05	3.86E-05	3.62E-05	3.01E-05	2.30E-05	1.46E-05	5.65E-06	2.13E-06
10,000-100,000	6.88E-07	7.00E-07	6.95E-07	6.88E-07	6.69E-07	6.39E-07	5.84E-07	4.56E-07	3.20E-07
100,000 - 500,000	3.20E-07	3.24E-07	3.22E-07	3.20E-07	3.13E-07	3.03E-07	2.84E-07	2.37E-07	1.81E-07

Demonstrated in Table 5.3 is the ability to more easily reduce the uncertainty in N for the smaller spill size ranges than the larger ranges, since the small ranges have a greater frequency of spill occurrence. In terms of an average spar (n_{spar} = infinity), the only factor in the variance for the number of occurrences is due to the uncertainty in the mean rate of occurrences, N. Therefore, Table 5.3 also represents the variance in the average number of occurrences, $Var(\overline{X_{Occur}})$, as presented in Equation 4.15. In order to demonstrate the effect this has on the total variance, then for each spill size range $Var(\overline{X_{Occur}})$ is multiplied by the square of the expected consequence to determine the term in the total variance represented by the variance in the number of occurrences, as shown in Equation 4.13. Figure 5.1 contains a plot of $StDev\left[\sum_{j=1}^{n}[Var(\overline{X_{occur}})_j E(Consequence)_j^2]\right]$ to demonstrate how the standard deviation in the total volume spilled will decrease with increased information and the ability to update the variance in N. Note, the x-axis is plotted on a log scale due to the large range in magnitude.

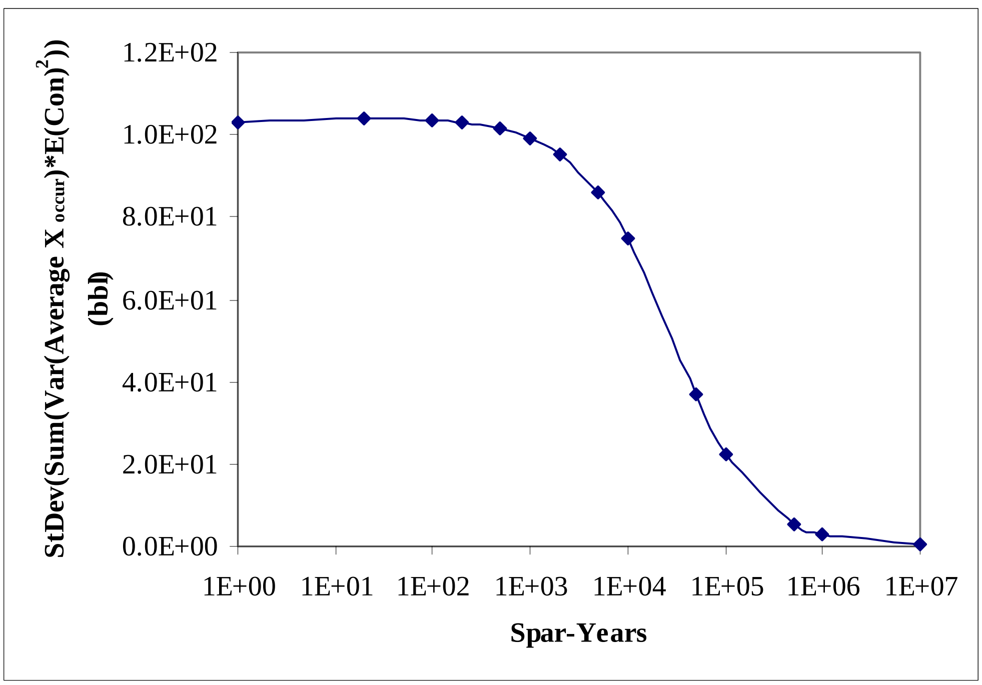


Figure 5.1. Effect of Var(N”) for An Average Spar-Shuttle Tanker

From this plot, it reveals that there is not enough information within the first 1000 spar-years to significantly reduce the total variance due to updating N. This makes sense since for 1000 spar-years only 0.5 spills are expected occur in the largest spill size range. As the number of spar-years increase, the variance reduces, as expected since the expected number of spills is also increasing. It appears that after about 1,000,000 spar-years, there is sufficient information to reduce the overall variance by one order of magnitude due to updating N; and to achieve any greater of a reduction, an even more substantial number of spar-years must be accumulated.

Assuming a spar operates in the Gulf of Mexico on average for 20 years, then this would equate to about 50,000 spars. This does not imply that 50,000 spars are all operating at the same time, but 50,000 spars operating for 20 years each is needed to obtain 1,000,000 spar-years worth of data. This value is highly unreasonable due to the need for immediate information. If a 50% reduction in the standard deviation of the total in terms of N is desired, then only about 40,000 spar-years of data are needed. This would equate to about 2000 spars operating for 20 years each in the Gulf of Mexico. If an average of 10 spars are operating per year, then to obtain data for 1,000 spar-years, it would take about 200 years.

A tremendous amount of information (spar-years) would be needed to reduce the uncertainty in the expected volumes of large, infrequent spills. The amount of information that is learned by increasing the spar-years of data is best depicted by dividing the updated standard deviation by the prior standard deviation of N for each spill size range, as presented in Figure 5.2. Several ideas demonstrated in the plot are 1) initially there is little knowledge learned, but with more time and data, the variance will reduce for each spill size range and 2) for a given number of spar-years, more information is learned for the smaller, more frequent spills, than the larger spills. The second observation reveals that it will take many spar-years to reduce the amount of uncertainty in the larger spill size ranges due to their lack of occurrences. Note the gap between the first four spill size ranges and the last two. This reflects the current knowledge on the mean rate of occurrences for the spill size distributions, indicating that more knowledge is currently known for the smaller spill size ranges than the larger ranges. Notice the ratio in standard deviations for the first four spill sizes reduces much faster than for the two biggest spill size ranges.

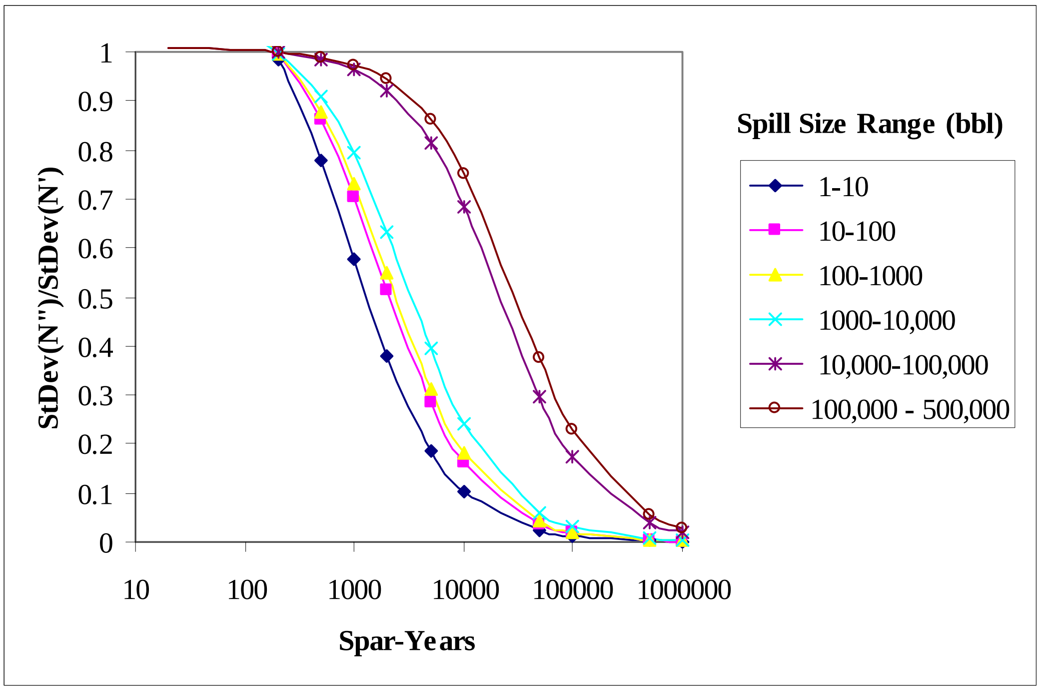


Figure 5.2. Ratio of Updated and Prior StDev(N) for each Spill Size Range

Again, considering a 50-percent reduction in the total variance due to N, at 40,000 spar-years the majority of the uncertainty is due to the two largest spill sizes ranges. Most of the uncertainty contributing from the four smaller ranges has been significantly reduced. Therefore, the two largest spill size ranges contribute more to the variance because of their increased uncertainty due to the limited amount of data.

5.1 Updating the Statistical Parameter, R

In Chapter 4, the spill size distribution was assumed to be unknown for the largest spill size range, except for spar-storage. The spill sizes were modeled by a beta distribution, where the statistical parameter describing the distribution, R, was modeled as a random variable with a uniform distribution. Again, it is desired to reduce the uncertainty in this variable by updating this parameter for the largest applicable spill size range based on the subsystem analyzed.

Unlike the case for updating N, analytical equations could not be derived. Therefore, numerical integration was applied in seeking a solution.

The need to update R is only applicable for the largest spill size range for spar-shuttle tanker, spar-export riser, and spar-pipeline. The posterior function of R depends on the size of the spill that occurs, along with the number of spills. Based on the expected frequency of a spill occurrence in the largest spill size range for these cases, there is a tendency that if a spill were to occur, only one spill would occur at a time. However, multiple spills are possible for a single realization.

In order to update R, the posterior function must be first defined in terms of R, by the following (Ang and Tang 1984):

$$f''(r) = kL(r)\,f'(r) \tag{5.5}$$

For this analysis, a uniform distribution is assumed for the prior distribution. Since R is allowed to vary between 1 and 5, this leads to the following equation:

$$f'(r) = 0.25 \qquad\qquad 1 \le r \le 5 \tag{5.6}$$

The likelihood function is the beta function given by the following equation:

$$L(r) = \frac{\Gamma(q+r)}{\Gamma(q)\Gamma(r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \quad a \le x \le b \tag{5.7}$$

where a = ln(lower bound) and b = ln(upper bound) for a spill size range, q and r are the statistical parameters of the function, and x = ln(spill size). The beta function is used since the spill size distribution was modeled with a beta distribution.

The constant k is then determined by numerical integration, with q equal to one for this analysis.

$$k = \frac{1}{\int_0^\infty \frac{\Gamma(1+R)}{\Gamma(1)\Gamma(R)} \frac{(b-x)^{R-1}}{(b-a)^R} dR} \quad a \le x \le b \tag{5.8}$$

Based on the frequency of spill occurrences for the largest spill size range, there exists the possibility of multiple spill occurrences. Therefore, the evaluation of the posterior function needs to incorporate the possibility of multiple spills. Since x represents the spill size, this portion of the likelihood function can be modified by the following equation. Note that for this analysis q equals one, thus, reducing the term in the numerator that is raised to the power of q minus one in Equations 5.7 and 5.8 to one. The remaining portion of the numerator involving a spill size is then expressed by the following for multiple spills:

$$y = \prod_{i=1}^{z} (b - x_i) \tag{5.9}$$

where z is the number of spill occurrences.

The constant of integration, k, is then rewritten to represent the inclusion of multiple spill occurrences and the range for which r will vary for this analysis:

$$k = \frac{1}{\int_1^5 \left(\frac{\Gamma(1+r)}{\Gamma(1)\Gamma(r)} \right)^z \frac{y^{r-1}}{(b-a)^{z*r}} dr} \qquad a \le x \le b \qquad (5.10)$$

Once y and k have been determined, the posterior function of R can then be evaluated based on the specific spill size(s) and number of occurrences by the following equation.

$$f''(r) = k \left(\frac{\Gamma(1+r)}{\Gamma(1)\Gamma(r)} \right)^z \frac{y^{r-1}}{(b-a)^{z*r}} \qquad a \le x \le b \qquad (5.11)$$

With the updated function, the expected value, E"(R), can be determined, but more importantly the variance of r, Var"(R), can be computed based on new information using the prior data. These two parameters are expressed as:

$$E''(R) = \int_1^5 f''(r)\ r\ dr \qquad (5.12)$$

$$Var''(R) = E''(R^2) - [E''(R)]^2 \qquad (5.13)$$

where $E''(R^2) = \int_1^5 f''(r)\ r^2 dr$ (5.14)

Note that Equations 5.11 thru 5.13 require numerical integration. This was accomplished, with the aid of MathCAD®, by creating a set of tables for the two largest spill size ranges being evaluated, 10,000 - 100,000 bbl and 100,000 – 500,000 bbl. The tables are presented in Appendix B for z = 1 to 6 based on y for k, $\int_1^5 RL(R) f'(R) dR$, and $\int_1^5 R^2 L(R) f'(R) dR$. The following example will demonstrate the use of this technique and the tables for a spill size range of 100,000-500,000, representing spar-shuttle tanker.

Given the number of spills and their respective magnitude, y can be computed, as displayed in Table 5.4. Next, using the lookup tables in Table B.3, k, $\int_1^5 RL(R)f'(R)dR$, and $\int_1^5 R^2 L(R)f'(R)dR$ can be determined, based on y, and the updated expected value and variance in R can be calculated (Table 5.5).

