PREDICTING THE RESPONSE CHARACTERISTICS OF LONG, FLEXIBLE CYLINDERS IN OCEAN CURRENTS

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ABSTRACT

Flow-induced vibration takes on many forms, only one of which is the phenomenon known as lockin. Case studies from 13 years of field and laboratory experiments are used to evaluate the criteria for the prediction of the various behaviors, from lockin to infinite cable response, with an emphasis on cylinders with large L/D ratios. The most useful and relevant dimensionless parameters are identified. The shear parameter and reduced damping are shown to be not very useful. Hydrodynamic damping is shown to be very important in regulating the dynamic behavior in sheared flows. The number of excited modes, the fractional variation in flow velocity over the cylinder length and the ratio of the half power bandwidth to the modal separation are shown to be especially important in determining response characteristics. Comparisons between predicted and measured response for cables in sheared flows are presented.

NOMENCLATURE

$y(x,t)$	= cross flow response displacement
m	= structural mass per unit length including added mass
T	= tension
x	= response measurement point
k	= wave number = ω/C
ω	= vibration frequency
ω_s	= local mean vortex shedding frequency

C = phase velocity of the cable ($=\sqrt{T/m}$)
L = the cylinder length
S.G. = specific gravity
ρ = water density
D = cylinder diameter
S_t = Strouhal number
$V(x)$ = flow velocity at x
V_R = reduced velocity
V_p = peak velocity in a linear shear
V_{rms} = turbulence standard deviation
N_s = the number of modes excited by the linear sheared flow
ω_n = natural frequency of mode n
ω_p = natural frequency closest to the peak shedding frequency
ζ_n = damping ratio for mode n
ζ_h = hydrodynamic modal damping ratio
ζ_s = structural damping ratio
β = velocity squared damping coefficient
γ = damping correction for response amplitude

OBSERVATIONS FROM FIELD AND LABORATORY EXPERIMENTS

By the mid 1970s a great deal of laboratory scale research had been completed by many investigators on fixed and moving cylinders in fluid flows. Concepts such as lockin, correlation length, and drag coefficient dependence on response amplitude were quite well developed. Strumming on long cables was a phenomena with centuries of observational history. Missing at the time were systematic experiments on long flexible cylinders which could rationally extend the observations made in laboratory scale tests to field applications of much larger scale.

The Early Castine Experiments

In the summers of 1975 and 1976 the author conducted experiments in a tidal flow at Castine, Maine. Most of the cylinders tested were synthetic and wire rope cables 75 feet in length with diameters varying from 1/4 to 5/8ths of an inch. Flow velocity was quite uniform from 1/2 to 2 1/2 feet per second. The typical vibration response was single mode lockin with response amplitudes of ±1 diameter at the antinodes. At certain flow velocities there were occurrences of a random vibration response which was named non-lockin behavior, Vandiver & Mazel, 1976 [16]. The Reynolds number range for these tests was from 800 to 10,000. Under these conditions the cables always vibrated.

An unusual observation was made, which still makes a significant point. On one occasion a wire rope .280 inches in diameter and 900 feet in length was stretched across the tidal basin from 2 points of land. The submerged portion of the cable was approximately 500 feet long and was exposed to flow which varied approximately 20% along the length. The response of the cable was essentially single mode lockin, at approximately the 50th mode. Response amplitudes of 1/2 to 1 diameter were observed. The significance of this experiment is that under favorable flow conditions lockin can happen at very high mode numbers. However, under different conditions the same cable can behave dynamically as if it is of infinite length. A purpose of this paper is to discuss the dimensionless parameters which govern this variation in behavior.

Another important lesson from these early tests was concluded from the test of several strumming suppression fairings. These fairings had constructions which varied from bristle brushes to a single row of plastic grass, 3 to 5 diameters in length, stitched into the covering of the cable. These strumming suppression devices have commonly been called hairy fairings. Each type of fairing tested in these early experiments exhibited a threshold speed above which the cable would begin to vibrate. In later flow visualization experiments conducted in the laboratory this speed appeared to coincide with the speed at which the fluid forces were able to bend the individual hairs back into the wake where they were of inadequate size to act as splitter plates, Pham, 1976 [7].

Castine, 1981

With these early learning experiences behind us, another much more ambitious experiment was conducted in the summer of 1981 at the same site at Castine, Maine. The experiment lasted 6 weeks and involved 8 people in the field with the research vessel Edgerton from the MIT Sea Grant Program. The experimental setup is shown in Figure 1.

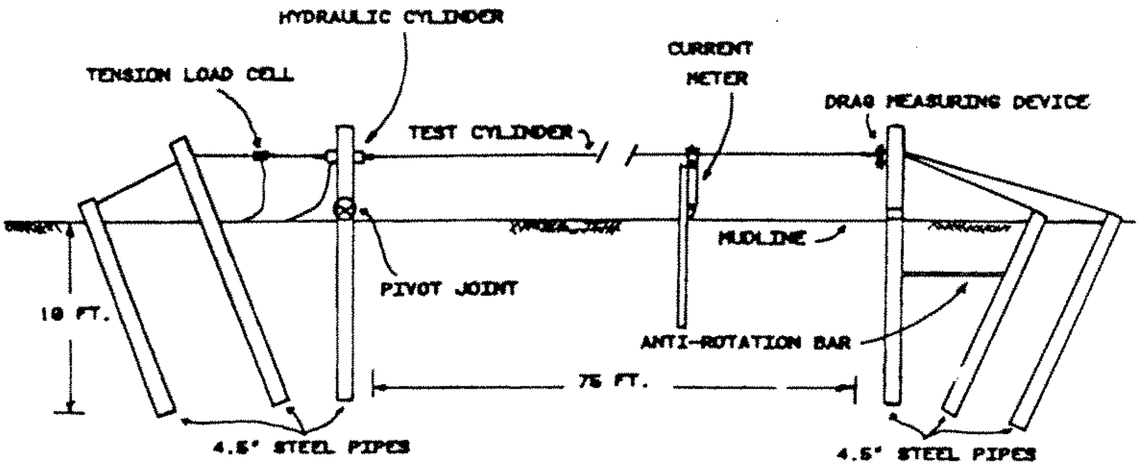


Figure 1. Castine Experiment, 1981

Two test cylinders were used in the experiments. The first was a cable 75 feet in length, 1 1/4 inches in diameter, containing 7 biaxial pairs of accelerometers. In the experiments tension, acceleration, current and drag coefficient were measured. This cable was also pulled inside of a 1 5/8th inch diameter steel pipe for use as the second test cylinder. The pipe was used to observe the flow-induced vibration of a cylinder with bending rigidity and tension. Properties of the test cylinders are given in Table 1.

The primary objectives of this field experiment were to (1) measure mean drag coefficients under field conditions and compare them to the very high values observed under laboratory conditions, (2) determine the differences in behavior of cables versus pipes with bending stiffness, and (3) test cable behavior with attached lumped masses. The behavior with lumped masses is reported in Griffin & Vandiver, 1984 [5].

TABLE 1 - MECHANICAL PROPERTIES AND DIMENSIONS OF TEST CYLINDERS

Cable Specifications

Length:	75.0 ± .1 feet
Diameter:	1.25 ± 0.02 inches
Weight per foot in air:	0.7704 pounds per foot
Specific Gravity:	1.408

Pipe Specifications

Length:	75.0 ± 0.02 feet
Outside diameter:	1.631 ± .003 inches
Inside diameter:	1.493 ± .003 inches
Weight per foot in air including weight of the internal cable:	2.001 pounds per foot
Weight per foot including cable and trapped water:	2.236 pounds per foot
Specific gravity of pipe with cable and trapped water:	2.40
Measured bending stiffness, EI:	$(3.016 \pm .05) \times 10^6$ pound inches^2

The field measurements of drag coefficients are presented in Figures 2 and 3 from Vandiver, 1983 [11]. Figure 2 shows the drag coefficient for the pipe measured over a period of 2 1/2 hours. The data is moving average data. Every data point in the plot is a sliding average of 8.55 seconds of observation. Also shown in the figure are the flow velocity and the rms vibration amplitude observed in the cross flow vibration direction and the in-line vibration direction at one location on the cylinder.

In the figure the drag coefficient exhibits periods of high plateaus adjacent to periods of relatively low drag coefficient. The plateaus are at times of lockin with

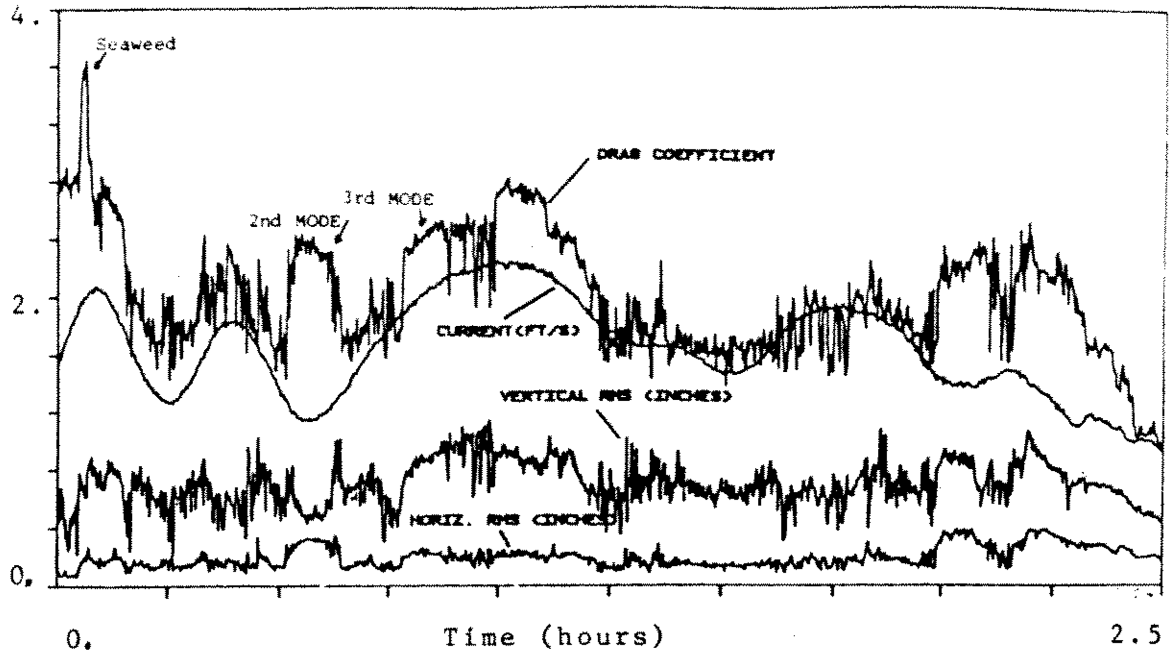


Figure 2. C_D, Current, RMS Displacement at L/6, for the Pipe at T = 670 - 920 LBS. C_D Error ± 10% at 2 ft/s, ± 15% at 1 ft/s.

