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cycles; if there are no cycles or if all cycles have pseudolength 0, then the index is taken to be 0. Typical results: (1) If in the graph G each connected component has a cycle whose pseudolength is not 0, then the set of all stable equivalences on G forms a lattice. (2) If G and H are connected graphs each of whose vertices is the endpoint of some directed edge, then the number of connected components of the graph $G \times H$ is the greatest common divisor of the index of G and the index of H. The second result was obtained earlier for strongly connected oriented graphs by McAndrew [Proc. Amer. Math. Soc. **14** (1963), 600–606; MR **27** #1932]. *G. N. Raney* (Storrs, Conn.)

Halin, Rudolf 65
Graphen ohne unendliche Wege.
Math. Nachr. **31** (1966), 111–123.
The author characterizes the graphs having no infinite paths. He finds that for a graph to be of this kind it must be decomposable into finite graphs whose intersections satisfy certain specified conditions. Moreover, these conditions exhibit the finite graphs as the vertices of a tree, and this tree must itself have no infinite path.
W. T. Tutte (Waterloo, Ont.)

Harary, Frank; Nash-Williams, C. St. J. A. 66
On eulerian and hamiltonian graphs and line graphs.
Canad. Math. Bull. **8** (1965), 701–709.
The line graph $L(G)$ is the incidence graph of the edges of G. Additional graphs $L_n(G)$ $(n \geqq 2)$ are defined, and some relationships between Eulerian and Hamiltonian properties of G, $L(G)$, and the graphs $L_n(G)$ are found. {The reader may find it helpful to note that $L_n(G) = L(M_n(G))$, where $M_n(G)$ is the graph obtained from G by replacing each edge of G by a path consisting of n edges; thus L_n is not the nth iterate of L.}
D. W. Walkup (Seattle, Wash.)

Havel, Ivan 67
On the completeness-number of a finite graph. (Czech and Russian summaries)
Časopis Pěst. Mat. **90** (1965), 191–193.
The author calls two distinct edges of a graph G quasi-neighbours if they both belong to some complete subgraph of G. He defines a graph G' in which the vertices correspond to the edges of G, and two vertices of G' are joined if and only if the corresponding edges of G are not quasi-neighbours. He shows that the minimum number of complete subgraphs of G whose union is G is equal to the chromatic number of G'. *W. T. Tutte* (Waterloo, Ont.)

Hoffman, A. J.; McAndrew, M. H. 68
The polynomial of a directed graph.
Proc. Amer. Math. Soc. **16** (1965), 303–309.
Let G be a directed graph and A the adjacency matrix of G. It is proved that there exists a polynomial $P(x)$ such that $P(A) = J$ (when J is the matrix consisting entirely of 1's) if and only if G is strongly connected and strongly regular. (G is strongly regular if for each vertex i the number of edges with initial vertex i equals the number of edges with terminal vertex i; G is strongly connected if for any vertices i, j $(i \neq j)$, there is a directed path from i to j.) The unique polynomial of least degree satisfying $P(A) = J$ (called the polynomial belonging to G) is characterized in terms of the minimum polynomial of A.

Does the polynomial belonging to G determine G up to isomorphism? This problem is studied for a particular class of directed graphs. Let t be a positive integer and let G_t be the graph whose vertices are all ordered pairs (i, j) of residues mod t and whose edges go from (i, j) to $(i, j+1)$ and $(i+1, j)$ for all i, j. Let $P_t(x)$ be the polynomial belonging to G_t. The following theorem is proved: If t is a prime or $t = 4$ and if H is a graph with t^2 vertices such that $P_t(x)$ belongs to H, then $H \cong G_t$.
J. K. Goldhaber (College Park, Md.)

Troy, D. J. 69
On traversing graphs.
Amer. Math. Monthly **73** (1966), 497–499.
A covering of a graph G is a cyclic edge sequence S such that consecutive edges in S are different and each edge in G appears exactly twice in S, once in each direction. The main result is that a graph in which all valences satisfy $\rho \leqq 3$, and the number of valences with $\rho = 3$ is divisible by 4, can have no covering. *O. Ore* (New Haven, Conn.)

Walther, H. 70
Ein kubischer, planarer, zyklisch fünffach zusammenhängender Graph, der keinen Hamiltonkreis besitzt.
Wiss. Z. Techn. Hochsch. Ilmenau **11** (1965), 163–166.
A graph is called cyclically n-connected if at least n edges must be deleted in order to separate it into two disjoint parts each of which contains a polygon. It is known that there exist cyclically 3-connected and cyclically 4-connected planar trivalent graphs which are non-Hamiltonian. The author constructs a cyclically 5-connected non-Hamiltonian planar trivalent graph.
W. T. Tutte (Waterloo, Ont.)

Harary, Frank; Palmer, Ed 71
The number of graphs rooted at an oriented line.
ICC Bull. **4** (1965), 91–98.
Let G be a graph with p points and let H be an induced (possibly oriented) subgraph of G with n points (i.e., a (possibly oriented) subgraph which contains all lines of G joining a pair of points in H). None of the lines in the graph $G - H$ is oriented. Let h_{pq} be the number (up to isomorphism) of such graphs G with p points and q unoriented lines. The generating function for these graphs is defined as $H_p(x) = \sum_q h_{pq} x^q$, where q goes from 0 to $(p_2 - n) + n(p-n)$. The main result of the paper is a formula for computing $H_p(x)$.

Specifically, $H_p(x) = Z(\Gamma(H) \circ S_{p-n}, 1+x)$, where $\Gamma(H)$ is the automorphism group of the oriented graph H, S_{p-n} is the symmetric group of degree $p-n$, and $Z(\cdot)$ is the cycle index of $(\cdot)$. *Leonard Weiss* (Providence, R.I.)

Harary, Frank; Palmer, Ed 72
Enumeration of mixed graphs.
Proc. Amer. Math. Soc. **17** (1966), 682–687.
A mixed graph is defined as a graph whose edges may be oriented or nonoriented. The problem is to derive an expression for the number m_{pqr} of mixed graphs on p