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Table 5.4. Summary of Calculations for y

a	b	R	Number of Spills	Spill Size (bbl), x		y
100,000	500,000	1.542	0	268,139	441,199	0
100,000	500,000	2.393	1	102,458	305,366	1.59
100,000	500,000	2.074	2	180,703	430,145	0.15

Table 5.5. Demonstration of Lookup Table and Calculation of Updated Parameters

R	y	k	integral r*L(r)f'(r)	E"(R)	integral r^2*L(r)f'(r)	Var"(R)
1.542	0	0	N/A	1.54	N/A	1.33
2.393	1.59	0.58	5.891	3.417	22.071	1.13
2.074	0.15	15.15	0.103	1.564	0.179	0.26

For zero spills, the updated expected value and variance of r are unchanged, since no new information is gained. Notice when a spill occurs, the variance in R decreases with respect to the prior variance of R, which was 1.33. This is as expected since more information is learned about R. In the case with two spills, the updated variance in R was greatly reduced compared to when only one spill occurred.

Table 5.6 shows the updated variance in R for the largest spill size range for Spar-Shuttle Tanker. Figure 5.3 contains a plot showing the change in $StDev\left[\sum_{j=1}^{n}[Var(\overline{Consequence})_j\, E(X_{Occur})_j]\right]$ to demonstrate how the standard deviation in the total volume spilled will decrease with increased information and the ability to update the variance in R.

Table 5.6. Summary of Var”(R) for Spar-Shuttle Tanker

Spar-Year	Var(R")
20	1.33
500	1.32
1000	1.30
10000	1.28

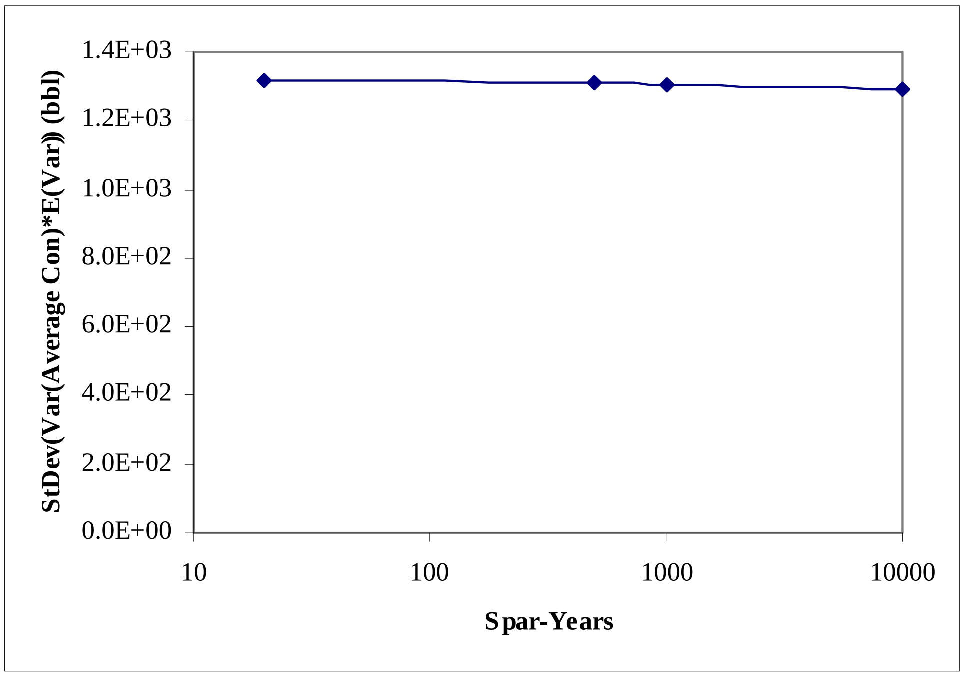

Figure 5.3. Effect of Var(R”) for Spar-Shuttle Tanker

As demonstrated in the plot, as the number of spar-years increases, the effect of reducing the variance in R is slight. This reflects the enormous amount of uncertainty for this parameter. This is in agreement with Figure 5.2 for the largest spill size range, 100,000-500,000 bbl. At 10,000 spar-years, the uncertainty in N was still very high, since only 0.5 spills were expected. Without enough spill occurrences, then the size of the spills cannot be predicted with better certainty. Based on the data from updating N, after about 1,000,000 spar-years, there should be enough information to also reduce the uncertainty in R effectively.

Overall, due to the increased knowledge from updating N and R, the standard deviation in the total volume spilled will decrease with the updated parameters with respect to the prior total standard deviation for a given number of spar-years. Figure 5.4 presents the updated standard deviation in the total spilled for an average spar. For spar-shuttle tanker, there is not a significant decline in the total standard deviation. This is in part shown in Figure 5.3, where the updated standard deviation due to R was barely affected, reflecting the uncertainty in the spill size distribution for the largest spill size range.

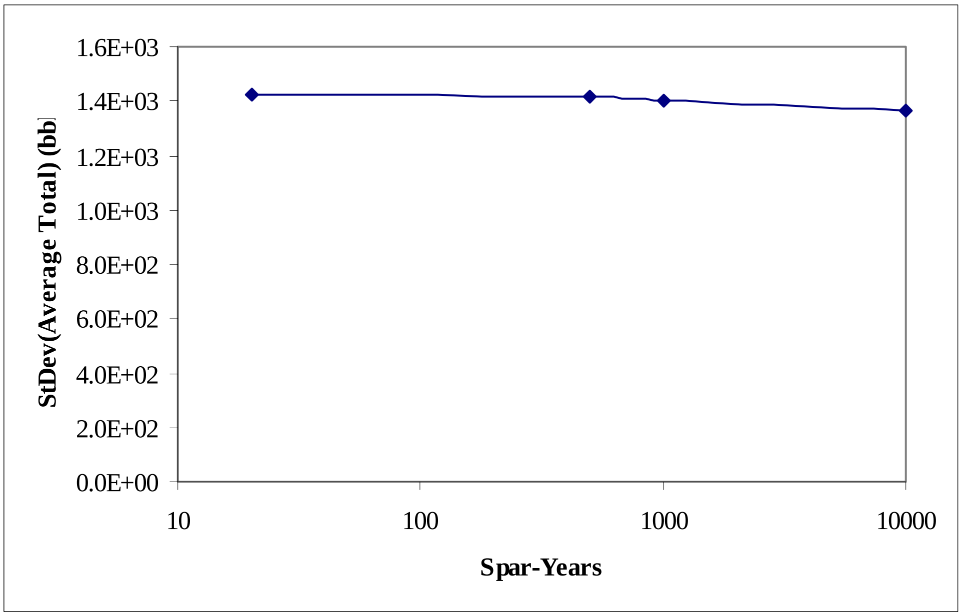

Figure 5.4. Overall Effect of Updating N and R for an Average Spar-Shuttle Tanker

5.2 Conclusion

In conclusion, as the number of spar-years increases, the updated variance in N and R decreases. The variance in N is easier to update for higher frequency events than lower frequency event, i.e., small spill sizes versus large spill sizes.

This can also be applied to updating R, since this parameter is only updated for the largest spill size range, which has the lowest rate of spill occurrence. Overall, the total standard deviation decreases with increasing spar-years, yet large amounts of uncertainty remain. From analysis of spar-shuttle tanker, it appears that it would take about one million spar-years to reduce the uncertainty in the total volume spilled effectively. Therefore, there will probably always be uncertainty in the average spar related to the largest spill size ranges.

Chapter 6: Comparison of Oil Transport for a Spar with and without Oil Storage

This chapter will compare the chronic environmental risk and its uncertainty for two spar options in regards to transportation of produced oil in the Gulf of Mexico: 1) offloading by means of export risers and pipelines without spar storage, and 2) spar storage with offloading via shuttle tankers. The updated variances for the random variables N and R will also be discussed with respect to these two transportation options in order to judge how much information is required before an informed decision can be made.

6.0 Expected Total Volume of Oil Spilled

Two analytical models were developed, as discussed in Chapters 3 and 4, to predict the total volume of oil spilled from a spar during transportation operations. Figure 6.1 compares these values from the two methods to the two options of oil transport from a spar.

Although allowing the variables for the mean rate of occurrence, N, and the statistical parameter describing the spill size distribution, R, to be random or unknown, the same mean value for the two parameters determined the expected total volume spilled. Thus, the value is the same for both models. The difference between the two models is better expressed in terms of the variance of the total volume spilled. Comparing the two systems of transportation reveals that the expected total volume spilled from storage-shuttle tanker is slightly less than for export riser-pipeline.

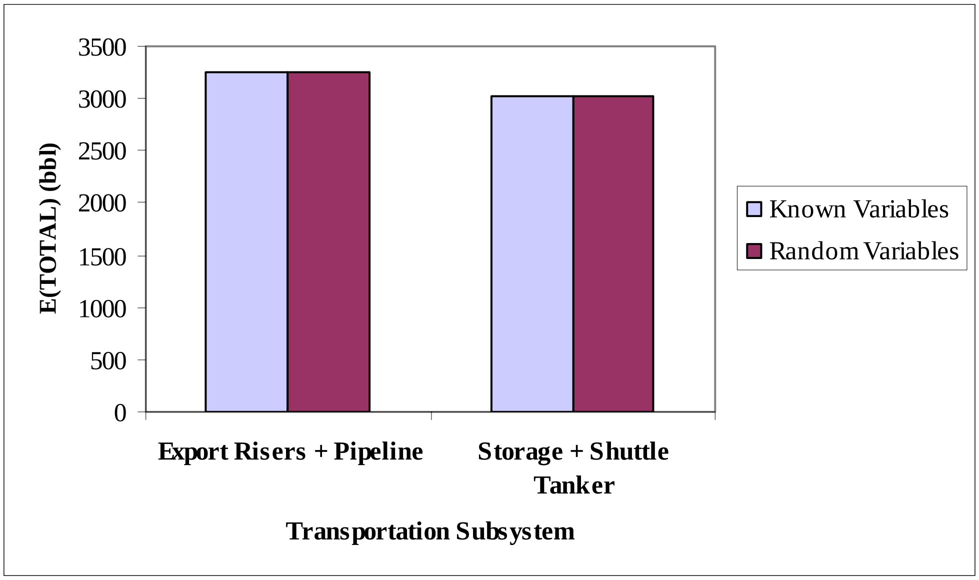

Figure 6.1. Analytical Expected Total Volume of Oil Spilled

6.1 Variance in the Total Volume of Oil Spilled

The analytical variance (square of the standard deviation) of the total volume spilled from an individual spar for each model is presented in Figure 6.2. The chart is presented on a log scale due to the large magnitudes of the variance. This can slightly distort the values, but a change in magnitude within an order of magnitude is not as significant as a change in order of magnitude.

Comparison of the two transportation options reveals that there is greater uncertainty in the volume spilled due to storage-shuttle tanker than by export risers-pipelines. This is due to the limited information on the number and volume of spills due to storage. In addition, a spill size range of 100,000 – 500,000 bbl and 500,000 – 1,000,000 bbl is used to represent the largest spill size range for shuttle tankers and storage, respectively. The largest spill size range for export risers and pipelines are both represented by a spill size range of 10,000 – 100,000 bbl. Therefore, the variance in the spill size range is greater for the larger volumes of oil, shown in Table 4.4.

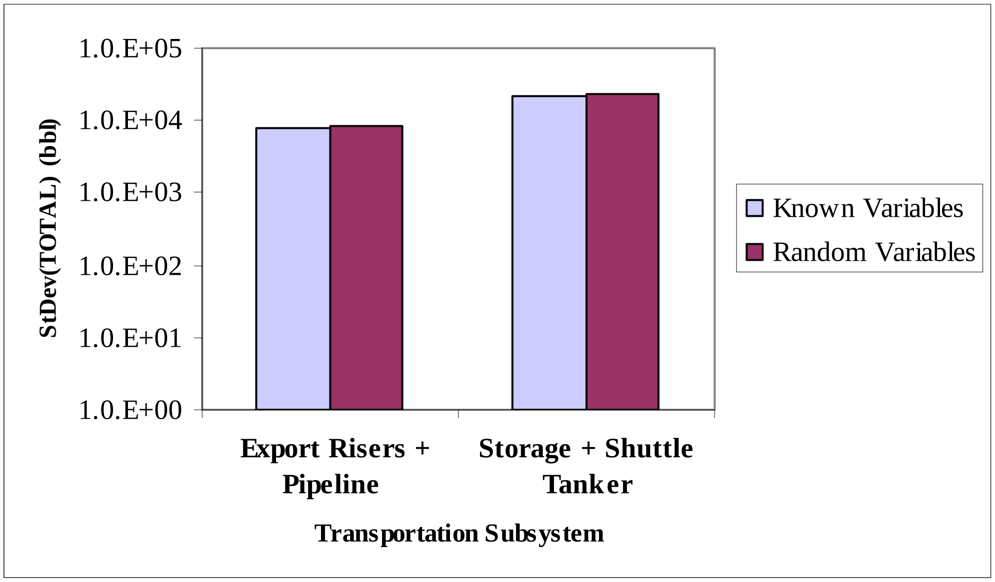

Figure 6.2. Analytical Variance in the Total Volume of Oil Spilled for an Individual Spar

In terms of an average spar, Figure 6.3 contains the difference in the standard deviation of the total volume spilled for each subsystem of transportation. Note, only the uncertainty due to the random variables N and R contributes to the uncertainty in the total, thus the standard deviation is less for an average spar.