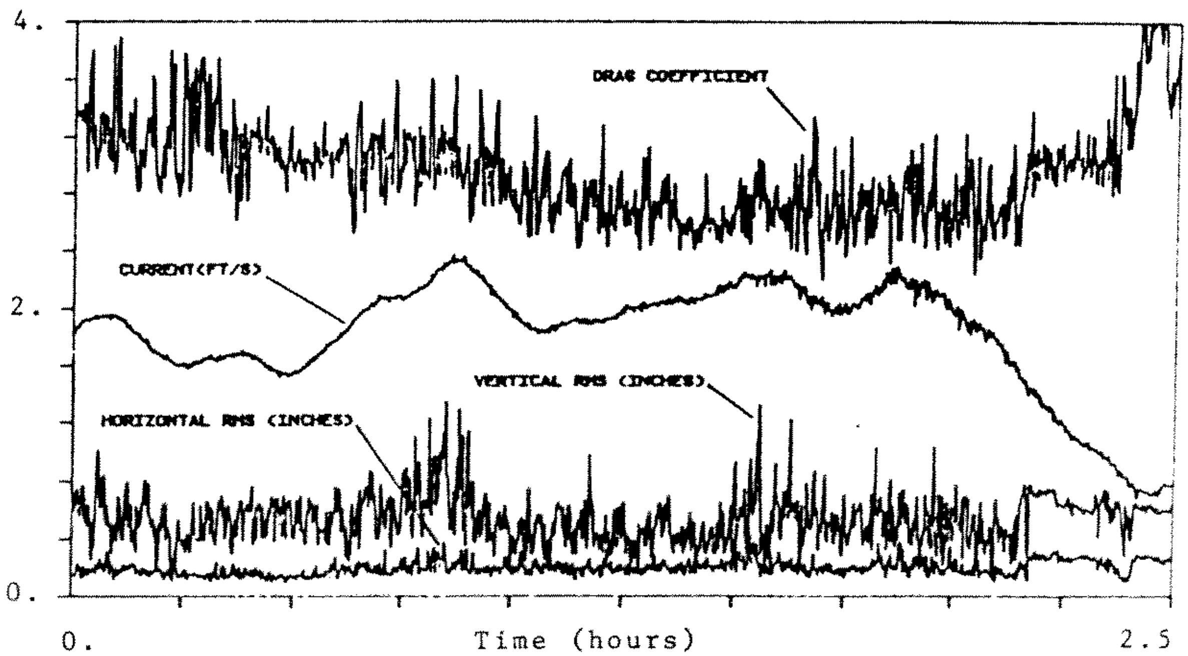


Figure 3. C_D, Current, RMS Displacement at L/6, for the Cable at T = 320 to 600 LBS. C_D + 0%, -20% Error Bound.

individual modes which are indicated in the figure; for example, 2nd and 3rd mode. At these times the natural frequency of the cylinder in that mode coincided with the vortex shedding frequency. The valleys in drag coefficient occurred at times when the flow velocity was such that wake synchronization could not occur. Vibration still occurred but with lower amplitudes and with wider vibration bandwidth than occurred during single mode lockin. Under lockin conditions the drag coefficients were greater than two times the value of a similar rigid cylinder at the same Reynolds number.

Figure 3 is also a 2 1/2 hour record, but for the cable, not the pipe. Again very high drag coefficients are observed. However, there is one remarkable difference between this data and the data shown for the pipe in Figure 2. There are no obvious plateaus and valleys in the drag coefficient. There are no clear demarcations between periods of lockin and non-lockin behavior. One of the objectives of the experiment was to determine differences in behavior between the pipe and the cable. That there is a difference is obvious from these two figures. However, the explanation which is given below, did not become evident until a few years later.

The Importance of Mass Ratio ($m/\rho D^2$)

The mass ratio is a measure of the mass per unit length of the cylinder (m) compared to the mass per unit length of the displaced fluid, ($\pi\rho D^2/4$). Some authors include the added mass in the mass per unit length. This should be avoided because the added mass is not constant, as will be discussed below. Cylinders in air tend to have very large mass ratios and cylinders in water tend to have mass ratios between about 1 and 3 except for solid or thick walled metal cylinders which have higher mass ratios.

Figure 4, reproduced from Chung, 1987 [2], shows data from a variety of sources. Response amplitude is plotted versus reduced velocity for a variety of cylinders with different mass ratios. The reduced velocity, V_R, in this figure is based on the natural frequency with an added mass coefficient of zero. The data for a mass ratio of 0.78 is for first mode vibration of a 120 mm diameter, 9.93 m long aluminum pipe in water [19]. The 1.77 mass ratio data is 1st mode vibration of a flexible tube in water [18].

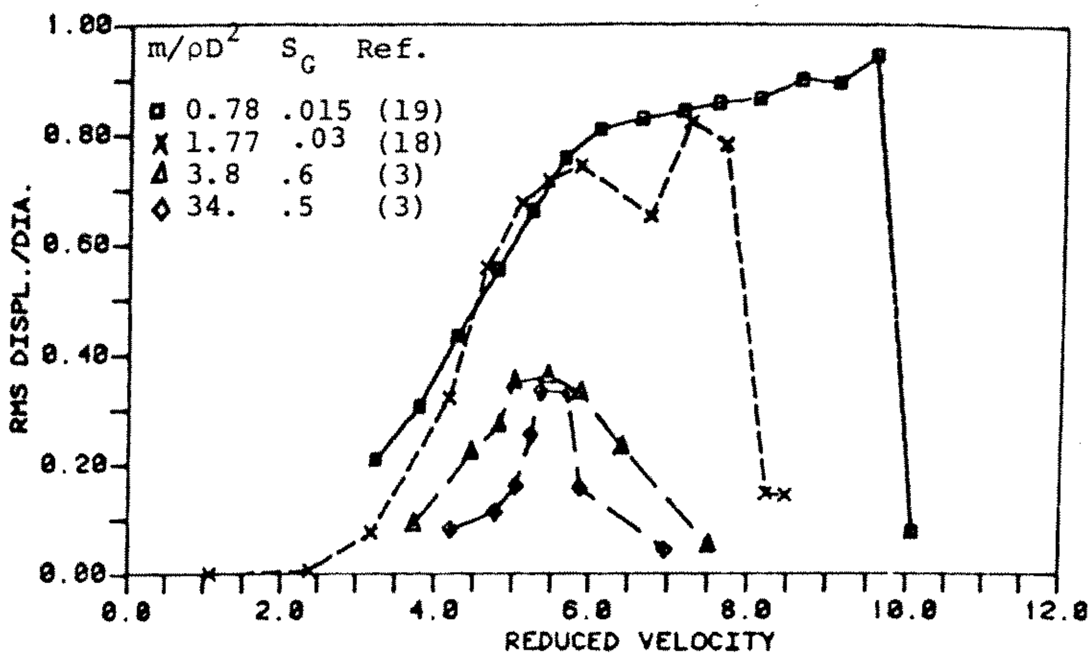


Figure 4. RMS Displacement versus Reduced Velocity for Various Mass Ratios; from [2].

The remarkable conclusion that one can draw from this figure is that low mass ratio cylinders have a much broader lockin range than high mass ratio ones when expressed in reduced velocity terms. I believe that this is due to the fact that the added mass coefficient decreases with increasing reduced velocity over the lockin range. There is driven cylinder data by Sarpkaya, 1977[8], to support this concept. His data reveals added mass coefficients which are much larger than 1 for reduced velocities less than 5, dropping to added mass coefficients of approximately -0.5 for reduced velocities above about 5.5. This observation is true for a wide range of vibration amplitudes. The result is that as the flow velocity increases the added mass decreases allowing the natural frequency to rise. This allows the lockin region to shift upward in flow speed as the speed increases.

This effect is less for high mass ratio cylinders than for low mass ratio ones, because the added mass is a lower percentage of the total mass per unit length. Equation 1 gives an expression for the natural frequencies of a beam under tension showing the effect of the added mass coefficient. For cables, the EI term is neglected. English units are indicated, but any other consistent set may be used in the formula.

$$\omega_n = [(n\pi/L)^2 T/m + (n\pi/L)^4 EI/m]^{1/2} \qquad (1)$$

where: L = length, feet
T = tension, lbs
m = mass/ft, slugs including added mass
EI = bending rigidity, lb-ft^2
n = mode number

This formula was used to evaluate the effect of added mass coefficient on the lockin bandwidth for the cylinders tested at Castine in 1981. Natural frequencies of the pipe and cable for added mass coefficients of 0.0 and 1.0 are given in Figure 5. The frequency axis is also scaled in the flow velocity, which would yield that frequency for a Strouhal number of .17.

The pipe had a specific gravity of 2.4, and a typical tension of 700 pounds. As the added mass coefficient for the pipe is varied from 1 to zero the natural frequency rises by only 19%. The consequence for the pipe was that the regions of lockin did not overlap for the low modes which were excited in these experiments. There were flow velocities which were between the lockin regions of adjacent modes. At these velocities the pipe exhibited non-lockin behavior, with characteristically lower response amplitudes and broader response spectra.

For the cable the specific gravity was 1.41, and a typical tension was 400 pounds. The increase in natural frequency, caused by a variation in the added mass coefficient from 1 down to zero, is approximately 31%, causing the natural frequencies to overlap above the second mode. The additional lockin bandwidth of about 20%, which results from the usual fluid structure interaction, caused the lockin bands of the cable at Castine to overlap at all flow velocities. In other words the lockin region of the 2nd mode overlapped that of the 3rd mode and the 4th mode etc. Therefore the cable in the Castine tests was capable of lockin at all flow speeds.

When two or more modes are within the lockin range, one usually dominates. The mechanism which determines which modes dominate is not well understood. The 900 foot long wire rope tested in 1975 had at least ten modes simultaneously capable of lockin. Somehow, one was able to dominate.

Mass ratio was the most significant source of difference in the behavior of the cable and the pipe at Castine. Broad lockin bandwidths such as those shown in Figure 4 are to be expected under ideal conditions for low mass ratio cylinders. However, there are many mechanisms for real ocean structures, which may prevent lockin. One of the more important ones is discussed below.

The Effect of Cylinder Motion on Lockin

In the late 70s a likely lockin prevention mechanism, which had not been previously studied was irregular motion of the cylinder. A laboratory experiment was conducted to test the hypothesis that the introduction of a small degree of random cylinder motion, while maintaining a mean frequency most favorable to lockin, could in fact prevent wake synchronization and, hence, lockin from occurring. A set of driven cylinder experiments were conducted on a rigid cylinder one half inch in diameter and 20 inches long in the MIT Ocean Engineering Department's circulating

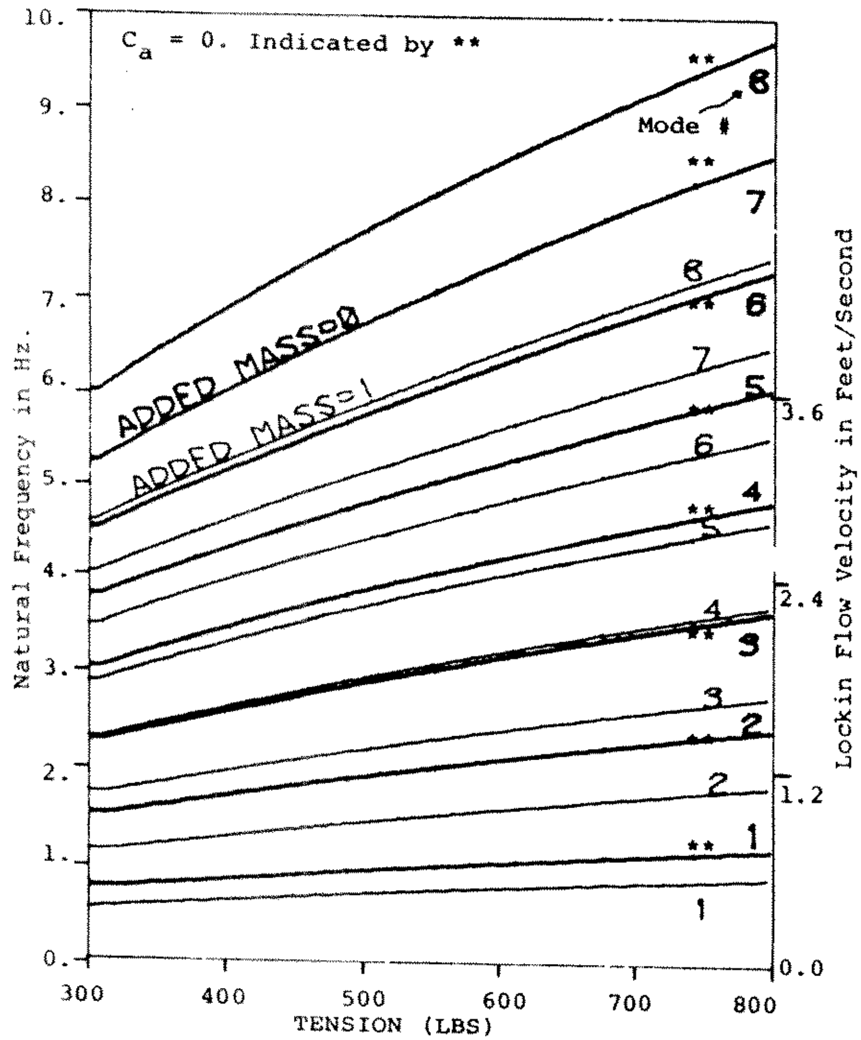


Figure 5A. **Cable Natural Frequencies for C_a = 0.0 and 1.0.**

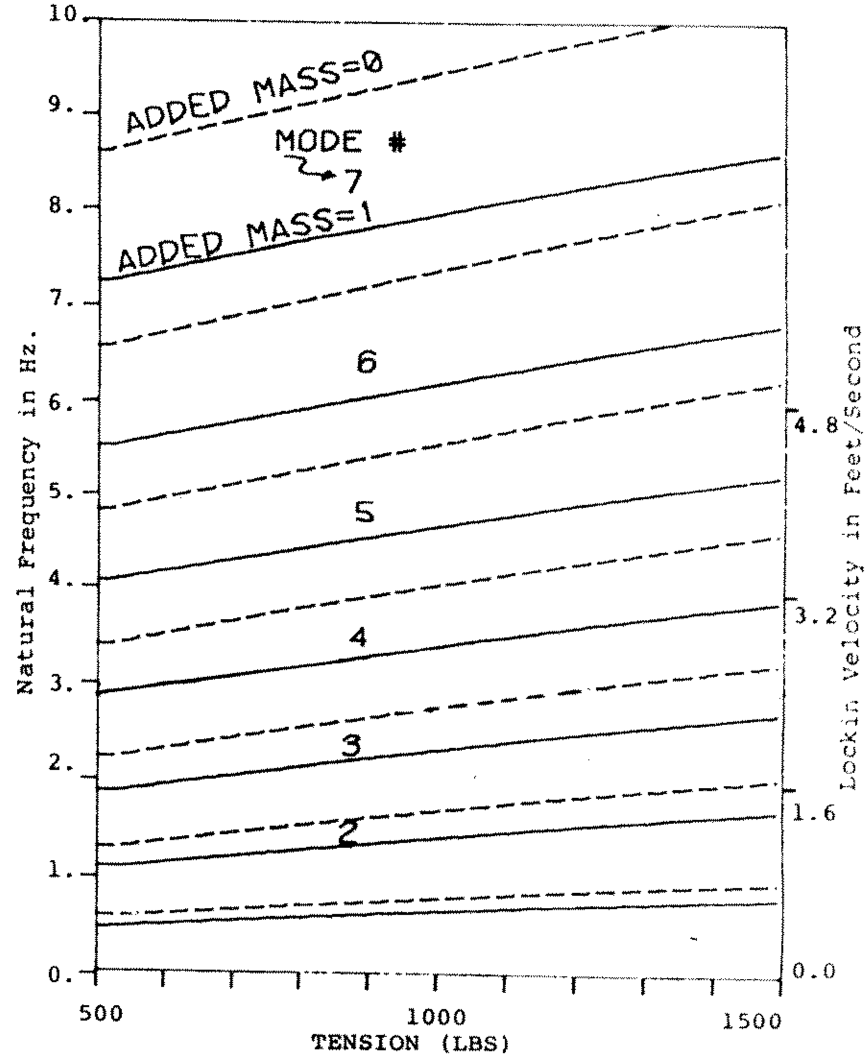


Figure 5B. **Pipe Natural Frequencies for C_a = 0.0 and 1.0.**

water tunnel. The cylinder was driven in cross flow vibration by an electromagnetic shaker. The cylinder vibration amplitude and spectral shape were varied in a controlled fashion. The spectral shape varied from sinusoidal to random with narrow to broad band characteristics. Flow velocity and hence reduced velocity were systematically varied. Cylinder motion and drag force were measured. Wake velocity components were measured with a laser doppler anemometer.

Under sinusoidal lockin conditions near unity coherence was observed between wake velocity measurements and cylinder motion. However when the cylinder motion was changed to a narrow band random process with a center frequency the same as would usually result in lockin, the coherence between cylinder motion and wake velocity dropped [Figures 6 and 7]. Even broader band cylinder vibration reduced the coherence to near zero. The mean drag coefficient was also measured. The disruption in the lockin process by random motions of the cylinder caused 50% reductions in drag coefficient, Shargel 1980 [9].

An important conclusion to be drawn from these experiments is that lockin is a rather fragile phenomenon which can be reduced or prevented by irregular cylinder motions. In more recent experiments at a test site in Lawrence, Massachusetts, high levels of turbulence (10 to 20%) were also found to prevent lockin from occurring. These experiments will be described later.

These conclusions have considerable importance when one considers the response prediction problem in sheared flow. Vibration, generated at one location, may propagate to another, where the flow is different, and prevent lockin. Therefore, the shear itself becomes the mechanism which prevents lockin. Several dimensionless parameters must be understood in order to predict whether or not lockin will occur, as illustrated below.

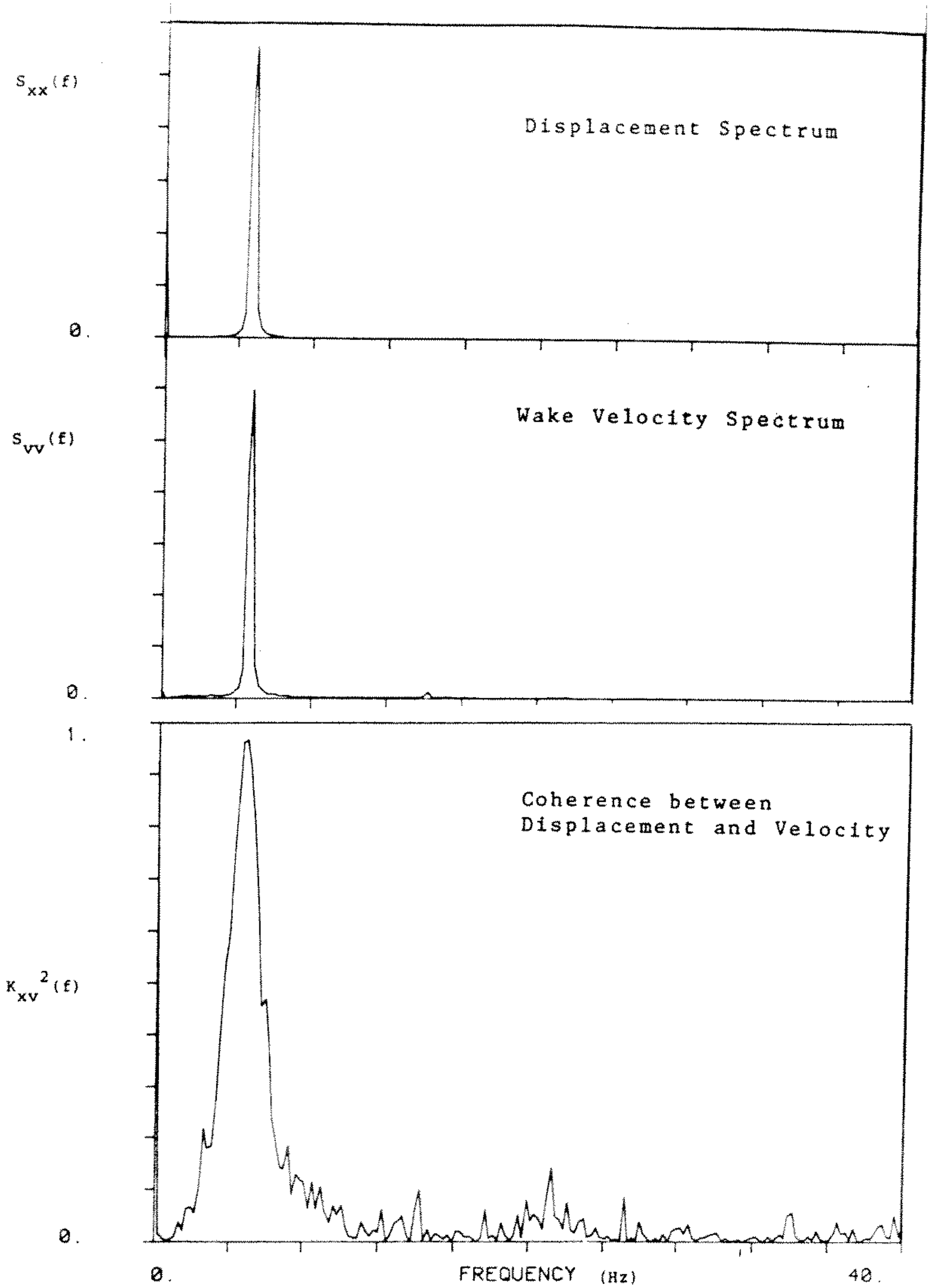


Figure 6. Displacement Spectrum, Wake Velocity Spectrum and Coherence for Sinusoidal Lockin Conditions: $R_e \approx 5000$, $A_{RMS}/D = .109$, from [9].