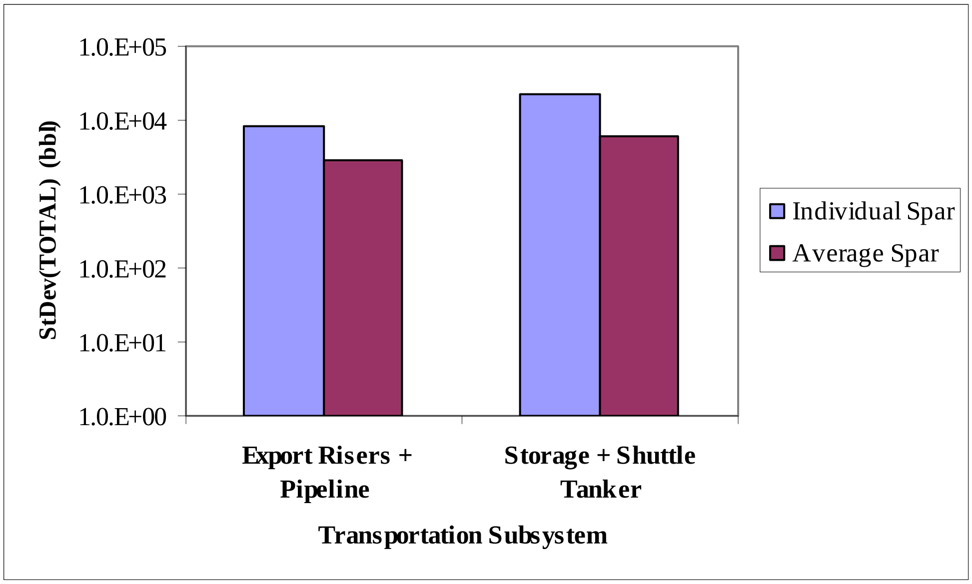

Figure 6.3. Analytical Standard Deviation in the Total Volume of Oil Spilled for an Individual Spar and an Average Spar

6.1.1 Comparison of Individual Spars

The following contains a comparison of the two transportation systems for an individual spar in the Gulf of Mexico based on results from the Monte Carlo simulation. The histogram in Figure 6.4 demonstrates the frequency for the total volume spilled from an individual spar for the two systems. Both systems show the tendency of the total volumes spilled to be more toward the middle range in magnitude. Both exhibit approximately the same frequency in zero volume spills and show very few total spills in the larger ranges. Note for export riser-pipeline, there is a larger frequency of total spills ranging between 10,000 – 100,000 bbl, then for storage-shuttle tanker, although storage-shuttle tanker contained the possibility of spills larger than 100,000 bbl. This reiterates the lack of data for spar-storage and spar-shuttle tanker in regard to the largest spills possible and their frequency.

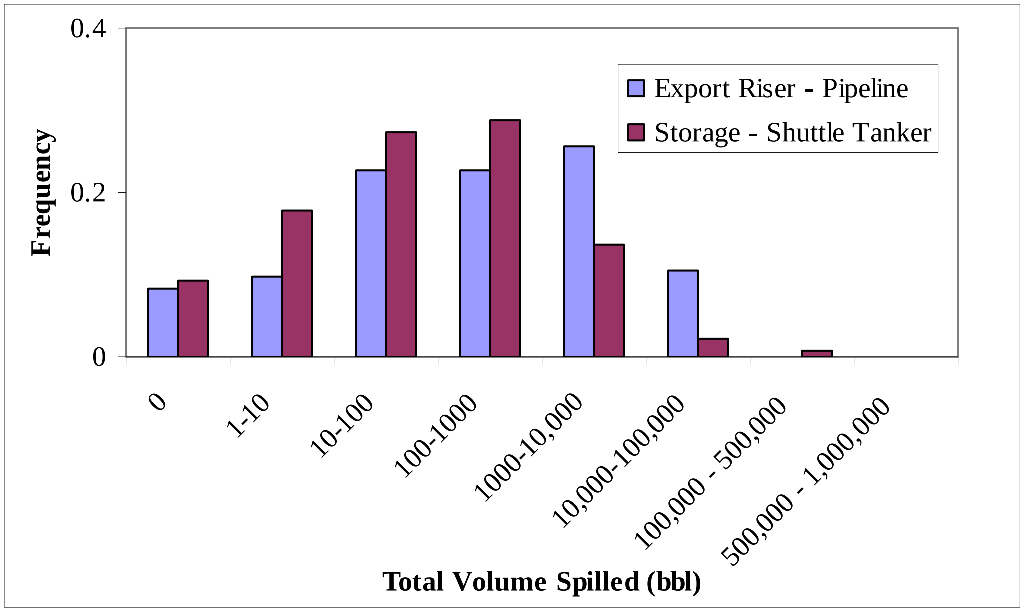

Figure 6.4. Frequency of Total Volume Spilled for an Individual Spar

Figure 6.5 and 6.6 demonstrate the contribution of each spill size range to the total volume spilled. Both figures display the small influence that the smaller spill size ranges contribute to the total volume spilled. Most noticeable from the figures are that the larger spill size ranges contribute the most to the total volume spilled for both systems. For a storage-shuttle tanker spar, the second to largest spill size range tends to dominate. This is because only spar-storage was considered to have the possibility of a spill in the range of 500,000 – 1,000,000 bbl and its frequency was small. However, both subsystems allowed for a spill within the range of 100,000 – 500,000 bbl, which is the dominant range. This indicates the importance that the larger spill sizes have in regard to the environmental risk and therefore cannot be ignored. More information is needed to improve the existing estimates.

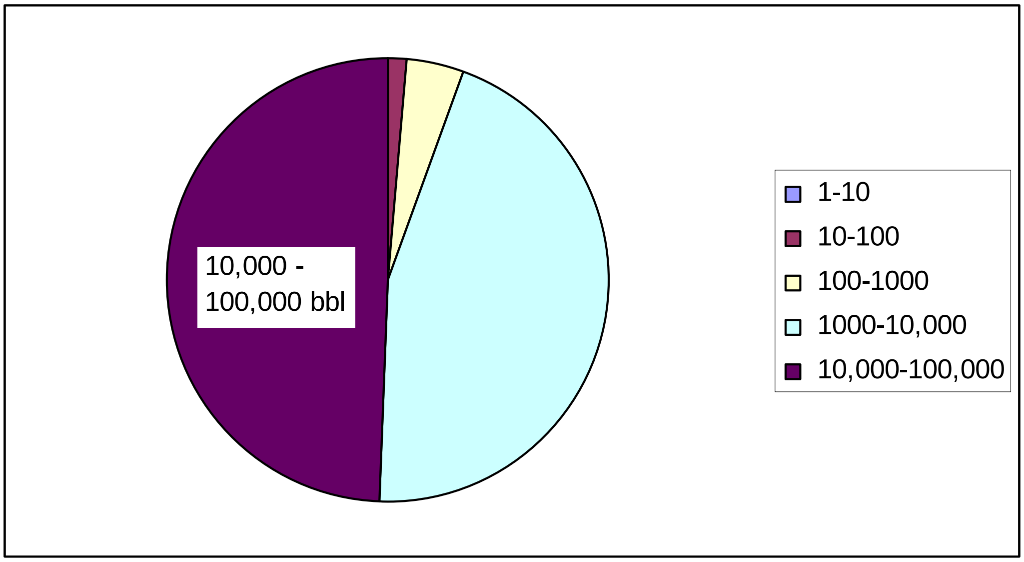

Figure 6.5. Contribution of Spill Size Range to Total Volume Spilled for an Individual Export Riser-Pipeline Spar

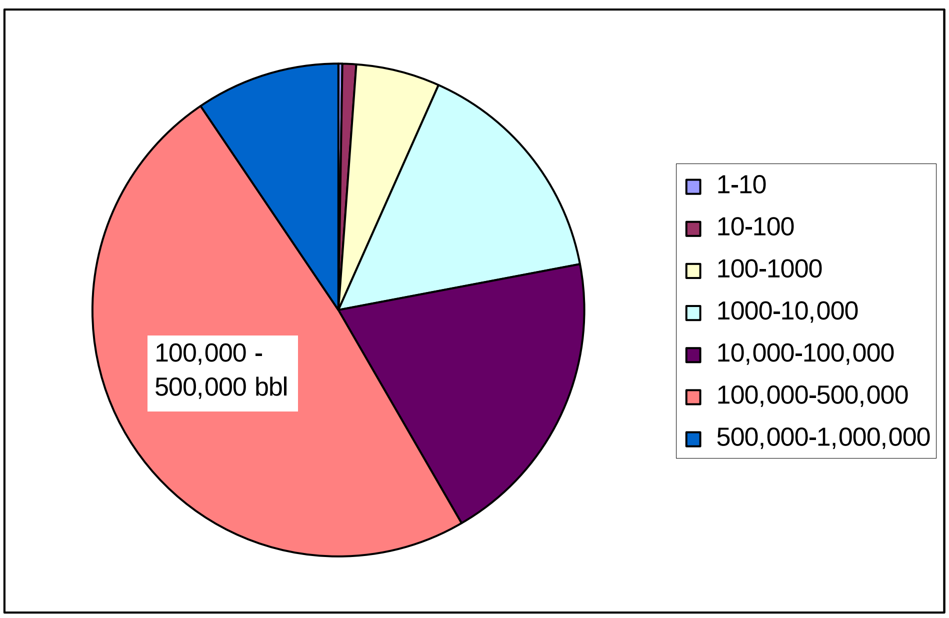

Figure 6.6. Contribution of Spill Size Range to Total Volume Spilled for an Individual Storage-Shuttle Tanker Spar

Comparing the different percentile ranges, Table 6.1, for the two transportation systems reveals that the 50th percentile values for both systems are significantly less than their expected values of 3212 bbl and 3027 bbl for export riser-pipeline and storage-shuttle tanker, respectively. This indicates that most spars will have relatively small volumes spilled. Notice that only 90-percent of the spills are less than the expected total volume spilled for storage-shuttle tanker and not for export riser-pipeline. This demonstrates that large total spill volumes are more likely from export riser-pipeline operations than from storage-shuttle tanker operations.

Table 6.1. Percentile Values for Total Volume Spilled for an Individual Spar

	Total Volume Spilled (bbl)	
Percentiles	**Export Riser-Pipeline**	**Storage-Shuttle Tanker**
50%	192	62
90%	9,385	1,981
95%	15,830	5,964
99%	39,479	75,398

It should also be noted the contribution that each subsystem has to the total volume spilled and the variance for an individual spar. As shown in Figure 6.7, operations from export risers and storage contribute very little to their overall system. Operations from pipelines and shuttle tankers contribute the most risk.

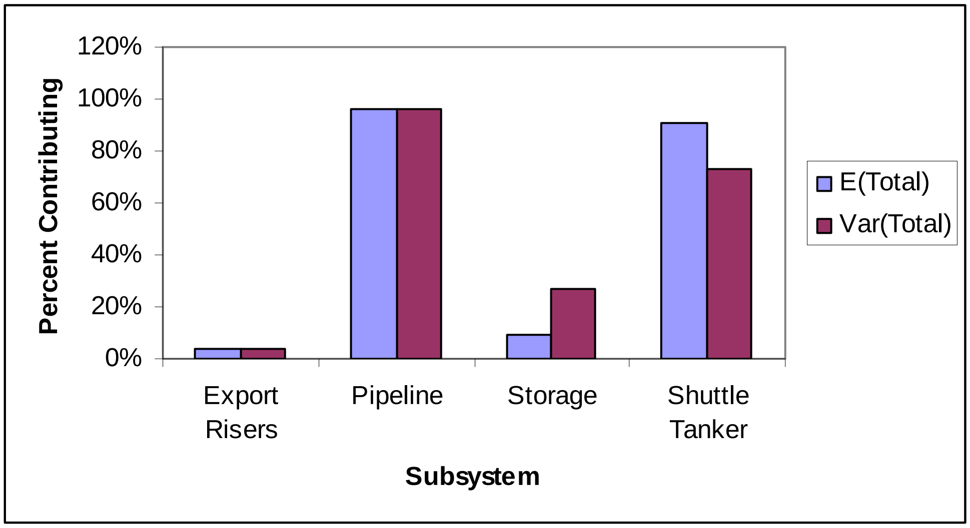

Figure 6.7. Contribution of each Subsystem to its Respective System for an Individual Spar

6.1.2 Contribution of Random Variables to Var(Total)

The random variables, N and R, created an additional uncertainty in the total volume spilled, therefore increasing the variance in the total. As expressed in Equation 4.10, the total variance is expressed in two parts: 1) the variance due to the spill size distribution and 2) the variance due to the number of occurrences. Figures 6.8 and 6.9 demonstrate the percent contributing to the total variance from the two variables for an export riser-pipeline system and storage-shuttle tanker system, respectively.

The two graphs express that the uncertainty due to the number of occurrences dominates the variance in the total volume spilled compared to the uncertainty due to the consequence, or spill size. Therefore, if more information was gained on the number of occurrences, then the overall variance would be reduced. Consequently, the magnitude of these spill sizes should also be known, therefore giving more certainty in the consequence.