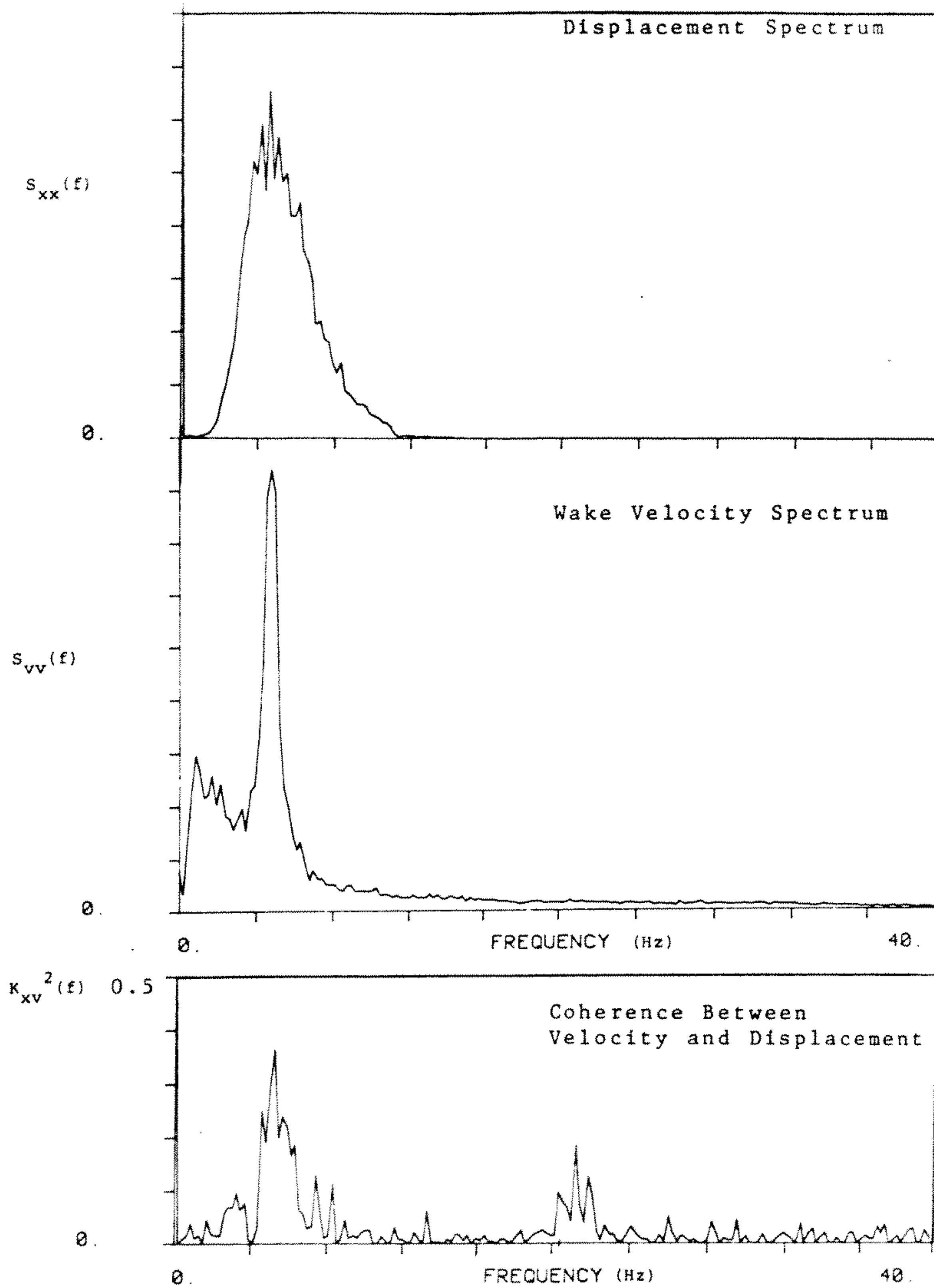


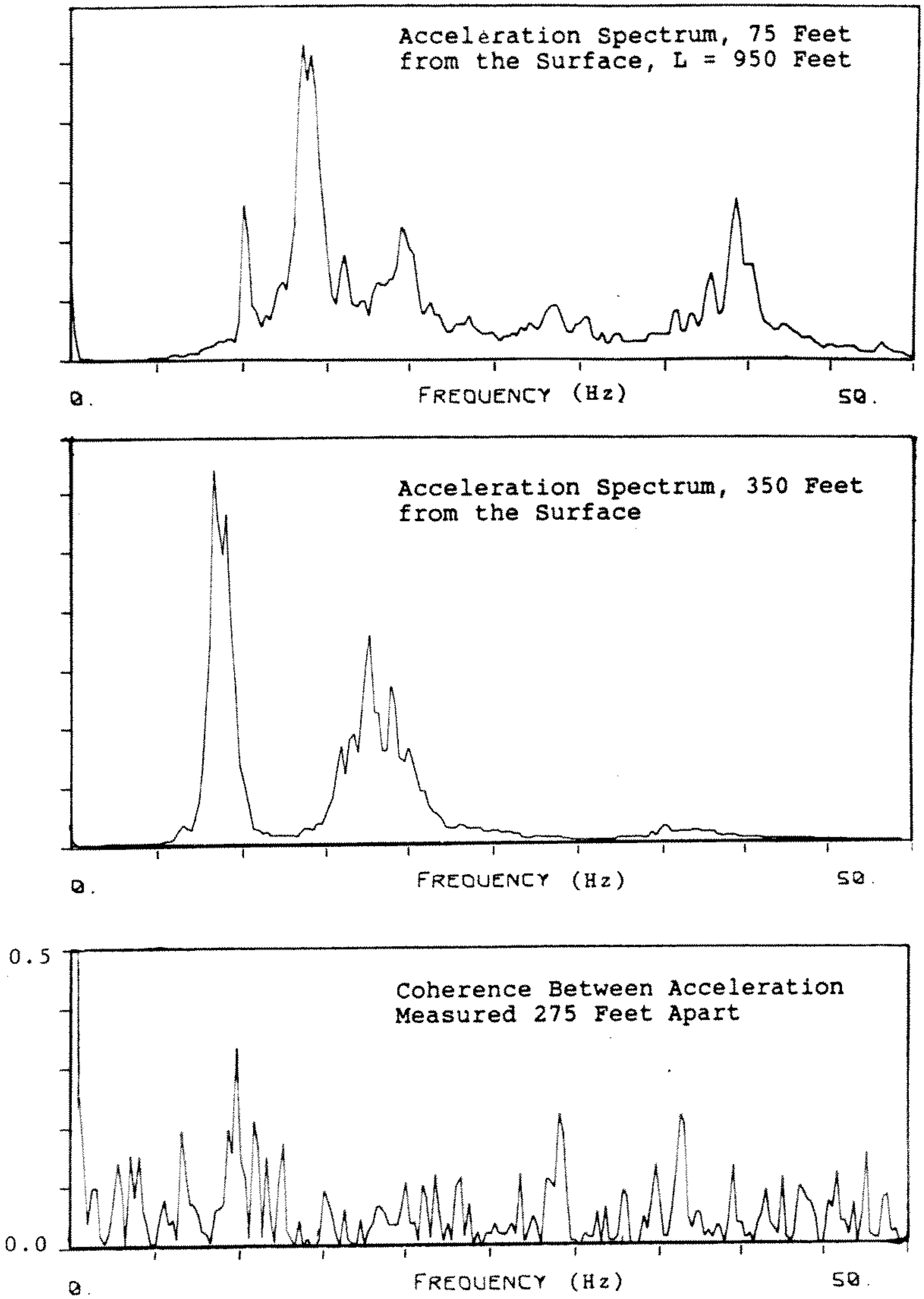
Figure 7. Displacement Spectrum, Wake Velocity Spectrum, and Coherence for Narrow Band Oscillations: $R_e \approx$, A_{RMS}/D = .106; from [9].

Sheared Flow Experiments in the Arctic and St. Croix

In 1982 a controversy developed regarding the correct drag coefficient to use for very long mooring cables exposed to realistic sheared ocean currents. Was it necessary as a design precaution to use drag coefficients measured under lockin conditions or were the reduced drag coefficients seen in the presence of random vibration more appropriate? In a sheared current, would lockin ever occur?

In 1983 two experiments were conducted on long, small diameter cables with the purpose of resolving the controversy. A shakedown experiment was first conducted on a vertical cable hung through the ice in the Arctic. Six months later a much more elaborate experiment was conducted from a United States Navy barge at St. Croix in the U.S. Virgin Islands. A braided Kevlar cable 0.16 inches in diameter was hung vertically at lengths up to 2000 feet under tensions of approximately 20 pounds. A 0.094 inch Kevlar cable was also tested at lengths up to 9000 feet. The current varied from a maximum of about 1.1 feet per second at the surface to a minimum of approximately 0.1 feet per second at depth, with substantial variations in between. Lockin never occurred. Broad band random vibration did occur as can be seen in Figures 8 and 9. Accelerometer measurements made as little as 275 feet apart were uncorrelated as shown in Figure 10. The cables responded to the vortex shedding as if they were of infinite length; Kim, Vandiver & Holler, 1985[6].

Drag coefficients were deduced from top tension and angle measurements to be approximately 1.5. The corresponding rigid cylinder value in the Reynolds number range of 200 to 2000 is about 1.2. High drag coefficients, typical of lockin conditions, were never observed. Rms response amplitudes of 1/4 to 1/2 a diameter were observed. For these particular cables and shear conditions, the controversy was resolved. To generalize the observations



Figures 8, 9, 10. Acceleration spectra and Coherence for Two Locations Separated by 275 feet; St. Croix 1983 [6].

requires the specification of appropriate dimensionless parameters, which can be used to predict the extreme variations of response: from single mode lockin seen in the Castine experiments to the broad band infinite cable behavior characteristic of the St. Croix experiments.

In the author's opinion the two most useful parameters for predicting the occurrence of lockin are the number of natural modes contained in the bandwidth of vortex shedding frequencies; hereafter referred to as N_s, and the dimensionless shear fraction, $\Delta V/V_{MAX}$. In some circumstances the turbulence level and damping may also be large enough to prevent lockin.

SHEAR FRACTION AND THE NUMBER OF EXCITED MODES CONTROL LOCKIN

Lockin occurs if and only if the separation of the natural frequencies of the cylinder is large compared to the bandwidth of vortex induced forces and large compared to the damping determined half power modal bandwidth. The separation between natural frequencies depends primarily upon the mechanical properties of the system such as mass per unit length, stiffness, tension, and length. The bandwidth of the exciting forces depends on the local vortex shedding phenomena, the shear fraction, $(\Delta V/V_{MAX})$, and the turbulence level (V_{RMS}/V_{MAX}).

One measure of the likelihood of lockin is then given by the ratio between the excitation bandwidth due to current shear and the separation in frequency between natural modes. This ratio is in fact the number of natural frequencies contained within the excitation bandwidth, and is here defined as N_S [Kim, Vandiver & Holler 1985].

The excitation bandwidth, Δf, due to shear can be estimated using a Strouhal number of approximately 0.17 and the variation in the velocity over the total length of the cylinder, ΔV, yielding:

$$\Delta f = S_t \cdot \Delta V/D \qquad (2)$$

For the constant tension string the separation in natural frequencies is the first natural frequency, yielding the following expression for N_s, the number nodes in the shear.

$$N_s = \Delta f/f_1 = S_t \cdot \Delta V/(f_1 \cdot D) \qquad (3)$$

If this number is large, lockin will not occur. If it is small, lockin is likely to occur. Three experimental examples are discussed here: the 950 foot long cable tested at St. Croix, the short cable tested at Castine, Maine in 1981, and the 900 foot long wire rope mentioned earlier.

St. Croix

The velocity variation at St. Croix was essentially 1.0 feet/second. The modal density ($1/f_1$) at a tension of 21 pounds was 10.6 modes per Hz. Letting St = 0.17, N_s is found to be 135 (For S_t=.2, N_s=158). Lockin was never observed. Infinite cable behavior was observed.

Castine

L = 75 feet	T = 350 lbs
D = 1.25 inches	$1/f_1$ = 1.0 modes/Hz

The maximum current at Castine was approximately 2.5 ft/sec. A spatial variation of approximately ± 3% of the flow speed was measured over the length of the test section. This yields a ΔV of approximately 0.15 ft/sec. The rms turbulence level was also very low.

For this case, N_s = 0.25 or .3 modes, depending on S_t being taken at .17 or .2. As a consequence, lockin was frequently observed at Castine. It happened whenever the mean flow velocity resulted in a shedding frequency which coincided closely with a natural frequency. Non-lockin response did occur when the mean shedding frequency fell outside the lockin bandwidth of any one natural frequency. As discussed earlier, this clearly happened for the pipe, but rarely for the 1.25" cable. At these times three or four modes were present in the cross-flow response. The in-line response would at the same time have several modes participating in the response; Vandiver & Jong, 1987 [15]. Under lockin conditions, the excitation bandwidth is very narrow. Under non-lockin conditions, even with very uniform flow the excitation bandwidth broadens substantially, and the lift force spectrum is best characterized as a random process.

Note that N_S approaches zero as the incoming flow becomes uniform. When N_S is less than one, the possibility to excite a single natural mode of the cable is very high and single mode lockin is very likely. Alternatively if N_S is very large, there is no chance to have lockin response, as more than one mode is always involved in the response. For the St. Croix test N_s was greater than 100, and lockin never occurred.

Between these extremes the prediction of lockin is not so clear. The .280 inch diameter, 900 foot long wire rope mentioned earlier is a good example. The relevant parameters were

TABLE 2: CASTINE LONG WIRE ROPE

V_{MAX}	= 1.3 ft/sec	ΔV	= .25 ft/sec
T	= 350 lbs.	L	= 900 feet
D	= .280 inches	m	= .00211 slugs/ft in air

ζ_s = .001, structural modal damping ratio
$1/f_1$ = 5.24 modes/Hz (using C_a = 1.0)

The relevant dimensionless groups are:

N_S = 9.9 modes, $\Delta V/V_{MAX}$ = .2±.1, $V_{RMS}/V_{MAX} \leq .03$

$2n\zeta_s$ = 2 x 50 x .001 = .1
= ratio of the half power bandwidth to modal separation.

In this case lockin occurred; one mode at a time dominated, even though approximately ten modes were simultaneously capable of lockin. The circumstances were such that no factor intervened to prevent lockin. The turbulence level was too low to interfere. The structural damping was too low to cause overlap between modes as indicated by the parameter $2n\zeta_s$.