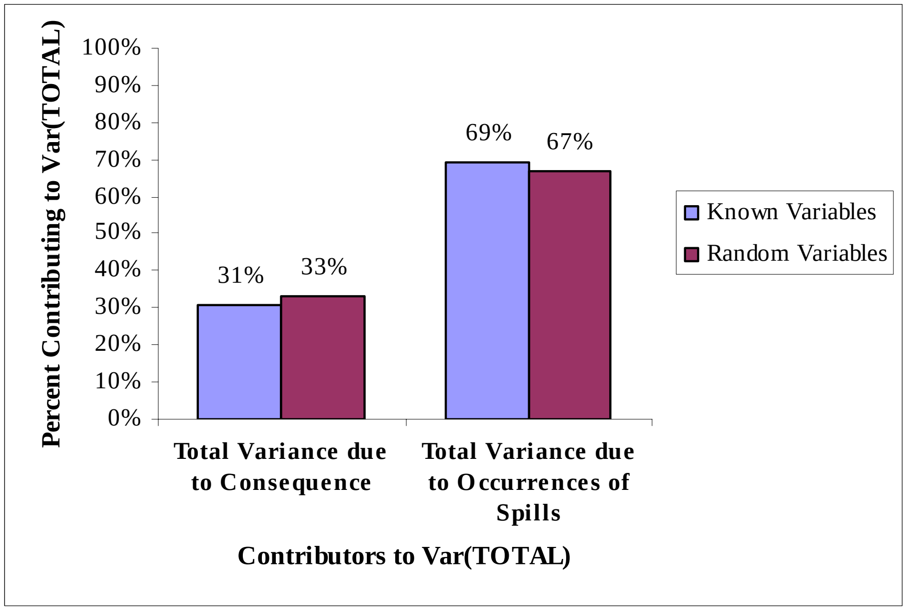

Figure 6.8. Percent Contributing to the Total Variance for an Individual Spar: Export Risers-Pipeline

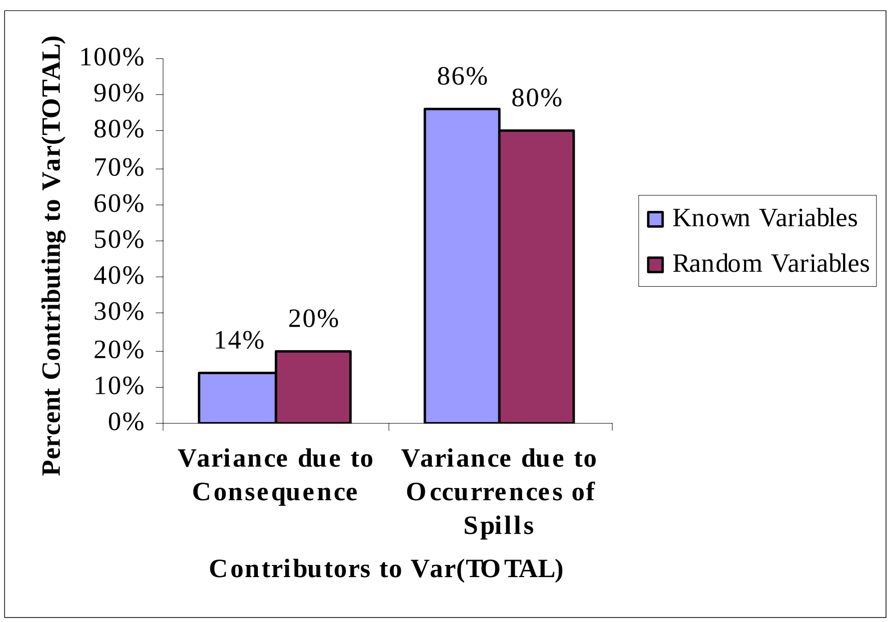

Figure 6.9. Percent Contributing to the Total Variance of an Individual Spar: Storage-Shuttle Tanker

By allowing N and R to be random variables, the overall variance in the consequence increases, whereas the variance in the occurrences decreases. Thus, it shows the impact of the uncertainty in the largest spill size range. However, the uncertainty in the number of occurrences still dominates. Therefore, the distribution of the largest spill size range should not be a major concern for an individual spar.

Comparison of the two options reveals that there is more uncertainty in the consequence for an export riser-pipeline system than for a storage-shuttle tanker system. This is due to the fact that the largest spill size for storage was assumed to be constant, i.e., R was not allowed to vary. Therefore, the only additional uncertainty in the consequence was from shuttle tankers. The lower variance in the occurrences for the export riser-pipeline system indicates that more information is known on the rate of spill occurrences for these two components.

In terms of the average spar, Figure 6.10 reveals the influence that N and R have on the variance in the total volume spilled. Its pattern is similar to those displayed in Figures 6.8 and 6.9, in that the variance in R or the consequence has a bigger influence on the total variance for a spar with storage and shuttle tanker transport.

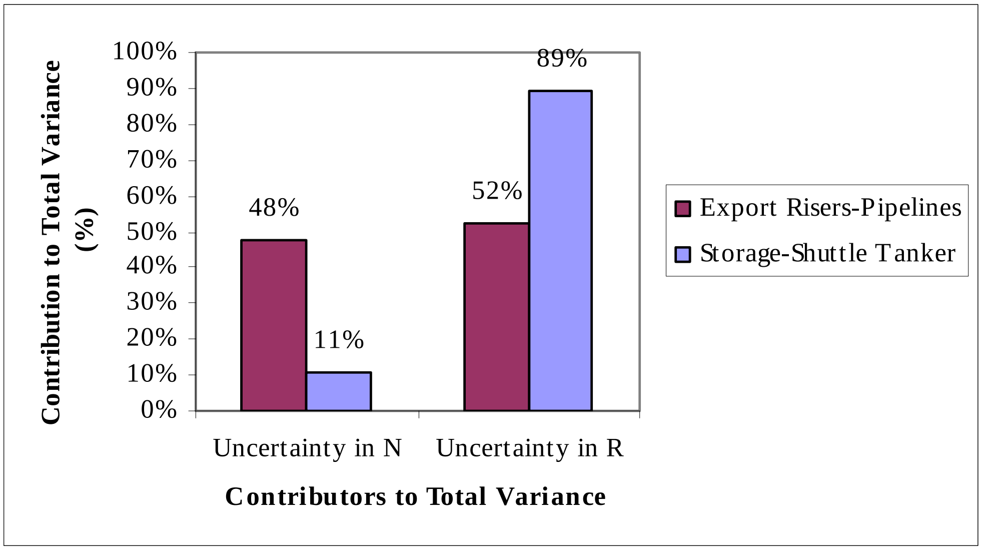

Figure 6.10. Contribution to Var(TOTAL) for Average Spar

The final results for the risk evaluation are shown as 90-percent confidence intervals on Figure 6.11. The confidence intervals were approximated using a gamma distribution for the analytical mean and variances calculated for an average spar whose N and R are random variables. The confidence intervals are wide indicating the large amount of uncertainty in the spill size distribution for the largest range. This is especially evident for a spar with storage and shuttle tanker transport. Although the expected values for the two transportation subsystems are similar, a spar with storage and shuttle tanker transport contains a greater risk as shown by the confidence intervals.

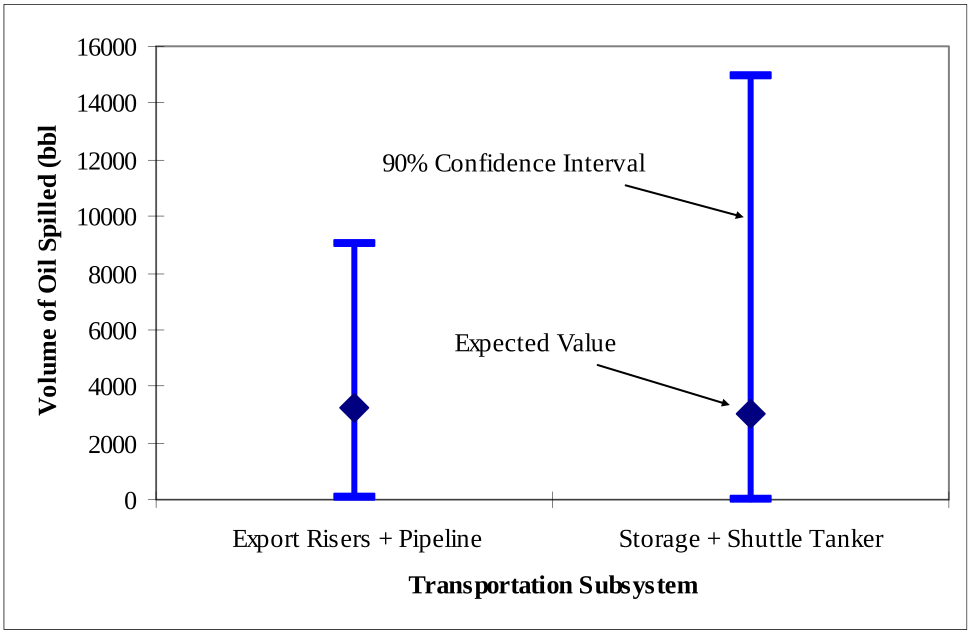

Figure 6.11. Risk Evaluation Results for an Average Spar

The frequency of spill occurrences for each spill size range also indicates that there is a significant amount of uncertainty for the larger spill ranges, as shown in Figure 6.12 with 90-percent confidence bounds. Again these bounds were determined based on a gamma distribution and the data in Tables 2.5 thru 2.8. Given the magnitude of uncertainty in the frequency of occurrence for the larger spill size range reveals that the typical size of the spill is also very uncertain due to the lack in occurrences.

The large confidence bounds for a spar with storage and shuttle tanker transport reflects the large amount of uncertainty in the combination of the two subsystems. The range for 500,000 – 1,000,000 bbls is smaller since it only includes the uncertainty for the storage subsystem. The shuttle tanker subsystem was not considered to contain a possible threat of a spill of that magnitude. In addition, the first three small spill size ranges for spar storage-shuttle tanker did not include the possibility of spills due to storage, reflecting their smaller confidence bounds compared to spar export riser-pipeline.

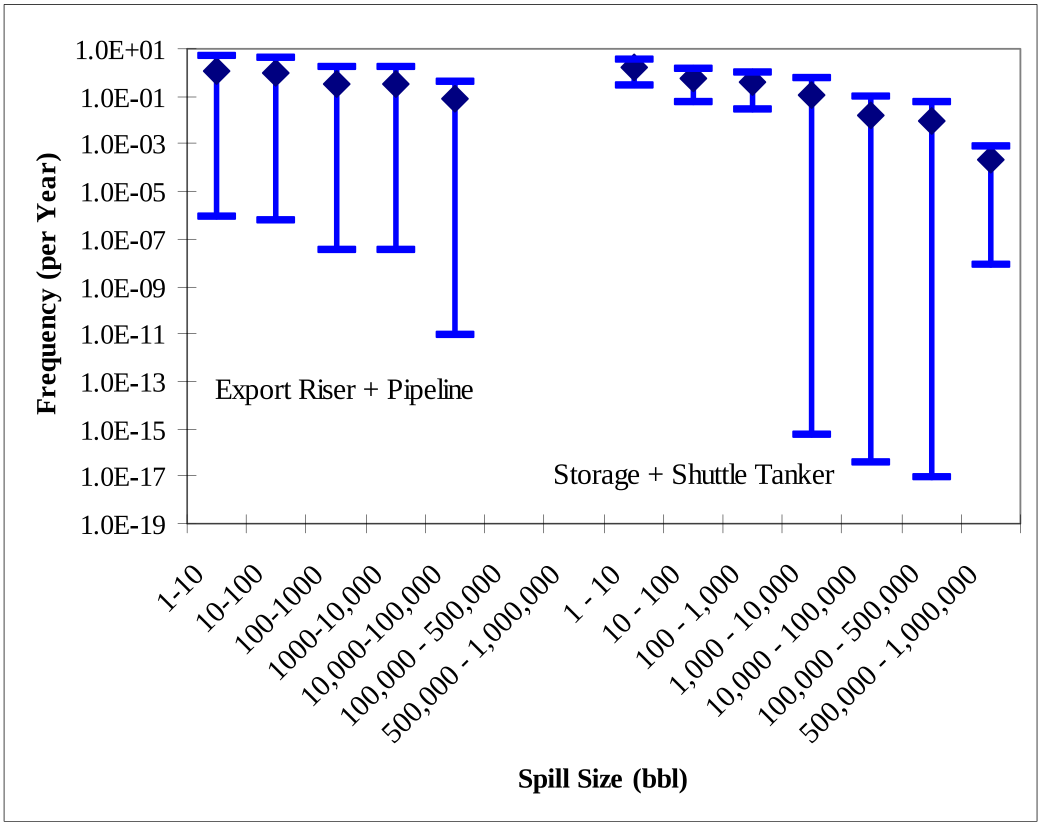

Figure 6.12. Estimated Spill Frequencies versus Spill Size

6.2 Updating Parameters

In Chapter 5, the two variable parameters determining the total volume of oil spilled, N and R, were updated. The following compares the results from the updated variances for the two spar options. The expected values were updated, but will not be discussed since they only provided a check on the procedure used for updating. The updated or posterior expected values were the same as the prior expected values, thus the method was performed correctly.

6.2.1 Variance (N”)

The effect of updating the variance in N in terms of its affect on the standard deviation of the total volume spilled is shown in Figure 6.13. As demonstrated, the standard deviation decreases with increasing spar-years, implying more information has been learned and that the uncertainty is decreasing. As noted previously, the standard deviation in the occurrences is less for the system with export risers-pipelines than for the system with storage-shuttle tankers. Therefore, the two systems for transporting oil cannot be compared for new knowledge, but both demonstrate the same trend.

From the data presented, there is a greater affect for the export riser-pipeline than storage-shuttle tanker. This would reflect ease in updating N when there are greater frequencies in spill occurrences. For the spar with storage-shuttle tanker, N for storage was only updated for the larger spill size ranges, which also correlate to lower frequencies of occurrences.

This process also reveals that there is not enough information within the first 1000 spar-years to significantly reduce the total variance due to updating N for either system. As noted in Chapter 5, the majority of the uncertainty is due to the limited quantity of data for the largest spill size ranges. It appears that after about 1,000,000 spar-years, there is sufficient information to reduce the overall variance by one order of magnitude due to updating N. After approximately 5,000 and 25,000 spar-years, the standard deviations have been reduced by 50-percent for export riser-pipeline and storage-shuttle tanker, respectively.

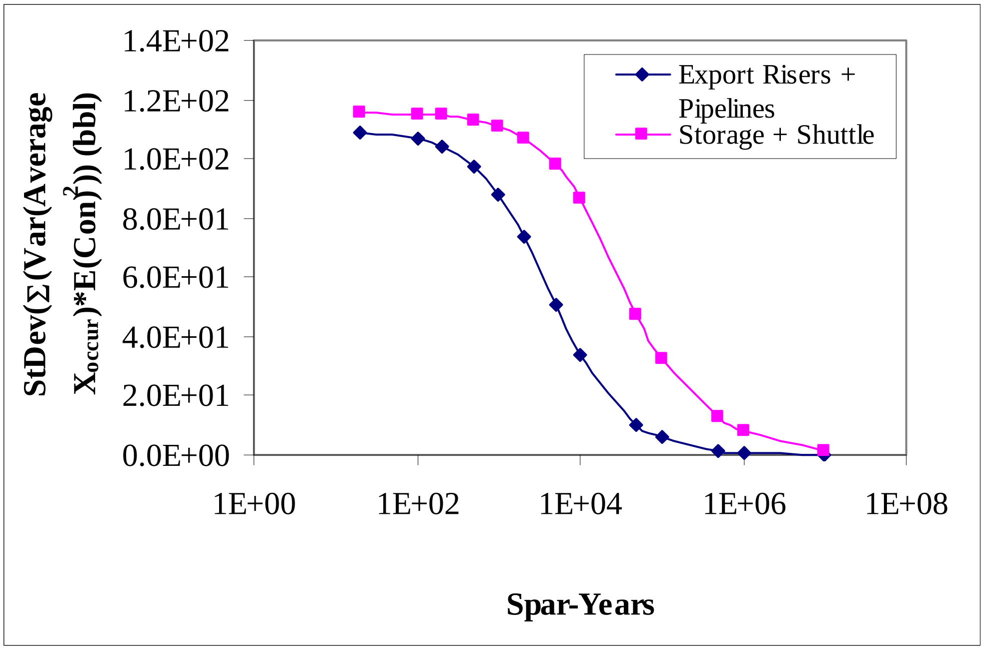

Figure 6.13. Effect of Var(N") for Both Transportation Systems

6.2.2 Variance (R")

The effect of updating the variance in R in terms of its affect on the standard deviation of the total volume spilled is shown in Figure 6.14. The standard deviation for a spar with storage-shuttle tanker is greater than for a spar with export riser-pipeline transport, reflecting the large amount if uncertainty in the spill size distribution for shuttle tanker. In addition, shuttle tanker considered a largest spill size range of 100,000 – 500,000 bbl compared to both export risers and pipelines, which considered a range of 10,000 – 100,000 to represent the possible largest spill sizes. Both options do not show much variation in the standard deviation, although there is a slight decrease (within one order of magnitude) with increasing spar-years for both systems. This also demonstrates the great lack in data for the largest spill size range and the enormous uncertainty in this parameter.

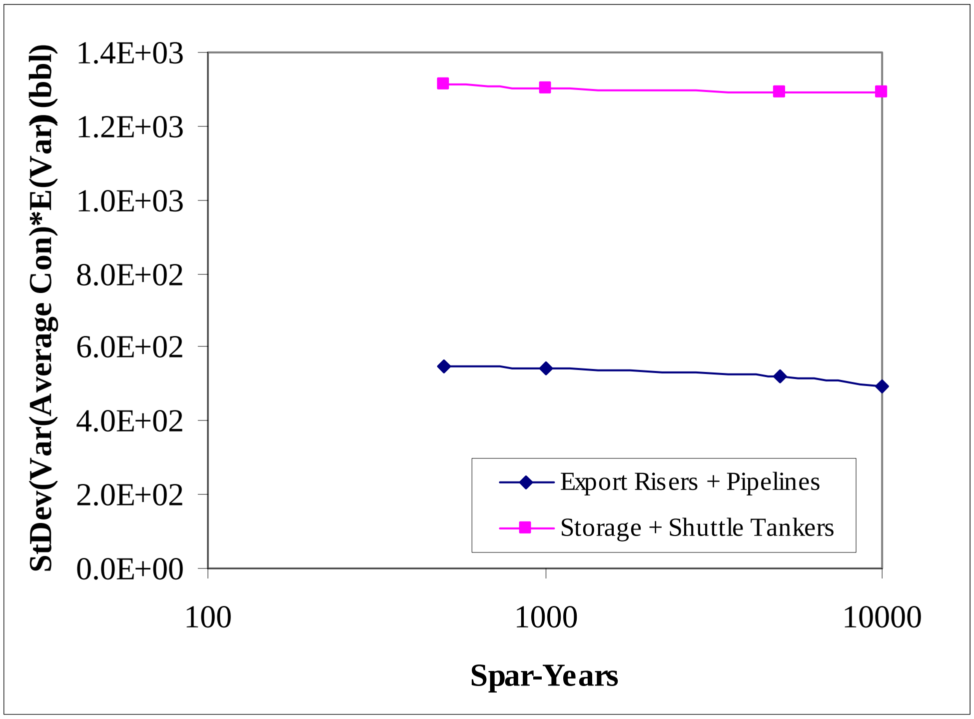

Figure 6.14. Effect of Var(R") for Both Transportation Systems

Overall, updating the variance for both parameters, N and R, decreases the standard deviation in the total volume spilled with increasing spar-years, as displayed in Figure 6.15. The subtle decrease is a reflection of the magnitude of the standard deviation for the major contributing factor, which is R. As shown in Figure 6.16, the major contributing parameter to the standard deviation for both systems of transportation is R. With increasing spar-years, the uncertainty in R is reduced faster for export riser-pipeline than for storage-shuttle tanker reflecting the enormous amount of uncertainty in the largest spill size range for the shuttle tanker.

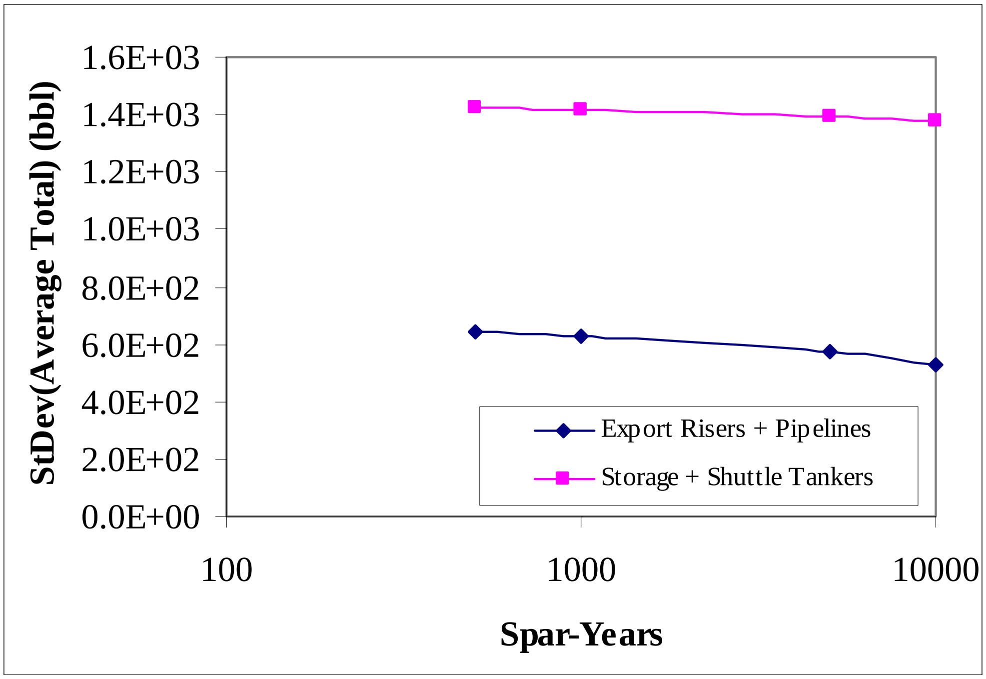

Figure 6.15. Overall Effect of Updating N and R for Both Transportation Systems

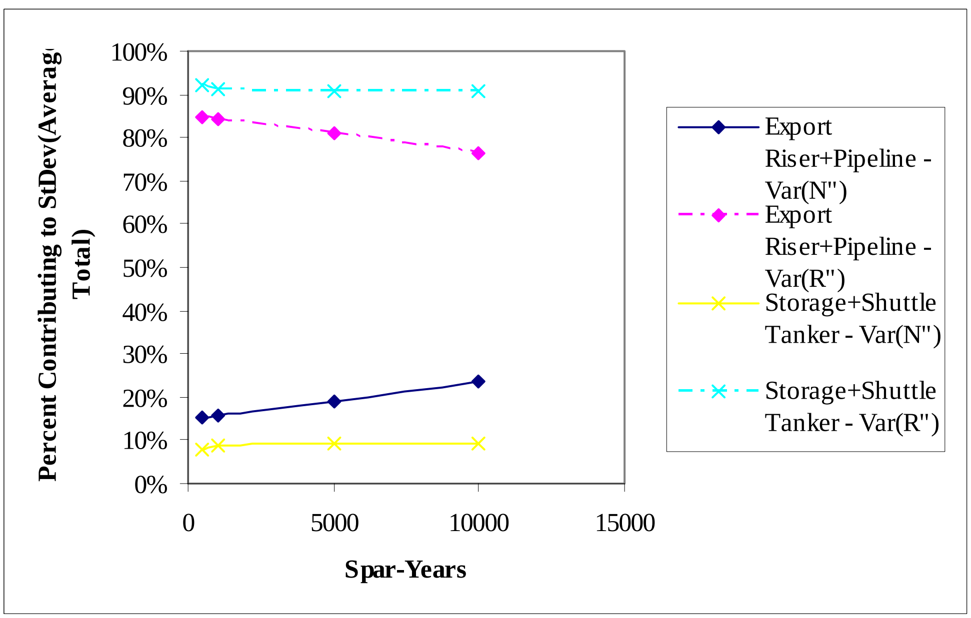

Figure 6.16. Contribution of N and R to StDev(Average Total)

6.3 Conclusion

The final results suggest that although both subsystems have a similar expected volume of oil spilled, the uncertainty is much greater for a spar with storage and shuttle tanker transport. The 90-percent confidence bounds range over an order of magnitude, reflecting the limited amount of historical data available to predict rare events.

Updating the mean rate of occurrence, N, and the statistical spill size distribution parameter, R, can aid in the reduction of the standard deviation in the total volume spilled for an average spar. However, for the spar-years analyzed, the reduction is small. This suggests that much more data, over 10,000 spar-years, is required from spar operations in the Gulf of Mexico before the uncertainties in the results will decrease with any significance. The uncertainty in the distribution of spill sizes for the largest spill size range contains the most uncertainty in terms of the standard deviation of the total volume spilled for an average spar.

Chapter 7: Conclusions and Recommendations

The following will present the conclusions and recommendations for future work.

7.0 Conclusions

A quantitative risk analysis was completed to analyze the risk to the environment associated with proposed oil storage on spars in the Gulf of Mexico. This risk was analyzed in terms of the environmental risk posed from transporting oil to the shore from a spar production facility. Two transportation systems were considered: the conventional system of export risers and a pipeline to the shore and an alternative system of storage in the spar with offloading and transport by shuttle tankers. The technique implemented was similar to the CRA model developed by Gilbert et al (2001a), but was extended to eliminate restrictions in the model and to incorporate those uncertainties. The additional uncertainties considered were in the mean rate of spill occurrence and in the spill size distribution for the largest spill size range. Input for analysis was based on data from the CRA report and compared to information from similar operations in the North Sea.

The following major conclusions have been drawn from the results of this analysis:

1. The risk due to spar storage with shuttle tanker transport is the same as a spar utilizing export risers and pipelines, based on the magnitude of the confidence bounds.
2. The major contribution to the total volume of oil spilled is from the largest spill size ranges. They also represent the greatest amount of uncertainty for the systems.

3. The confidence intervals for the total volume spilled from an average spar each system range over several orders of magnitude, reflecting the uncertainty due to the limited quantity and quality of historical data for the larger spill size ranges.
4. Updating the input parameters pertaining to the mean rate of occurrence and the spill size distribution can help to reduce the uncertainty in the average total volume spilled. However, it appears that it would take approximately one million spar-years before the uncertainty could be reduced effectively.

7.1 Recommendations

The following recommendations have been developed from this research:

1. The data should be periodically updated to incorporate new information. This will improve the uncertainties in the average total volume spilled.
2. This study could be extended to include oil spill risks from other subsystems, acute oil spill risks, and risk to personnel. Gilbert et al. (2001a) provides a basis for making this extension.