The shear fraction was the pivotal parameter in this case. The shear fraction of approximately 20% was smaller than the lockin bandwidth for this cable, thus allowing lockin to occur over the entire length of the cable. At this point in time the upper bound on shear fraction which still allows lockin is not known. It is certainly related to the maximum lockin bandwidth. The maximum lockin bandwidth is probably close to that which one would measure in a driven oscillating cylinder experiment at the same Reynolds number and vibration amplitude. For sinusoidal cross-flow oscillation at subcritical Reynolds numbers and amplitudes of 0.5 to 1.0 diameters, the lockin range corresponds to reduced velocities of approximately 5.0 to 6.5, a range of about 20% of the maximum. This suggests that had the shear been much greater in the experiment, lockin would not have occurred. In fact a review of the raw data indicates that about half of the time lockin did not occur.

Griffin, 1985 [4], has suggested that the lockin bandwidth might be as large as 70% of the natural frequency, based on data which, as of the writing of this paper, the author did not have available to evaluate. Griffin also introduces a parameter which serves the same purpose as N_s.

An important subtle point should be noted with regard to lockin bandwidth. As discussed earlier, a low mass ratio cable is capable of sustaining lockin over an extended range of reduced velocity in a time varying uniform flow, because changes in added mass with reduced velocity can cause large changes in the natural frequency. Under uniform flow lockin conditions the added mass coefficient changes simultaneously over the entire cylinder in response to a variation in flow speed. Under lockin conditions in a sheared flow, the added mass will vary along the span of the cylinder in relation to the local reduced velocity. The natural frequency is a function of the modal added mass which is a weighted average of the sectional added mass along the entire length of the cable. Under sheared conditions the modal added mass will vary much less than the local added mass. Therefore the natural frequency will not vary nearly as much as in the uniform flow case. The lockin band will be increased by only small amounts due to adjustments in the natural frequency. Therefore the appropriate shear lockin bandwidth may be comparable to that observed in similar driven cylinder experiments, typically about 20% to 30%.

Stansby, 1976 [10], has shown greater lockin bandwidths in shears on driven cylinders with L/D's of 8 and 16. In his experiment the shear fraction was approximately 33%, and in one scenario lockin occurred over the entire length. However, his results show that the extent of the lockin region in a sheared flow on a driven cylinder is very amplitude dependent, and is related to the formation of a single coherent vortex cell in the wake. The presence of vibration nodes on a cable and the spatially varying amplitudes, suggest that such long lockin regions will not occur on realistic flexible structures with large L/D and high mode number.

When lockin occurs with one of the lowest modes of the structure, it may happen without requiring the wake to synchronize over the entire structure. This is a very typical occurrence on cantilevers, which may exhibit very large tip deflections under lockin conditions. Often with such cylinders there are regions which are not locked in and act as hydrodynamic damping regions. Although vortex shedding is happening in these non-locked-in regions, the frequency of the resulting lift force does not correspond to any other system natural frequency, and the resulting response amplitudes are not sufficient at these frequencies to disrupt the lockin process. For long cables and risers, which respond at higher mode numbers, this is not as likely to occur, because non-lockin regions will generate lift forces which coincide with other system natural frequencies, and will prevent pure lockin response from occurring. Therefore, for long cylinders it is unlikely that lockin will occur when the shear fraction exceeds 20 to 30% and two or modes are included in the excitation bandwidth.

Two parameters are conspicuously missing from the previous discussion: the shear parameter β and the reduced damping or stability parameter, S_G. The shear parameter β is not particularly useful. It is usually defined as:

$$\begin{aligned} \beta &= (D/V_{REF})dV/dx \\ &= (D/L)\cdot\Delta V/V_{REF}, \text{ for linear shears} \end{aligned} \qquad (4)$$

where V_{REF} is inconsistently defined in the literature. Letting it be V_{MAX} for the purposes of this discussion then

$$\beta = (D/L)\cdot\Delta V/V_{MAX} \qquad (5)$$

This is just the ratio of the shear fraction, which separately has usefulness, and L/D, which has little to do with dynamic response with the exception of L/D values less than approximately 20 to 30. β can in fact be quite misleading. The short Castine cable had an L/D of 720 and a shear fraction of .06 yielding a $\beta = 8.3 \times 10^{-5}$. The 900 foot long wire rope with 500 feet in the water had an in water L/D of 21,430 and a shear fraction of 0.2 yielding a $\beta = 9.3 \times 10^{-6}$. In the later case the shear was a much more important factor, even though β was an order of magnitude smaller. In the St. Croix experiment the 0.16 inch diameter, 950 foot long cable had a $\beta \approx 10^{-5}$ and yet had a 135 responding modes. In this case the shear fraction was .91, and is a much better indicator of the observed response than β.

Alone the shear parameter or the shear fraction give no indication as to the dynamics of the cable. That this is so can be simply proven. Consider a cable with a fixed L/D exposed to exposed to two different linear shear flows; one from 0.0 to 2.0 feet per second and one from 0.0 to 4.0 feet per second. Both cases have the same shear fraction, (i.e. 100%) and both have the same β, (i.e. D/L). Either representation of the shear needs an additional dynamics parameter, such as the number of excited modes, N_s, to describe the participation of modes in the response.

The stability parameter has usefulness in predicting response amplitude under lockin conditions but is not particularly useful in predicting the occurrence of lockin. This parameter will be discussed in greater detail in the next section on damping.

DAMPING CONTROLS CABLE DYNAMIC BEHAVIOR

The structural damping for most marine structures susceptible to flow-induced vibration is very small and is not usually the deciding factor in the determination of whether or not lockin occurs. However, when lockin does not occur or occurs over only a portion of the structure then hydrodynamic sources of damping can be large and become very important in determining dynamic response behavior. Whether or not a cable responds dynamically as if it is of infinite length or at the other extreme is capable of being dominated by a single mode is controlled by damping.

The Reduced Damping Parameter

Response amplitude prediction under lockin conditions has long been based on a dimensionless parameter known variously as the "reduced damping", the "stability parameter", or simply the "response parameter". This is sometimes written as S_G or ζ_s/μ. Written out in one of its common forms it looks like; Griffin & Ramberg, 1982 [3]

$$S_G = \zeta_s/\mu = 2\pi S_t^2(2m\delta/\rho D^2) \qquad (6)$$

where: $\delta = 2\pi\zeta_s$, $\qquad 2\pi S_t = \omega_n D/V$

$\zeta_s = r/(2\omega_n m)$, and r is the damping per unit length

This parameter is both useful and very often misinterpreted. As this parameter increases, response decreases. From equation 6 one can see that S_G becomes large for large values of damping ratio or for small values of mass ratio. The common erroneous conclusion is that low density cables hence ones with small mass ratio are likely to respond more than high density ones. In fact mass ratio has little to do with the response amplitude. When one replaces the Strouhal number, the damping ratio, and the mass ratio, in equation 6 with their definitions as given with the equation the cylinder mass per unit length drops out and the following expression for S_G results.

$$S_G = r\omega_n/(\rho V^2) \qquad (7)$$

S_G is essentially the ratio of dissipative forces on the cable to hydrodynamic exciting forces. It is essentially a statement of dynamic equilibrium between the average power injected into the cable by the fluid through lift forces and the power dissipated by damping. Such an equilibrium exists for all cases including shears. One must however properly account for the hydrodynamic and structural sources of damping in each case, and must also properly account for the fluid excitation regions on the structure. Vandiver, 1985 [12], addresses this topic in some detail.

How Long Is Long?

The Green's function of a cable is the response of the cable to a unit harmonic exciting force at a specified location. Figure 11 shows the magnitude squared of the Green's function of a constant tension cable of length L to a unit harmonic force applied at its center. Figure 11B shows the response when the excitation frequency is equal to the natural frequency of the 5th mode and the damping ratio is 1% of critical damping. The response shown is the same as a standing wave mode shape for the 5th mode. Single mode resonant response dominates this case. A single mode approximation to the total response would be adequate. In Figure 11C the excitation frequency is equal to the 99th natural frequency and the damping ratio is 10%. The Green's function reveals that the vibration never reaches the cable ends. This is an example of infinite cable behavior. In Figure 11D the natural frequency of the 9th mode is equal to the excitation frequency and the damping ratio is again 10%. In this case the Green's function reveals intermediate dynamic behavior. Some attenuation of the response exists between the point of excitation and the ends of the cable. Some standing wave behavior is also exhibited.

The parameter of importance which distinguishes between single mode dominated resonant behavior and infinite cable behavior is the product of the mode number, n, and the damping ratio, ζ_n, for that mode. When this product $n\zeta_n$ is less than 0.2, single mode resonant behavior is dominant. When this parameter is greater than approximately 3 infinite cable behavior is the dominant characteristic. In between 0.2 and 3 mixed behavior results and significant attenuation is observed between the point of application of the excitation and the ends of the cable, but vibration does reach the ends of the cable and standing wave behavior is observed. Strictly speaking, the above discussion applies only to constant tension cables. However, the general concept also applies to other cylinders as well. Minor modifications to the interpretation of the parameter $n\zeta_n$ are all that would be required.

The problem for the designer is to estimate both the mode number, n, and the damping ratio, ζ_n. The mode number, n, is easy. One gets it by comparing the natural frequencies of the cable to the vortex shedding frequencies in the shear. The damping ratio for a given mode however is not so obvious. Does one use structural damping or does one include the hydrodynamic sources of damping? This was a very controversial point in the mid to late 1970s. Ultimately most researchers, including the author, agreed that the structural non-hydrodynamic sources were the only important ones. When considering response in sheared flows that conclusion is completely false. The flaw is that in the 1970s our thinking was narrowly focused on lockin. Shear flow phenomena was not being seriously considered. Lockin over the entire structure was the focus of most discussions. Under such lockin conditions the important damping is the structural damping.

Under non-lockin shear flow conditions hydrodynamic damping must be considered, as it is often many times greater than the structural damping. This will be discussed in the next section.

Under unusual conditions lockin may occur but not over the entire structure. In these cases hydrodynamic damping from the non-lockin regions should be included in the damping, when computing $n\zeta_n$, or for that matter the reduced damping. S_G.

Damping Estimation

The practical, but approximate, hydrodynamic damping model used here is developed in detail in Vandiver and Chung, 1987 [13]. A brief summary is presented here. At any specific location an instantaneous drag force per unit length may be defined as the force in the direction of the instantaneous relative fluid flow, as shown in Figure 12.