Appendix A –Monte Carlo Simulation Code

```
Sub Simulation()
Dim num As Integer, spars As Integer, all As Integer, n As Integer, x As Integer,
a As Integer, b As Integer
Dim i As Integer, J As Integer, k As Integer, ri As Integer, size As Integer,
realization As Integer, freq As Integer
Dim total As Single, max As Single, random As Single

Sheets("simulation").Select
'Determine number of spill size ranges
num = Range("B2").Value
'Determine number of spars/realizations in GoM
spars = Range("B3").Value

'Repeat generation of everything 10 times to improve the total expected spill
size including all spill size ranges
For all = 1 To 1

'Repeat generation of 100 realizations 100 times; repeating each ν 10,000
times
For ri = 1 To 1

'Generate random number for new ν
For n = 1 To num
Sheets("Poisson Distribution").Select
'Change ν for each spill size range
Range(Cells(5 + n, 13), Cells(5 + n, 13)) = Rnd
Range(Cells(5 + n, 14), Cells(5 + n, 14)).Select
Selection.Copy
Range("G2").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False

'Random value of R
Range("G21") = Rnd * (5 - 1) + 1
Range("G21").Select
Selection.Copy
Sheets("Update r").Select
Range(Cells(2 + ri + (100 * (all - 1)), 1), Cells(2 + ri + (100 * (all - 1)), 1)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
```

```
'Generate 100 random numbers for 100 realizations
Sheets("simulation").Select
For x = 1 To spars
   Range(Cells(x + 5, 2), Cells(x + 5, 2)) = Rnd
Next x

'Generate random spill size for range for each realization
For a = 1 To spars
   For b = 1 To 20
      random = Rnd
      If n <> num Then
      'Uniform Distribution
      Range(Cells(5 + a, 3 + b), Cells(5 + a, 3 + b)) = Exp((random * (Log(10 ^ n) - Log(10 ^ (n - 1))) + Log(10 ^ (n - 1))))
      Else
      'Beta Distribution
      Sheets("Poisson Distribution").Select
      Range("G22") = Rnd
      Range("G23").Select
      Selection.Copy
      Sheets("Simulation").Select
      Range(Cells(5 + a, 3 + b), Cells(5 + a, 3 + b)).Select
         Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False
      'Copy number of occurrences and spill sizes to update R
       Range(Cells(5 + a, 3), Cells(5 + a, 7)).Select
       Selection.Copy
       Sheets("Update r").Select
       Range(Cells(2 + ri + (100 * (all - 1)), 2 + (a * 9 - 9)), Cells(2 + ri + (100 * (all - 1)), 6 + (a * 9 - 9))).Select
         Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False
      End If
   Next b
Next a

'Add number of total spilled for each realization
Sheets("Simulation").Select
For i = 1 To spars
J = 0
k = 0
```

```
total = 0
    If Range(Cells(i + 5, 3), Cells(i + 5, 3)) = 0 Then
    Range(Cells(i + 5, 24), Cells(i + 5, 24)) = 0
    Range(Cells(i + 5, 25), Cells(i + 5, 25)) = 0
    End If
    If Range(Cells(i + 5, 3), Cells(i + 5, 3)) = 1 Then
    Range(Cells(i + 5, 25), Cells(i + 5, 25)) = Range(Cells(i + 5, 4), Cells(i + 5, 4))
    End If
        Do While Range(Cells(i + 5, 3), Cells(i + 5, 3)) > J
            total = total + Range(Cells(i + 5, J + 4), Cells(i + 5, J + 4))
            J = J + 1
            Range(Cells(i + 5, 24), Cells(i + 5, 24)) = total
        Loop
                    'Find maximum spilled
                    'max = Range(Cells(i + 5, 4), Cells(i + 5, 4))
                    'Do While k <= J - 2
                    '   If Range(Cells(i + 5, k + 5), Cells(i + 5, k + 5)) < max Then
                    '       max = max
                    '   Else
                    '       max = Range(Cells(i + 5, k + 5), Cells(i + 5, k + 5))
                    '   End If
                    '   k = k + 1
                    '   Range(Cells(i + 5, 25), Cells(i + 5, 25)) = max
                    'Loop

'Copy total spilled(random)for each realization to be added to all spill sizes for each realization
Range(Cells(i + 5, 24), Cells(i + 5, 24)).Select
Selection.Copy
Range(Cells(i + 5, n + 26), Cells(i + 5, n + 26)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False

'Copy total spilled(average)for each realization to be added to all spill sizes for each realization
Range(Cells(i + 5, 3), Cells(i + 5, 3)).Select
Selection.Copy
Range(Cells(i + 5, n + 37), Cells(i + 5, n + 37)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False
```

'Copy maximum spilled for each realization to be added to all spill sizes for each realization

```
'Range(Cells(i + 5, 25), Cells(i + 5, 25)).Select
'Selection.Copy
'Range(Cells(i + 5, n + 47), Cells(i + 5, n + 47)).Select
'Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Next i
Next n
```

'Results
'Copy calculations from 100 realizations performed 100 times
'Copy results from random spill sizes

```
Sheets("Simulation").Select
Range("BQ13:BQ16").Select
Selection.Copy
Sheets("Results").Select
Range(Cells(3 + ri, 2), Cells(3 + ri, 5)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=True
```

'Copy results from random spill size ranges

```
'Sheets("Simulation").Select
'Range("BQ20:BQ33").Select
'Selection.Copy
'Sheets("Ranges").Select
'Range(Cells(4 + ri, 2), Cells(4 + ri, 15)).Select
'    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone,
     SkipBlanks:=False, Transpose:=True
```

'Copy results (random spill size) for each spar

```
For realization = 1 To spars
Sheets("Simulation").Select
Range(Cells(5 + realization, 27), Cells(5 + realization, 33)).Select
Selection.Copy
If realization = 1 Then
    Sheets("spar1").Select
    Else
        If realization = 2 Then
            Sheets("spar2").Select
    Else
```

```
    If realization = 3 Then
        Sheets("spar3").Select
  Else
    If realization = 4 Then
        Sheets("spar4").Select
  Else
    If realization = 5 Then
        Sheets("spar5").Select
  Else
    If realization = 6 Then
        Sheets("spar6").Select
  Else
    If realization = 7 Then
        Sheets("spar7").Select
  Else
    If realization = 8 Then
        Sheets("spar8").Select
  Else
    If realization = 9 Then
        Sheets("spar9").Select
  Else
    If realization = 10 Then
        Sheets("spar10").Select
End If
End If
End If
End If
End If
End If
End If
End If
End If
End If
Range(Cells(4 + ri + (100 * all - 100), 2), Cells(4 + ri + (100 * all - 100),
8)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False

'Copy results (# of occurrences) for each spar
Sheets("Simulation").Select
Range(Cells(5 + realization, 38), Cells(5 + realization, 44)).Select
```

```
Selection.Copy
If realization = 1 Then
    Sheets("spar1").Select
    Else
        If realization = 2 Then
            Sheets("spar2").Select
    Else
        If realization = 3 Then
            Sheets("spar3").Select
    Else
        If realization = 4 Then
            Sheets("spar4").Select
    Else
        If realization = 5 Then
            Sheets("spar5").Select
    Else
        If realization = 6 Then
            Sheets("spar6").Select
    Else
        If realization = 7 Then
            Sheets("spar7").Select
    Else
        If realization = 8 Then
            Sheets("spar8").Select
    Else
        If realization = 9 Then
            Sheets("spar9").Select
    Else
        If realization = 10 Then
            Sheets("spar10").Select
End If
End If
End If
End If
End If
End If
End If
End If
End If
End If
```

```
Range(Cells(4 + ri + (100 * all - 100), 11), Cells(4 + ri + (100 * all - 100),
17)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
```

'Copy results (max random spill) for each spar

```
'Sheets("Simulation").Select
'Range(Cells(5 + realization, 48), Cells(5 + realization, 54)).Select
'Selection.Copy
'If realization = 1 Then
'    Sheets("spar1").Select
'    Else
'        If realization = 2 Then
'            Sheets("spar2").Select
'    Else
'        If realization = 3 Then
'            Sheets("spar3").Select
'    Else
'        If realization = 4 Then
'            Sheets("spar4").Select
'    Else
'        If realization = 5 Then
'            Sheets("spar5").Select
'   Else
'        If realization = 6 Then
'            Sheets("spar6").Select
'    Else
'        If realization = 7 Then
'            Sheets("spar7").Select
'    Else
'        If realization = 8 Then
'            Sheets("spar8").Select
'    Else
'        If realization = 9 Then
'            Sheets("spar9").Select
'    Else
'        If realization = 10 Then
'            Sheets("spar10").Select
'End If
'End If
'End If
```

```
'End If
'End If
'End If
'End If
'End If
'End If
'End If
'Range(Cells(4 + ri + (100 * all - 100), 20), Cells(4 + ri + (100 * all - 100), 26)).Select
'    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone,
     SkipBlanks:=False, Transpose:=False
Next realization
```

'End Spars

'Copy results from assumed spill sizes

```
Sheets("Simulation").Select
Range("BR13:BR16").Select
Selection.Copy
Sheets("Results").Select
Range(Cells(3 + ri, 6), Cells(3 + ri, 9)).Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone,
    SkipBlanks:=False, Transpose:=True
```

'Copy results from average spill size ranges

```
'Sheets("Simulation").Select
'Range("BR20:BR33").Select
'Selection.Copy
'Sheets("Ranges").Select
'Range(Cells(4 + ri, 16), Cells(4 + ri, 29)).Select
'    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone,
     SkipBlanks:=False, Transpose:=True
```

'Copy average number of occurrences from average spill size ranges

```
'Sheets("Simulation").Select
'Range("BU4:BU10").Select
'Selection.Copy
'Sheets("Ranges").Select
'Range(Cells(4 + ri, 62), Cells(4 + ri, 68)).Select
'    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone,
     SkipBlanks:=False, Transpose:=True
```

Next ri

'Copy overall expected value for 10 iterations
Sheets("Results").Select
Range(Cells(104, 2), Cells(104, 9)).Select
Selection.Copy
Range(Cells(3 + all, 12), Cells(3 + all, 19)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False

'Copy overall expected value for 10 iterations for all spill size ranges
'Sheets("Ranges").Select
'Range(Cells(109, 2), Cells(109, 29)).Select
'Selection.Copy
'Range(Cells(4 + all, 32), Cells(4 + all, 59)).Select
' Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False

'Copy overall average number of occurrences from assumed spill size ranges
'Sheets("Ranges").Select
'Range(Cells(105, 62), Cells(105, 68)).Select
'Selection.Copy
'Range(Cells(4 + all, 71), Cells(4 + all, 77)).Select
' Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False
Next all

'Copy individual spar info to spar summary sheet
Sheets("spar1").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(4, 2), Cells(4, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False, Transpose:=False
Sheets("spar2").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(5, 2), Cells(5, 34)).Select

```
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar3").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(6, 2), Cells(6, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar4").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(7, 2), Cells(7, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar5").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(8, 2), Cells(8, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar6").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(9, 2), Cells(9, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar7").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(10, 2), Cells(10, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar8").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
```

```
Range(Cells(11, 2), Cells(11, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar9").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(12, 2), Cells(12, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar10").Select
Range(Cells(1005, 2), Cells(1005, 34)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(13, 2), Cells(13, 34)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False

'Copy frequency of total spilled for individual spar to spar summary sheet
Sheets("spar1").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45, 2), Cells(45, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar2").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(46, 2), Cells(46, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar3").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(47, 2), Cells(47, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar4").Select
```

```
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(48, 2), Cells(48, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar5").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(49, 2), Cells(49, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar6").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(50, 2), Cells(50, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar7").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(51, 2), Cells(51, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar8").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(52, 2), Cells(52, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar9").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(53, 2), Cells(53, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
```

```
Sheets("spar10").Select
Range(Cells(1010, 11), Cells(1010, 16)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(54, 2), Cells(54, 7)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
```

'Copy frequency in each spill size range

```
For freq = 1 To num
Sheets("spar1").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar2").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar3").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar4").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar5").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
```

```
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar6").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar7").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar8").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar9").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Sheets("spar10").Select
Range(Cells(1022, 20 + freq), Cells(1032, 20 + freq)).Select
Selection.Copy
Sheets("spar summary").Select
Range(Cells(45 + (freq * 14 - 14), 24), Cells(55 + (freq * 14 - 14), 24)).Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False
Next freq
```

```
'Copy variance in r to summary sheet
Sheets("Update r").Select
Range("C1006:C1015").Select
Selection.Copy
Sheets("spar summary").Select
Range("AJ4:AJ13").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:=False,
Transpose:=False

End Sub
```