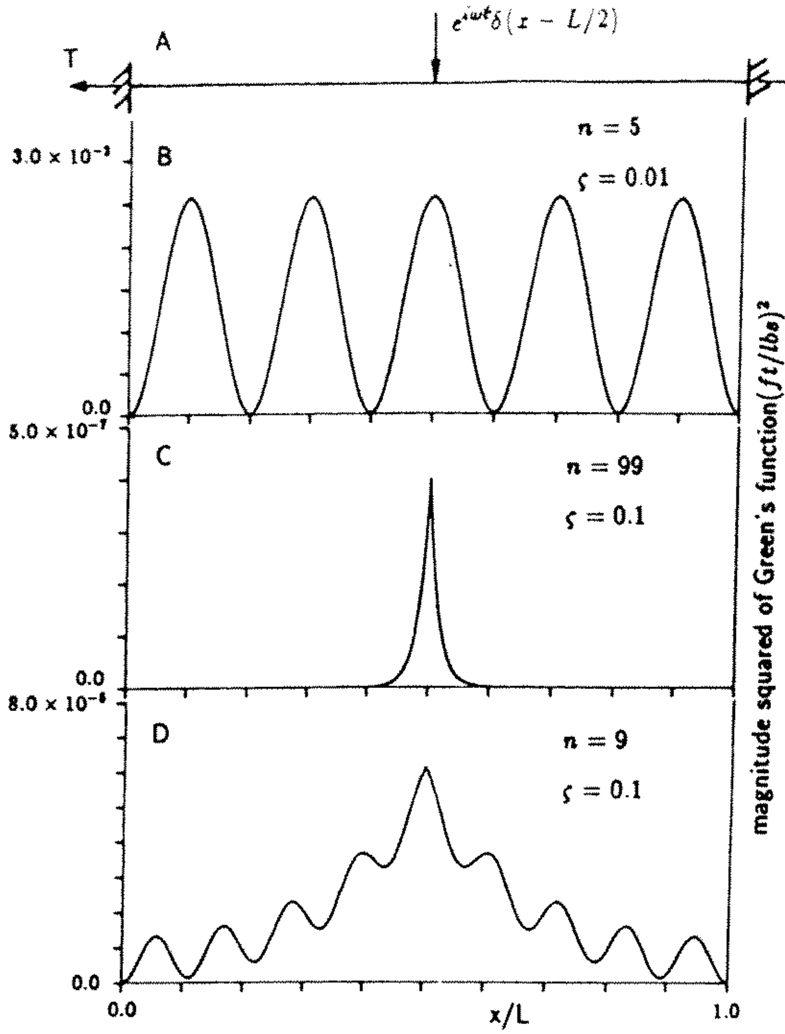


Figure 11. Magnitude of Response to a Unit Harmonic Force at L/2

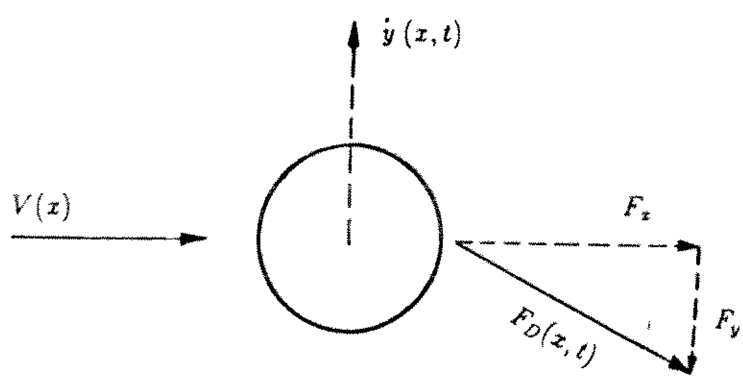


Figure 12. Damping Force Vector Components

The fluid velocity relative to the cable is the vector sum of the free stream velocity V(x) and the negative of the local cross-flow cable velocity $\dot{y}(x,t)$. The in-line cable velocity $\dot{z}(x,t)$ is assumed small and is neglected (it could be included if a more precise estimate was required). If one assumes the drag force to be proportional to the relative velocity squared, then the magnitude of the drag force takes the form given below.

$$F_D(x,t) = \tfrac{1}{2}\rho C_D \cdot D \cdot \{V^2 + \dot{y}^2\} \qquad (8)$$

Letting, $B = \tfrac{1}{2}\rho C_D \cdot D$, the component of the drag force in the y direction is by simple trigonometry given by:

$$F_y(x,t) = -B \cdot \dot{y}\{V^2 + \dot{y}^2\}^{\frac{1}{2}} \qquad (9)$$

The damping force in equation (9) is a non-linear function of $\dot{y}$. It is helpful to find a linear equivalent damping constant r(x) per unit length which dissipates the same energy per cycle as the non-linear one. This derivation is done explicitly in Vandiver and Chung, 1987 [13]. Only the final result is presented here.

$$r(x) = \gamma B V(x) \qquad (10)$$

γ is a correction factor which accounts for the effect of $\dot{y}(x,t)$ on the damping. If $\dot{y}$ is small compared to V(x) then $\gamma = 1.0$. γ falls in the range of 1.0 to 1.2 for most non-lockin flow induced vibration situations.

In order to calculate r(x) for a specific mode, a shear profile must be specified. For the purpose of example, consider a linear shear profile given by

$$V(x) = V_{MAX} \cdot x/L \qquad (11)$$

If ω_p is the natural frequency of a constant tension cable which most closely corresponds to the vortex shedding frequency at the velocity, V_{MAX}, then the damping ratio from hydrodynamic sources for any natural frequency ω_n, less than or equal to ω_p is approximately given by:

$$\zeta_{h,n} = \gamma C_D V_R (\omega_p/\omega_n)/\{4\pi^2(S.G.+C_a)\} \qquad (12)$$

where V_R is assumed to be around 5, S.G. is the specific gravity of the cable and C_a is the assumed added mass coefficient.

This is a rather remarkable conclusion. In a linear sheared flow, under non-lockin conditions, many modes respond. The one with the lowest hydrodynamic damping is the one with the highest natural frequency ω_p. All other modes have higher damping by the ratio ω_p/ω_n. To get a feeling for the magnitude of the hydrodynamic damping, let $\gamma = 1.0$, S.G. = 1.34, $C_a = 1.0$, and $C_D = 1.0$. These correspond to the experimental results from the Lawrence experiments, to be presented later. With these values the highest excited mode has a damping of about 6%. This is very large compared to typical structural damping ratios of 0.1 to 0.3%.

Under actual shear flow conditions, each excited mode will have a region of the cable where the local vortex shedding frequency and the natural frequency coincide as depicted in Figure 13. In these regions net power flows into the cable in that mode. These regions should be excluded in the calculation of hydrodynamic damping. This would cause a corresponding reduction in the damping ratio predicted by Equation 12.

The specification of this power in and damping exclusion range, is at the present very uncertain. It is discussed in Brooks 1987 [1], Wang et al, 1985 [17], and Vandiver and Chung, 1987 [13]. It is arguably dependent on the correlation length of the exciting forces, the half power bandwidth of the mode under consideration, and the distance on the cable which separates the points of coincidence of the shedding frequency and two adjacent natural frequencies. The author has used the last criteria with some success; Vandiver and Chung, 1988 [14].

Regardless of the selected criterion, if the power in region for a particular mode is small compared to the total cable length, this correction to the damping ratio is unnecessary. This is the case when the number of responding modes exceeds about 10.

The damping model described above may also be extended to cables which respond as if of infinite length. The derivation may be found in Vandiver and Chung, 1987 [13]. The result is described briefly below.

The linear equivalent damping ratio on an infinite cable is given by

$$\zeta_h = \gamma 5 C_D \, \omega_s / \{2\pi^2 \{S.G. + C_a)\omega\} \qquad (13)$$

where ω is the frequency of a wave travelling in the cable and ω_s is the local vortex shedding frequency.

If one wished to predict the attenuation between two points on a cable, separated by a distance L, the correct expression would be

$$y(x+L)/y(x) = e^{-\zeta_h kL} = e^{-\zeta_h \omega L/C} \qquad (14)$$

where k is the wave number, C is the wave propagation velocity, and ω is the vibration frequency of interest. For a fixed distance L, over which the shedding frequency varies a small amount so that the average shedding frequency can be used in equation 13, then by substitution for ζ_h in equation 14, an expression for attenuation between the two points is arrived at which is independent of ω, the vibration frequency, Vandiver and Chung, 1987 [13].

A verification of these damping models was accomplished in a field experiment in a sheared flow conducted in the summer of 1986, reported on below.

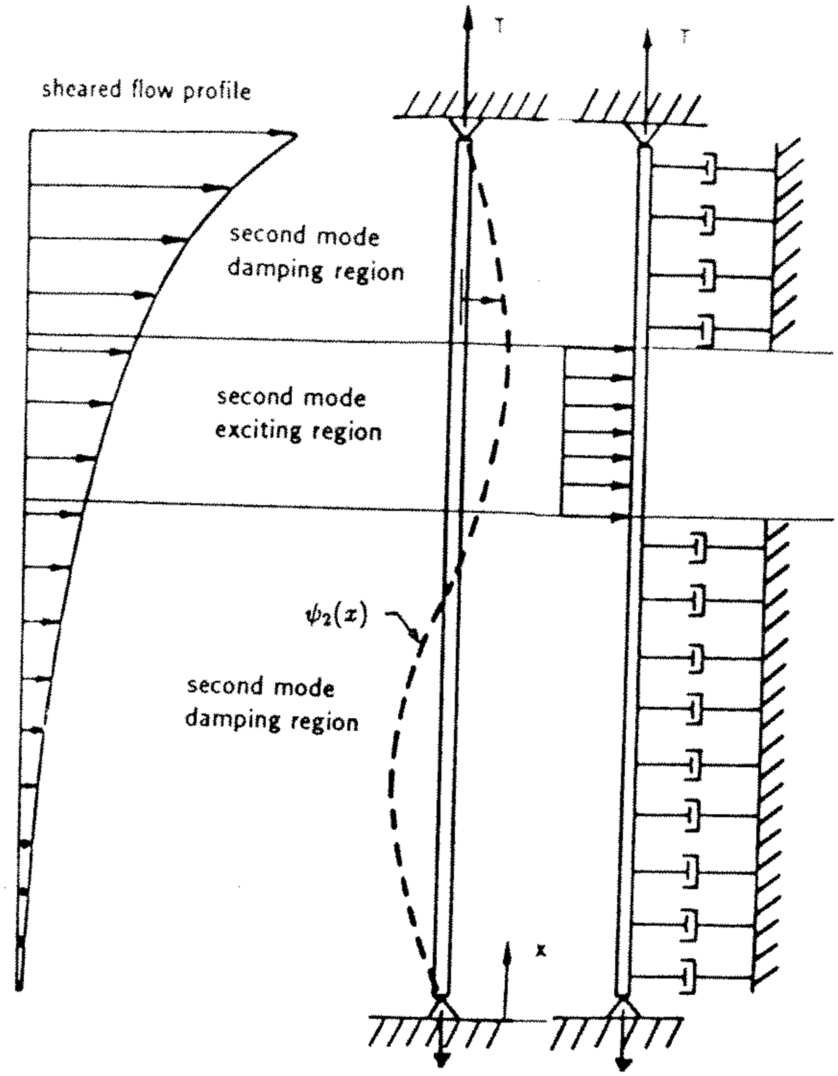


Figure 13. Power Flow Model for a Flexible Cylinder

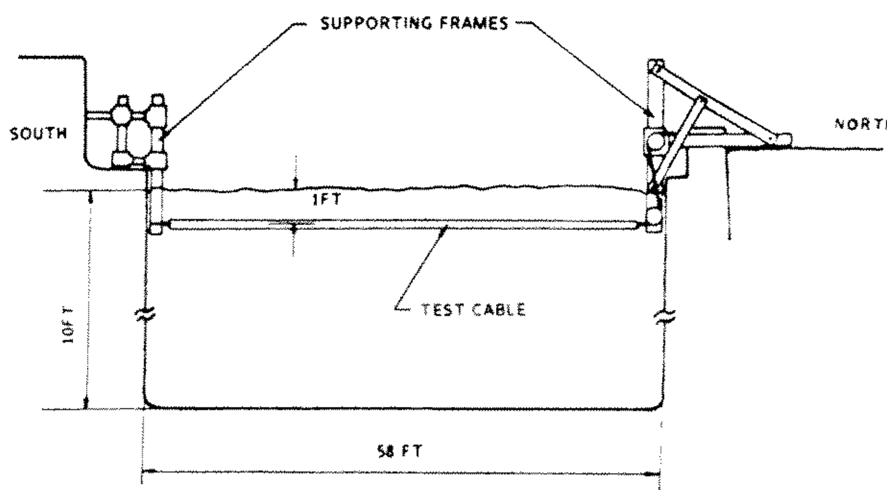


Figure 14. Lawrence Test Configuration, 1986

THE SHEARED FLOW EXPERIMENTS AT LAWRENCE, 1986

Experiments were conducted during the summer of 1986. A complete description, including many figures, may be found in References Chung, 1987 [2], and Vandiver and Chung, 1987 [13]. The test site was a mill canal, built in 1848 in Lawrence, Massachusetts. A dam diverts the water from the Merrimack River into the canal. The flow is controlled by four submerged gates, which are spaced at equal horizontal intervals beneath a gate house at the head of the canal. By controlling the various gate openings a sheared flow can be developed horizontally across the width of the canal, which is approximately 58 feet. The average depth of the canal is ten feet and the average flow rate for the experiments was from 200 to 750 cubic feet per second.