Appendix B – Tables for Updating R

Table B.1. Updating R for 10,000-100,000 bbl; z = 1 to 3

z = 1				z = 2				z = 3			
y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR
0.01	42.32	0.115	0.143	0.01	97.14	0.050	0.062	0.01	222.82	0.022	0.027
0.05	27.97	0.188	0.261	0.10	51.64	0.106	0.154	0.10	121.04	0.045	0.063
0.10	21.91	0.255	0.382	0.20	38.69	0.152	0.243	0.30	76.55	0.078	0.128
0.15	18.41	0.318	0.508	0.30	31.42	0.199	0.342	0.50	57.57	0.113	0.202
0.20	15.97	0.382	0.646	0.40	26.45	0.248	0.456	0.70	45.94	0.150	0.291
0.25	14.11	0.450	0.801	0.50	22.73	0.303	0.589	0.90	37.82	0.193	0.399
0.30	12.61	0.523	0.975	0.60	19.80	0.363	0.744	1.10	31.75	0.242	0.530
0.35	11.36	0.601	1.173	0.70	17.41	0.430	0.925	1.30	27.02	0.298	0.687
0.40	10.29	0.687	1.397	0.80	15.42	0.505	1.135	1.50	23.22	0.362	0.875
0.45	9.37	0.779	1.649	0.90	13.74	0.588	1.377	1.70	20.12	0.435	1.096
0.50	8.57	0.880	1.934	1.00	12.29	0.680	1.655	1.90	17.55	0.518	1.355
0.55	7.85	0.990	2.255	1.10	11.04	0.783	1.973	2.10	15.39	0.612	1.657
0.60	7.21	1.110	2.614	1.20	9.95	0.897	2.334	2.30	13.56	0.717	2.005
0.65	6.64	1.241	3.015	1.30	8.99	1.023	2.742	2.50	12.00	0.836	2.404
0.70	6.13	1.383	3.461	1.40	8.15	1.162	3.201	2.70	10.65	0.969	2.858
0.75	5.66	1.538	3.956	1.50	7.40	1.315	3.716	2.90	9.49	1.116	3.373
0.80	5.24	1.706	4.504	1.60	6.73	1.482	4.290	3.10	8.48	1.280	3.953
0.85	4.85	1.888	5.108	1.70	6.14	1.666	4.929	3.30	7.61	1.462	4.604
0.90	4.50	2.085	5.773	1.80	5.61	1.866	5.636	3.50	6.84	1.662	5.331
0.95	4.18	2.298	6.502	1.90	5.13	2.085	6.416	3.70	6.16	1.881	6.139
1.00	3.88	2.528	7.299	2.00	4.70	2.323	7.275	3.90	5.57	2.122	7.035
1.05	3.61	2.776	8.168	2.10	4.31	2.581	8.217	4.10	5.04	2.386	8.023
1.10	3.36	3.042	9.114	2.20	3.97	2.860	9.247	4.30	4.58	2.673	9.109
1.15	3.14	3.328	10.141	2.30	3.65	3.162	10.370	4.50	4.16	2.985	10.301
1.20	2.92	3.635	11.253	2.40	3.37	3.487	11.591	4.70	3.80	3.323	11.603
1.25	2.73	3.963	12.456	2.50	3.11	3.837	12.917	4.90	3.47	3.690	13.023
1.30	2.55	4.314	13.753	2.60	2.87	4.214	14.352	5.10	3.17	4.086	14.567
1.35	2.39	4.690	15.149	2.70	2.66	4.618	15.902	5.30	2.91	4.513	16.241
1.40	2.23	5.090	16.649	2.80	2.46	5.050	17.574	5.50	2.67	4.972	18.052
1.45	2.09	5.516	18.259	2.90	2.29	5.513	19.371	5.70	2.46	5.464	20.008
1.50	1.96	5.969	19.983	3.00	2.12	6.006	21.302	5.90	2.26	5.993	22.115
1.55	1.84	6.450	21.826	3.10	1.98	6.533	23.371	6.10	2.09	6.558	24.380
1.60	1.73	6.961	23.793	3.20	1.84	7.093	25.585	6.30	1.93	7.162	26.811
1.65	1.62	7.503	25.890	3.30	1.72	7.689	27.951	6.50	1.79	7.806	29.416
1.70	1.53	8.076	28.122	3.40	1.60	8.322	30.474	6.70	1.66	8.492	32.201
1.75	1.44	8.682	30.495	3.50	1.50	8.993	33.162	6.90	1.54	9.222	35.175
1.80	1.35	9.323	33.013	3.60	1.40	9.703	36.020	7.10	1.43	9.997	38.346
1.85	1.27	9.999	35.684	3.70	1.31	10.455	39.056	7.30	1.33	10.820	41.721
1.90	1.20	10.712	38.512	3.80	1.23	11.250	42.277	7.50	1.24	11.691	45.308
1.95	1.13	11.462	41.503	3.90	1.15	12.089	45.689	7.70	1.16	12.613	49.117
2.00	1.07	12.253	44.664	4.00	1.08	12.973	49.300	7.90	1.08	13.588	53.155
2.05	1.01	13.083	47.999	4.10	1.02	13.906	53.116	8.10	1.01	14.617	57.431
2.10	0.96	13.956	51.516	4.20	0.96	14.887	57.145	8.30	0.95	15.703	61.953
2.15	0.91	14.872	55.221	4.30	0.90	15.918	61.395	8.50	0.89	16.847	66.731
2.20	0.86	15.833	59.119	4.40	0.85	17.002	65.873	8.70	0.83	18.051	71.773
2.25	0.81	16.840	63.217	4.50	0.80	18.140	70.586	8.90	0.78	19.318	77.089
2.30	0.77	17.895	67.522	4.60	0.75	19.334	75.542	9.10	0.73	20.648	82.686
2.35	0.73	18.999	72.039	4.70	0.71	20.585	80.749	9.30	0.69	22.045	88.576
2.40	0.69	20.153	76.776	4.80	0.67	21.895	86.215	9.50	0.65	23.510	94.767
				4.90	0.64	23.265	91.948	9.70	0.61	25.046	101.268
				5.00	0.60	24.699	97.955	9.90	0.58	26.654	108.090
				5.10	0.57	26.196	104.246	10.10	0.55	28.337	115.242
				5.20	0.54	27.760	110.829	10.30	0.52	30.097	122.734
				5.30	0.51	29.392	117.711	10.50	0.49	31.935	130.576
				5.40	0.49	31.093	124.902	10.70	0.46	33.855	138.779
								10.90	0.44	35.859	147.352
								11.10	0.42	37.948	156.306
								11.30	0.39	40.125	165.651
								11.50	0.37	42.392	175.399
								11.70	0.36	44.752	185.559
								11.90	0.34	47.207	196.144
								12.10	0.32	49.759	207.163
								12.30	0.31	52.411	218.627

Table B.2. Updating R for 10,000-100,000 bbl; z = 4 to 6

z = 4				z = 5				z = 6			
y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR
0.01	510.84	0.009	0.012	0.01	1170.616	0.004	0.005	0.01	2681.380	0.002	0.002
0.20	219.11	0.026	0.038	0.93	242.344	0.027	0.048	1.54	435.100	0.016	0.029
0.65	121.87	0.053	0.094	2.03	135.176	0.056	0.117	4.04	193.092	0.042	0.095
1.10	84.79	0.084	0.165	3.13	88.512	0.094	0.221	6.54	111.084	0.082	0.209
1.55	63.54	0.121	0.260	4.23	62.220	0.145	0.371	9.04	70.936	0.141	0.391
2.00	49.52	0.165	0.383	5.33	45.636	0.212	0.579	11.54	48.164	0.222	0.662
2.45	39.58	0.219	0.540	6.43	34.476	0.298	0.859	14.04	34.120	0.333	1.042
2.90	32.20	0.284	0.737	7.53	26.640	0.405	1.224	16.54	24.960	0.477	1.559
3.35	26.55	0.360	0.980	8.63	20.956	0.538	1.688	19.04	18.736	0.662	2.237
3.80	22.14	0.451	1.276	9.73	16.736	0.700	2.268	21.54	14.364	0.893	3.105
4.25	18.63	0.556	1.632	10.83	13.544	0.894	2.980	24.04	11.216	1.178	4.195
4.70	15.80	0.679	2.055	11.93	11.084	1.125	3.842	26.54	8.892	1.523	5.537
5.15	13.49	0.820	2.553	13.03	9.164	1.396	4.871	29.04	7.152	1.936	7.166
5.60	11.60	0.982	3.133	14.13	7.644	1.713	6.088	31.54	5.820	2.426	9.117
6.05	10.02	1.167	3.804	15.23	6.428	2.079	7.512	34.04	4.788	3.000	11.427
6.50	8.71	1.375	4.575	16.33	5.448	2.499	9.164	36.54	3.976	3.667	14.135
6.95	7.60	1.610	5.454	17.43	4.648	2.979	11.067	39.04	3.332	4.435	17.281
7.40	6.67	1.874	6.450	18.53	3.992	3.522	13.241	41.54	2.816	5.315	20.907
7.85	5.87	2.168	7.574	19.63	3.448	4.134	15.711	44.04	2.396	6.315	25.056
8.30	5.19	2.494	8.834	20.73	2.996	4.821	18.500	46.54	2.052	7.446	29.773
8.75	4.61	2.856	10.242	21.83	2.612	5.588	21.633	49.04	1.768	8.717	35.105
9.20	4.10	3.255	11.806	22.93	2.292	6.440	25.137	51.54	1.532	10.140	41.100
9.65	3.66	3.693	13.539	24.03	2.020	7.384	29.036	54.04	1.336	11.725	47.806
10.10	3.28	4.174	15.450	25.13	1.784	8.424	33.358	56.54	1.172	13.482	55.276
10.55	2.95	4.699	17.552	26.23	1.584	9.568	38.130	59.04	1.028	15.424	63.560
11.00	2.66	5.271	19.855	27.33	1.412	10.822	43.382	61.54	0.912	17.563	72.712
11.45	2.40	5.893	22.372	28.43	1.264	12.192	49.142	64.04	0.808	19.909	82.789
11.90	2.18	6.567	25.113	29.53	1.132	13.684	55.440	66.54	0.720	22.476	93.845
12.35	1.98	7.297	28.093	30.63	1.020	15.305	62.306	69.04	0.644	25.276	105.939
12.80	1.80	8.084	31.323	31.73	0.920	17.062	69.772	71.54	0.576	28.322	119.131
13.25	1.64	8.932	34.816	32.83	0.832	18.962	77.869	74.04	0.520	31.627	133.480
13.70	1.50	9.843	38.585	33.93	0.756	21.012	86.631	76.54	0.468	35.205	149.049
14.15	1.38	10.821	42.643	35.03	0.688	23.219	96.090	79.04	0.424	39.069	165.901
14.60	1.26	11.868	47.005	36.13	0.624	25.591	106.281	81.54	0.384	43.233	184.101
15.05	1.16	12.988	51.684	37.23	0.572	28.135	117.237	84.04	0.348	47.712	203.714
15.50	1.07	14.183	56.694	38.33	0.524	30.859	128.995	86.54	0.316	52.519	224.808
15.95	0.99	15.457	62.049	39.43	0.480	33.770	141.590	89.04	0.292	57.671	247.452
16.40	0.91	16.813	67.765	40.53	0.440	36.877	155.059	91.54	0.264	63.182	271.714
16.85	0.84	18.254	73.856	41.63	0.408	40.188	169.439	94.04	0.244	69.066	297.667
17.30	0.78	19.784	80.337	42.73	0.376	43.710	184.768	96.54	0.224	75.341	325.383
17.75	0.73	21.406	87.224	43.83	0.348	47.453	201.084	99.04	0.208	82.022	354.935
18.20	0.68	23.123	94.532	44.93	0.320	51.424	218.426	101.54	0.192	89.125	386.399
18.65	0.63	24.938	102.277	46.03	0.296	55.633	236.835	104.04	0.176	96.666	419.850
19.10	0.59	26.856	110.475	47.13	0.276	60.088	256.351	106.54	0.164	104.662	455.367
19.55	0.55	28.879	119.143	48.23	0.256	64.798	277.015	109.04	0.152	113.130	493.027
20.00	0.51	31.012	128.297	49.33	0.240	69.771	298.869	111.54	0.140	122.088	532.912
20.45	0.48	33.258	137.954	50.43	0.224	75.018	321.954	114.04	0.132	131.552	575.102
20.90	0.45	35.621	148.131	51.53	0.208	80.547	346.314	116.54	0.120	141.541	619.680
21.35	0.42	38.105	158.846	52.63	0.196	86.368	371.993	119.04	0.112	152.072	666.729
21.80	0.40	40.712	170.115	53.73	0.184	92.490	399.034	121.54	0.104	163.164	716.334
22.25	0.37	43.448	181.958	54.83	0.172	98.922	427.482	124.04	0.100	174.835	768.582
22.70	0.35	46.316	194.391	55.93	0.160	105.676	457.384	126.54	0.092	187.104	823.559
23.15	0.33	49.320	207.433	57.03	0.152	112.760	488.784	129.04	0.088	199.990	881.355
23.60	0.31	52.465	221.103	58.13	0.140	120.185	521.729	131.54	0.080	213.512	942.060
24.05	0.29	55.753	235.419	59.23	0.132	127.960	556.267	134.04	0.076	227.689	1006.000
24.50	0.28	59.189	250.400	60.33	0.124	136.097	592.446	136.54	0.072	242.542	1073.000
24.95	0.26	62.778	266.066	61.43	0.116	144.605	630.314	139.04	0.068	258.091	1143.000
25.40	0.25	66.524	282.435	62.53	0.112	153.495	669.919	141.54	0.064	274.355	1216.000
25.85	0.24	70.430	299.528	63.63	0.104	162.778	711.312	144.04	0.060	291.355	1292.000
26.30	0.22	74.502	317.364	64.73	0.100	172.464	754.543	146.54	0.056	309.111	1373.000
26.75	0.21	78.743	335.964					149.04	0.052	327.646	1456.000
27.20	0.20	83.157	355.348								
27.65	0.19	87.750	375.535								
28.10	0.18	92.526	396.548								

Table B.3. Updating R for 100,000-500,000 bbl; z = 1 to 3

z = 1				z = 2				z = 3			
y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R)dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R)dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R)dR
0.010	27.332	0.180	0.228	0.010	40.412	0.123	0.158	0.010	59.548	0.084	0.110
0.030	20.492	0.254	0.347	0.050	25.016	0.219	0.320	0.040	39.068	0.140	0.202
0.060	16.240	0.339	0.498	0.100	18.700	0.316	0.507	0.110	25.212	0.241	0.397
0.090	13.784	0.417	0.649	0.150	15.152	0.413	0.715	0.180	19.020	0.344	0.623
0.120	12.068	0.495	0.810	0.200	12.728	0.518	0.957	0.250	15.180	0.459	0.898
0.150	10.752	0.575	0.986	0.250	10.920	0.633	1.240	0.320	12.488	0.591	1.234
0.180	9.688	0.659	1.181	0.300	9.492	0.761	1.572	0.390	10.468	0.741	1.642
0.210	8.804	0.749	1.398	0.350	8.336	0.904	1.960	0.460	8.896	0.915	2.133
0.240	8.044	0.845	1.638	0.400	7.372	1.064	2.411	0.530	7.636	1.113	2.720
0.270	7.388	0.948	1.906	0.450	6.556	1.242	2.932	0.600	6.604	1.340	3.415
0.300	6.808	1.058	2.204	0.500	5.856	1.440	3.533	0.670	5.752	1.598	4.229
0.330	6.292	1.177	2.534	0.550	5.252	1.661	4.219	0.740	5.036	1.891	5.177
0.360	5.832	1.305	2.900	0.600	4.724	1.906	5.001	0.810	4.428	2.221	6.273
0.390	5.412	1.443	3.304	0.650	4.264	2.177	5.887	0.880	3.912	2.592	7.531
0.420	5.036	1.592	3.749	0.700	3.856	2.477	6.884	0.950	3.468	3.008	8.965
0.450	4.688	1.752	4.238	0.750	3.496	2.808	8.004	1.020	3.088	3.471	10.591
0.480	4.376	1.924	4.775	0.800	3.176	3.171	9.255	1.090	2.756	3.986	12.426
0.510	4.084	2.108	5.363	0.850	2.892	3.569	10.647	1.160	2.468	4.557	14.485
0.540	3.820	2.307	6.004	0.900	2.636	4.004	12.190	1.230	2.216	5.186	16.786
0.570	3.576	2.519	6.703	0.950	2.408	4.479	13.895	1.300	1.992	5.878	19.346
0.600	3.348	2.747	7.462	1.000	2.204	4.996	15.772	1.370	1.800	6.638	22.183
0.630	3.140	2.991	8.286	1.050	2.020	5.558	17.832	1.440	1.628	7.468	25.316
0.660	2.944	3.252	9.177	1.100	1.856	6.166	20.086	1.510	1.476	8.374	28.763
0.690	2.764	3.530	10.140	1.150	1.708	6.824	22.546	1.580	1.340	9.360	32.544
0.720	2.600	3.827	11.178	1.200	1.572	7.534	25.224	1.650	1.220	10.430	36.679
0.750	2.444	4.143	12.294	1.250	1.448	8.299	28.131	1.720	1.112	11.589	41.187
0.780	2.300	4.479	13.494	1.300	1.340	9.122	31.280	1.790	1.020	12.841	46.091
0.810	2.164	4.837	14.781	1.350	1.236	10.004	34.683	1.860	0.932	14.191	51.411
0.840	2.040	5.216	16.158	1.400	1.148	10.951	38.353	1.930	0.856	15.644	57.168
0.870	1.924	5.619	17.631	1.450	1.064	11.963	42.303	2.000	0.788	17.205	63.386
0.900	1.812	6.045	19.203	1.500	0.988	13.044	46.547	2.070	0.724	18.878	70.087
0.930	1.712	6.496	20.878	1.550	0.916	14.197	51.097	2.140	0.668	20.670	77.293
0.960	1.616	6.973	22.661	1.600	0.852	15.426	55.968	2.210	0.616	22.584	85.028
0.990	1.528	7.477	24.556	1.650	0.796	16.732	61.173	2.280	0.572	24.627	93.316
1.020	1.444	8.009	26.568	1.700	0.740	18.120	66.728	2.350	0.528	26.804	102.182
1.050	1.368	8.569	28.702	1.750	0.692	19.593	72.646	2.420	0.492	29.120	111.651
1.080	1.296	9.159	30.961	1.800	0.648	21.153	78.942	2.490	0.456	31.580	121.746
1.110	1.224	9.781	33.351	1.850	0.604	22.804	85.631	2.560	0.424	34.191	132.495
1.140	1.160	10.434	35.876	1.900	0.568	24.550	92.729	2.630	0.396	36.958	143.923
1.170	1.100	11.120	38.541	1.950	0.532	26.394	100.251	2.700	0.368	39.887	156.057
1.200	1.044	11.840	41.352	2.000	0.500	28.339	108.213	2.770	0.344	42.984	168.924
1.230	0.992	12.596	44.313	2.050	0.468	30.389	116.630	2.840	0.320	46.254	182.550
1.260	0.944	13.388	47.429	2.100	0.440	32.547	125.519	2.910	0.300	49.705	196.965
1.290	0.896	14.217	50.705	2.150	0.412	34.817	134.896	2.980	0.280	53.341	212.195
1.320	0.852	15.085	54.146	2.200	0.388	37.203	144.778	3.050	0.264	57.170	228.269
1.350	0.808	15.993	57.759	2.250	0.368	39.708	155.182	3.120	0.248	61.197	245.217
1.380	0.772	16.942	61.547	2.300	0.348	42.336	166.124	3.190	0.232	65.429	263.068
1.410	0.732	17.933	65.517	2.350	0.328	45.091	177.623	3.260	0.220	69.873	281.851
1.440	0.700	18.967	69.674	2.400	0.308	47.977	189.696	3.330	0.208	74.535	301.597
1.470	0.668	20.046	74.023	2.450	0.292	50.998	202.361	3.400	0.196	79.421	322.336
1.500	0.636	21.171	78.570	2.500	0.276	54.157	215.635	3.470	0.184	84.539	344.099
1.530	0.608	22.343	83.321	2.550	0.260	57.458	229.537	3.540	0.172	89.896	366.918
1.560	0.580	23.563	88.282	2.600	0.248	60.906	244.085	3.610	0.164	95.497	390.825
1.590	0.552	24.832	93.457					3.680	0.156	101.351	415.850
1.620	0.528	26.153	98.854					3.750	0.148	107.464	442.028
								3.820	0.140	113.844	469.391
								3.890	0.132	120.497	497.972
								3.960	0.124	127.432	527.805
								4.030	0.120	134.654	558.925
								4.100	0.112	142.173	591.364
								4.170	0.108	149.995	625.160

Table B.4. Updating R for 100,000-500,000 bbl; z = 4 to 6

z = 4				z = 5				z = 6			
y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R)dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R)dR	y	k	integral R*L(R)f'(R)dR	integral R^2*L(R)f'(R) dR
0.010	87.420	0.058	0.076	0.010	127.864	0.040	0.053	0.010	186.312	0.028	0.038
0.040	56.076	0.099	0.145	0.200	34.132	0.199	0.373	0.300	34.284	0.215	0.438
0.110	35.444	0.174	0.289	0.400	19.364	0.403	0.881	0.600	17.324	0.495	1.185
0.180	26.472	0.250	0.455	0.600	12.636	0.684	1.666	0.900	10.388	0.919	2.448
0.250	21.012	0.334	0.654	0.800	8.816	1.065	2.816	1.200	6.772	1.530	4.409
0.320	17.236	0.429	0.892	1.000	6.408	1.568	4.428	1.500	4.660	2.375	7.269
0.390	14.444	0.537	1.176	1.200	4.800	2.217	6.605	1.800	3.332	3.505	11.252
0.460	12.284	0.659	1.512	1.400	3.680	3.039	9.459	2.100	2.452	4.976	16.596
0.530	10.564	0.797	1.908	1.600	2.872	4.061	13.110	2.400	1.852	6.845	23.562
0.600	9.164	0.953	2.369	1.800	2.276	5.311	17.687	2.700	1.424	9.176	32.425
0.670	8.008	1.128	2.903	2.000	1.832	6.821	23.324	3.000	1.116	12.034	43.479
0.740	7.044	1.325	3.516	2.200	1.488	8.623	30.164	3.300	0.888	15.489	57.033
0.810	6.224	1.544	4.217	2.400	1.224	10.749	38.357	3.600	0.716	19.613	73.416
0.880	5.528	1.787	5.013	2.600	1.016	13.236	48.059	3.900	0.584	24.482	92.967
0.950	4.928	2.057	5.912	2.800	0.852	16.119	59.435	4.200	0.480	30.176	116.046
1.020	4.408	2.356	6.923	3.000	0.720	19.437	72.654	4.500	0.400	36.777	143.025
1.090	3.956	2.685	8.053	3.200	0.612	23.228	87.894	4.800	0.336	44.371	174.293
1.160	3.564	3.046	9.313	3.400	0.524	27.534	105.337	5.100	0.284	53.047	210.252
1.230	3.216	3.442	10.710	3.600	0.448	32.396	125.173	5.400	0.244	62.897	251.320
1.300	2.912	3.874	12.255	3.800	0.388	37.857	147.600	5.700	0.208	74.015	297.930
1.370	2.644	4.345	13.956	4.000	0.340	43.963	172.819	6.000	0.180	86.500	350.527
1.440	2.404	4.857	15.823	4.200	0.296	50.758	201.039	6.300	0.156	100.452	409.573
1.510	2.192	5.412	17.867	4.400	0.260	58.291	232.475	6.600	0.136	115.975	475.542
1.580	2.004	6.013	20.098	4.600	0.228	66.610	267.348	6.900	0.120	133.176	548.921
1.650	1.836	6.661	22.525	4.800	0.204	75.763	305.884	7.200	0.104	152.164	630.213
1.720	1.684	7.359	25.161	5.000	0.180	85.803	348.316	7.500	0.092	173.051	719.933
1.790	1.548	8.111	28.015	5.200	0.160	96.782	394.884	7.800	0.084	195.953	818.611
1.860	1.424	8.917	31.099	5.400	0.144	108.752	445.831	8.100	0.072	220.988	926.787
1.930	1.312	9.781	34.424	5.600	0.132	121.769	501.408	8.400	0.064	248.276	1045.000
2.000	1.212	10.705	38.002	5.800	0.116	135.888	561.871	8.700	0.060	277.940	1174.000
2.070	1.124	11.693	41.844	6.000	0.104	151.166	627.482	9.000	0.052	310.106	1314.000
2.140	1.040	12.746	45.964	6.200	0.096	167.662	698.509	9.300	0.048	344.905	1466.000
2.210	0.964	13.867	50.372	6.400	0.088	185.435	775.223	9.600	0.044	382.466	1630.000
2.280	0.896	15.059	55.081	6.600	0.080	204.544	857.905	9.900	0.039	422.924	1807.000
2.350	0.832	16.325	60.104	6.800	0.072	225.053	946.837	10.200	0.036	466.416	1998.000
2.420	0.776	17.668	65.455	7.000	0.068	247.023	1042.000	10.500	0.033	513.081	2204.000
2.490	0.724	19.091	71.145	7.200	0.060	270.519	1145.000	10.800	0.030	563.061	2424.000
2.560	0.676	20.596	77.188	7.400	0.056	295.606	1254.000	11.100	0.027	616.501	2660.000
2.630	0.632	22.187	83.598	7.600	0.052	322.350	1371.000	11.400	0.025	673.548	2912.000
2.700	0.592	23.866	90.389	7.800	0.048	350.819	1496.000	11.700	0.023	734.351	3181.000
2.770	0.556	25.637	97.574	8.000	0.044	381.080	1628.000	12.000	0.021	799.063	3468.000
2.840	0.520	27.503	105.168	8.200	0.040	413.203	1769.000	12.300	0.020	867.838	3773.000
2.910	0.488	29.467	113.184	8.400	0.037	447.260	1919.000	12.600	0.018	940.835	4098.000
2.980	0.460	31.532	121.638	8.600	0.035	483.322	2078.000	12.900	0.017	1018.000	4442.000
3.050	0.432	33.701	130.543	8.800	0.032	521.461	2246.000	13.200	0.016	1100.000	4807.000
3.120	0.408	35.978	139.916	9.000	0.030	561.752	2424.000	13.500	0.014	1187.000	5193.000
3.190	0.384	38.366	149.771	9.200	0.028	604.270	2612.000	13.800	0.013	1278.000	5602.000
3.260	0.360	40.868	160.123	9.400	0.026	649.091	2811.000	14.100	0.013	1375.000	6034.000
3.330	0.340	43.488	170.989	9.600	0.024	696.292	3020.000	14.400	0.012	1477.000	6489.000
3.400	0.324	46.230	182.383	9.800	0.023	745.951	3241.000	14.700	0.011	1584.000	6969.000
3.470	0.304	49.096	194.321	10.000	0.021	798.147	3473.000	15.000	0.010	1696.000	7475.000
3.540	0.288	52.090	206.820	10.200	0.020	852.962	3717.000	15.300	0.010	1815.000	8007.000
3.610	0.272	55.216	219.896	10.400	0.019	910.476	3973.000	15.600	0.009	1939.000	8565.000
3.680	0.260	58.478	233.566	10.600	0.018	970.773	4242.000	15.900	0.008	2070.000	9152.000
3.750	0.248	61.878	247.846	10.800	0.017	1034.000	4524.000	16.200	0.008	2207.000	9768.000
3.820	0.232	65.422	262.752					16.500	0.007	2350.000	10410.000
3.890	0.220	69.112	278.303					16.800	0.007	2500.000	11090.000
3.960	0.212	72.952	294.515					17.100	0.007	2657.000	11800.000
4.030	0.200	76.946	311.406					17.400	0.006	2821.000	12540.000
4.100	0.192	81.098	328.994								
4.170	0.180	85.412	347.295								

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