The test cable location was approximately 250 feet downstream of the gate house. The cable was tensioned horizontally across the width of the canal about one foot under the surface, as shown in Figure 14. Heavy steel pipe supports transferred the cable loads to the walls of the canal. Tension was applied to the cable via a system of pulleys and a hand-operated winch. For a given winch position the cable had essentially constant arc length. The tension then varied slowly with mean drag force on the cable. Tension was measured with a tension cell connected in series between the cable and winch.

Five feet upstream of the test cable, a simple traversing mechanism was constructed to carry a Neil Brown Instruments DRCM-2, two-axis acoustic current meter. The transducer was located about one foot under water and was oriented so that the instantaneous velocity was resolved into two components in the horizontal plane. The velocity was measured at two samples per second.

The 58-foot long test cable is shown in Figure 15. It consisted of a 1.125 inch rubber hose with a 0.5 inch inside diameter. Seven 0.16 inch diameter braided kevlar cables were carried inside of the hose. Each kevlar cable had seven conductors inside of it. Three kevlar cables were used solely as load carrying members, three cables were used to carry accelerometer signals and power, and one cable was used as a spare.

Six biaxial pairs of force balance accelerometers were placed on the centerline of the cable at locations shown in the figure. Each biaxial pair was 0.5 inches in diameter and 3 inches long. Space was created for the kevlar cables to pass around the accelerometers at these locations, with no change in the outside diameter of the hose. The accelerometers, tension cell, and current meter were the same as used in previous experiments conducted at Castine, Maine.

The 12 accelerometer outputs, tension, and current data were carried via a multi-conductor cable from the test cable to the gatehouse, where a Digital Equipment MINC-23 data acquisition computer was located. Fourteen data channels were digitized and stored on floppy disks.

Sheared Current Profiles

The current profiles were measured prior to response tests. The results of three different profiles are shown in Figure 16. They are designated shear flow profile 1, 2, and 3 (SFP1, etc.). SFP3 was the steepest shear with a peak flow velocity at times exceeding 4 feet per second and a minimum flow velocity of -0.5 feet per second. The minus indicates reverse flow. SFP2, a milder shear, ranged from 2 feet per second down to zero in a nearly linear profile. SFP1 was made as close to uniform as possible by careful positioning of the gates.

For all profiles the flow was highly turbulent. The rms turbulence level was from 10 to 20 percent of the maximum current in the profile. The time scale of the turbulence was up to several seconds in length, and was associated with large eddies, which were carried

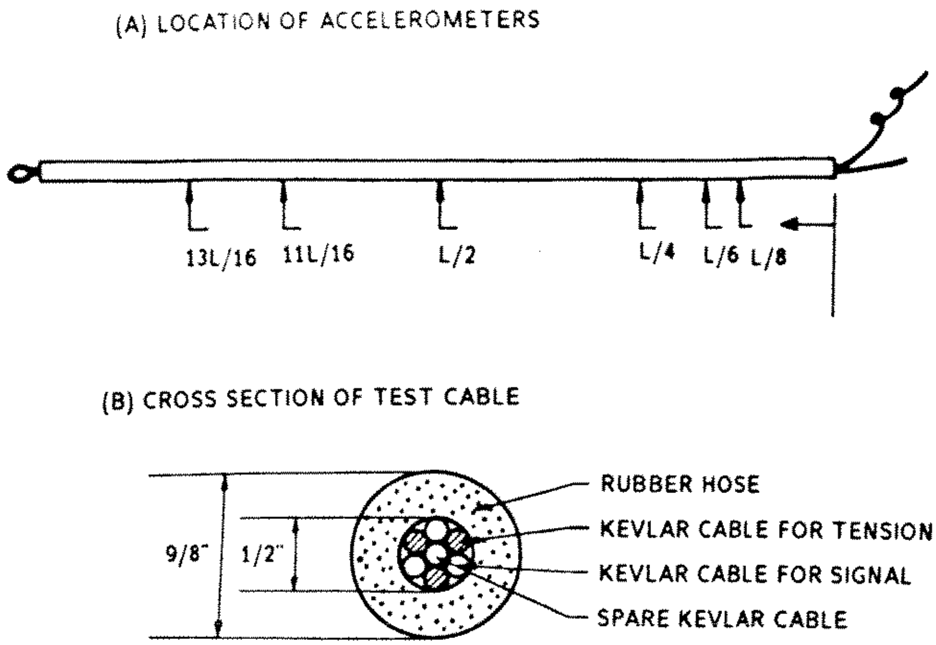


Figure 15. Lawrence Test Cable Construction

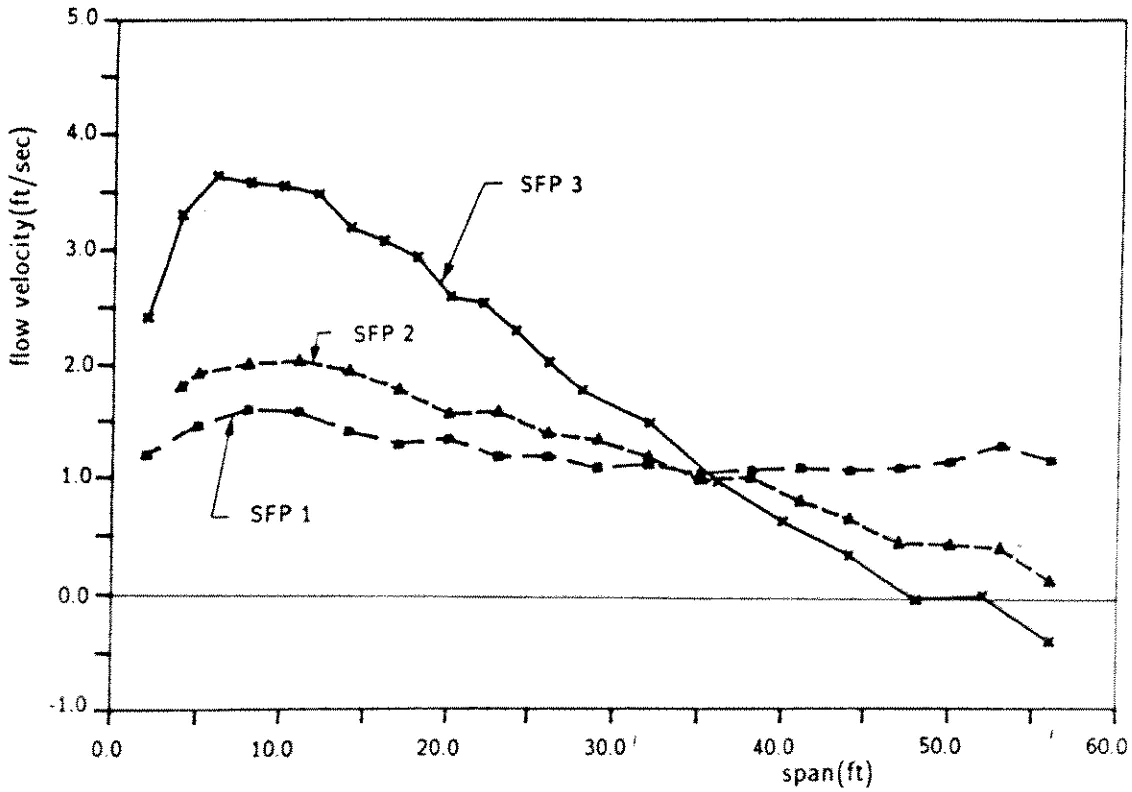


Figure 16. Shear Flow Profiles for the Lawrence Experiments

downstream from the gatehouse.

An important conclusion is that the turbulence was able to prevent pure, constant amplitude single mode lockin from occurring, even with the nearly uniform profile, SFP1.

EXPERIMENTAL OBSERVATIONS OF DAMPING

The structural damping measured by free vibration decay tests in air for the test cable in this experiment was about 0.3% of critical for the frequency ranges and tensions later experienced in the water.

Response at Low Tension in a Highly Sheared Flow

Figure 17 is a sample time history of the response of the test cable in the highly sheared flow, for which $\Delta V/V_{MAX}$ was 1.125. Simultaneous sample time histories of cross-flow acceleration are given for all six measurement locations. It is quite obvious that the high velocity locations had higher response than the low velocity regions. In this case the rms displacement was 0.3 diameters at x=13L/16 and 0.5 diameters at x=L/8. The tension was 151 pounds and the natural modes were 0.6 Hz apart. The peak vortex shedding frequency corresponded to about the 10th natural frequency. Enough modes are involved in the response that variation in mode shape is not a particularly important factor when comparing the rms response of one location to another.

The observed attenuation would require that the highest excited mode have from 4% to 6% damping, which is consistent with the prediction made earlier in this paper. To conclusively demonstrate this requires the use of a prediction model, which includes the effects of current shear and hydrodynamic damping. Such a model is proposed and used in Chung, 1987 [2], and Vandiver and Chung, 1988 [14]. Comparisons between measured and predicted response for the shear flow experiments at Lawrence are presented. One example is given here.

Figure 18 is a comparison between the predicted and measured acceleration response spectrum of the cable at x = 13L/16. The cable was exposed to the intermediate shear profile (SFP2). The predictive model includes the effects of shear, turbulence, hydrodynamic damping, correlation length, and higher order harmonics of the vortex shedding frequencies.

Impulse Response Under High Tension in a Turbulent Uniform Flow

With large hydrodynamic damping the vibration excited at one location is attenuated as it travels through the cable to distant points. This was confirmed by an independent measurement. Under steady state flow-induced vibration conditions, the cable was struck impulsively with a wooden pole, at a location near one end. The impulse generated, propagated through the cable. Figure 19 shows the simultaneous time histories at all six accelerometer locations. The impulse can be seen to travel from one location to the next with a travel time delay and an attenuation due to damping. By independently examining the spectrum of each accelerometer time history it was possible to estimate the frequency content of the impulse and the effective damping coefficient. The cable tension was 450 pounds, the current was approximately uniform (SFP1). The cable VIV response was dominated by third mode response at a natural frequency of 3.0 Hz. The impulse had most of its energy in the 15 to 24 Hz range.

By assuming an exponential decay with distance traveled, it was possible to estimate the effective damping by comparing the magnitudes of the acceleration response spectra in the 15 to 24 Hz band. Figure 20 is an example. Two spectra are shown. The locations were separated by 19.3 feet. If one assumes that the ratios of the two spectra are exponentially related and in proportion to the square of Equation 14, then it is possible to estimate the effective damping. Choosing a typical frequency of f=18

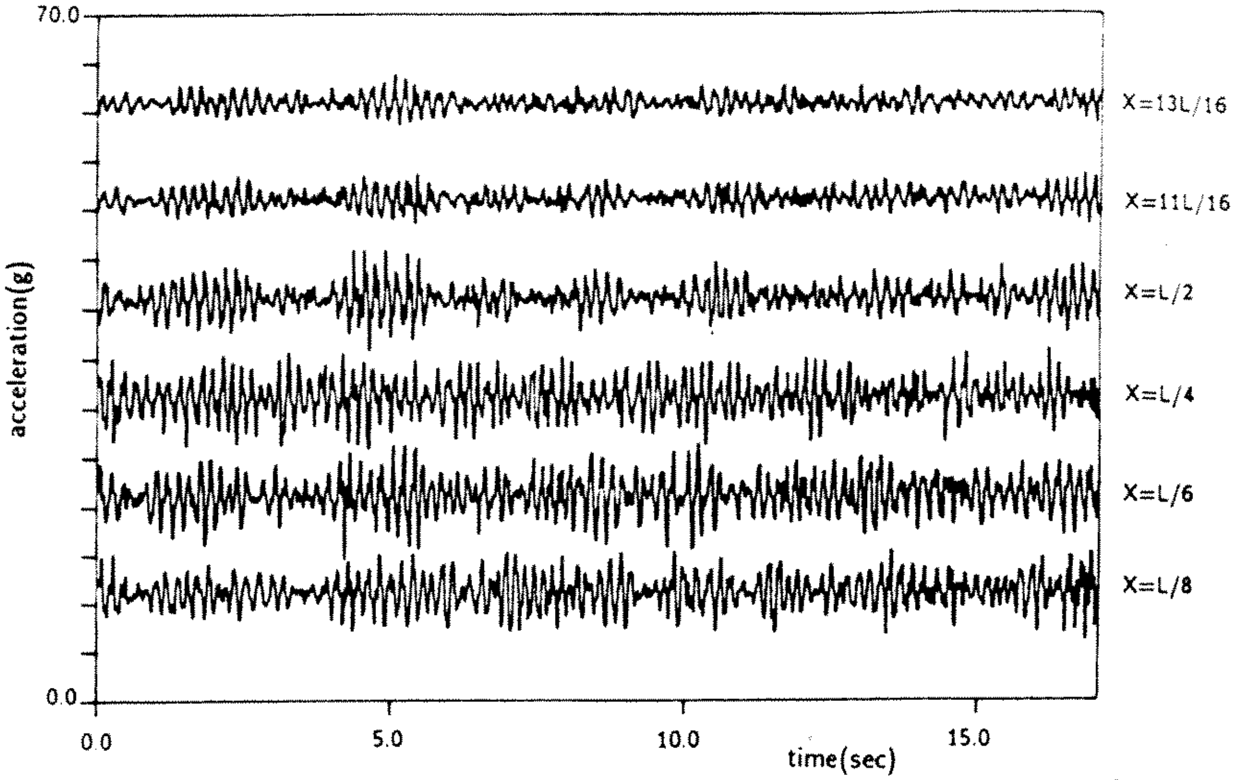


Figure 17. Cross-Flow Acceleration Time Histories for All Six Accelerometer Locations for the High Shear Case.

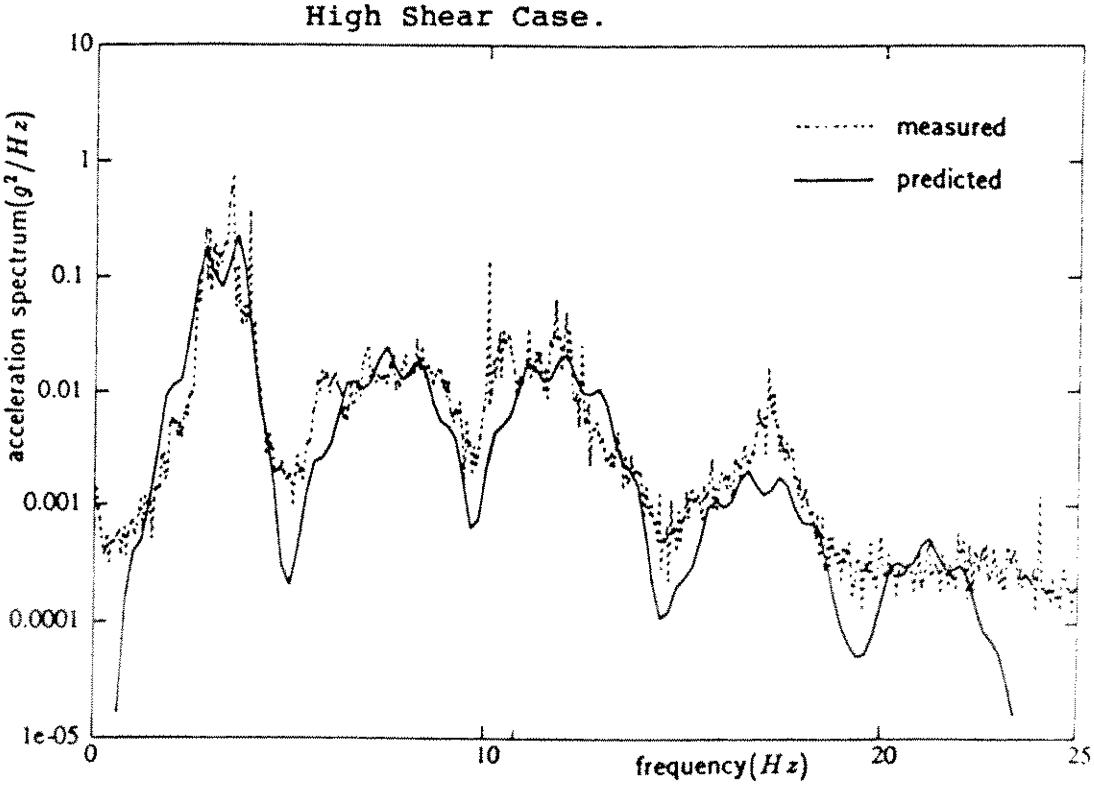


Figure 18. Measured and Predicted Acceleration Response Spectra at 13L/16 (Low Current Side) in SFP2.

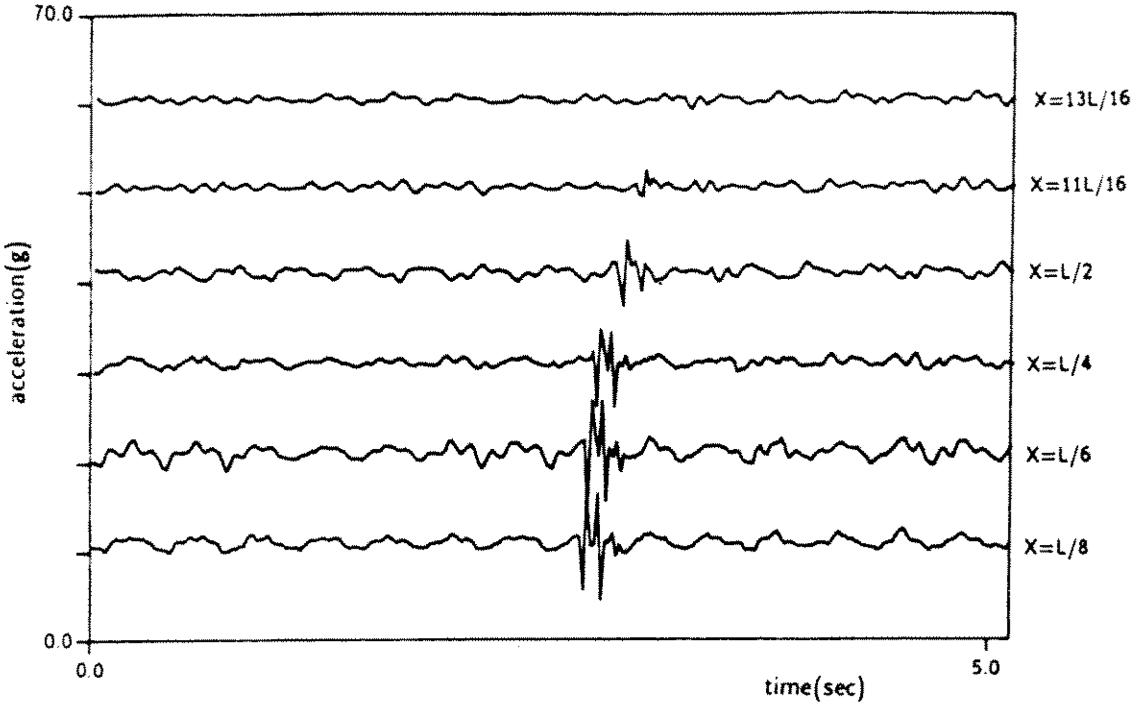


Figure 19. Measured Impulse Response Propagation Time Histories

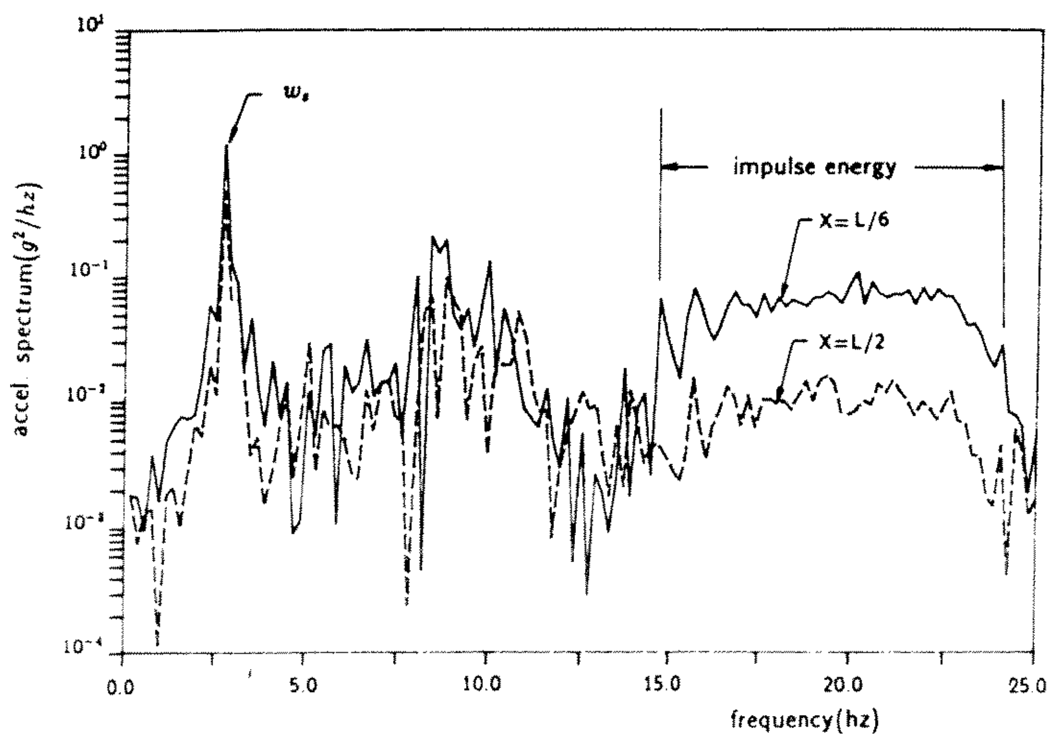


Figure 20. Measured Impulse Response Spectra at L/2 and L/6 in a Very Turbulent Uniform Flow

Hz, and noting that $\omega=2\pi f$, L = 19.3 feet, C = 120 feet/sec, and the ratio of the spectra is approximately six to one, results in an estimate of ζ = 0.049 or 4.9%. Many similar calculations were performed between different locations and for different impulse events. The results fell into a range of 4 to 6% damping, Vandiver & Chung, 1987 [13].

The earlier prediction for the infinite cable damping applies in the case of propagation of an impulse. Substituting for C_a = 1.0, S.G. = 1.34 and ω_s/ω = 1/6 into equation 13 yields $\zeta_h = .018\gamma \cdot C_D$. For C_D = 2.0 and γ = 1.2, ζ_h = .043 or 4.3% compared to the 4% to 6% observed. The response in this case was near lockin and in excess of 0.5 diameters rms, which would justify using the larger values of γ and C_D.

SUMMARY AND CONCLUSIONS

The design of moorings, ROV tethers, pipelines, and petroleum drilling and production risers all depend on the expected magnitude and frequency of vortex-induced vibration. Lockin usually results in the largest amplitudes of vibration and the largest mean drag coefficients, and, therefore is considered in most situations to be the worst case. Establishing whether or not it will occur is usually of great concern. This paper has attempted to reveal those parameters which have greatest influence over the occurrence of lockin for flexible cylinders with large L/D, and has provided case studies to support the conclusions.

The parameters of greatest importance are the shear fraction, the mass ratio, the turbulence level, the number of responding modes, and the ratio of the half power bandwidth to the modal separation. Furthermore, the response in sheared flows has been shown to be highly dependent on the hydrodynamic damping. Hydrodynamic damping is usually the parameter which determines if the cable will be dominated by single modes or will take on the characteristics of an infinitely long cable.

Many conclusions require further refinement, largely through experimental work. In many cases the critical parameters are known, but the values which mark the transition from one type of behavior to another need refinement. For example, what combination of values of N_S, the number of potential responding modes, and $\Delta V/V_{MAX}$, the shear fraction, are just sufficient to prevent lockin? These and many other similar problems require experimental data before they can be resolved.

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