EFFECTIVE STRESS CONCEPTS, DEFINITIONS, AND RESTRICTIONS

by

Nobuo Morita
and
K. E. Gray

Center for Earth Sciences and Engineering
University of Texas at Austin
Austin, Texas

Preface

Blowout prevention is an important activity in the oil industry. A blowout may develop using either low density or high density drilling fluid. When a high pressure zone is encountered with a low density fluid, a kick occurs. A blowout may develop if the kick is not properly controlled. Since the progress of the kick is relatively slow, the drilling engineer usually has sufficient time to detect and control it. However, it is better to detect the high pressure zone before it is encountered. There are several techniques to detect high pressure zones, mainly based upon porosity change (logging, from cuttings, etc.) and effective stress change (higher drill-ability due to both small effective stress and porosity).

A blowout may also occur using a high density drilling fluid, especially in offshore drilling where tectonic stresses at shallow depths are very small with respect to depth from the drilling mud line. When high density mud is used during deep offshore operations, the mud may induce fracturing at shallow depth close to the bottom of the ocean. Drilling fluid rapidly leaks off and control of high pressure zones at greater depths is reduced. This can develop very rapidly and thus is more catastrophic than the previous case where lower density drilling fluid is used. To prevent rupture of the borehole, the drilling engineer should know approximately the tectonic stress. There are three methods developed so far to determine the horizontal tectonic stress. These are: leak-off test, deformation test of the borehole, and core deformation test. These three techniques are not continuous methods, and the tectonic stress can be known only at the position where these tests are performed.

Before 1970, petroleum engineers often used Young's modulus and Poisson's ratio to estimate horizontal tectonic stress from vertical tectonic stress. However, this technique proved to be useless since the ratio of horizontal tectonic stress to vertical stress is not directly related to these instantaneous elastic moduli when one applies them to sedimentary rocks buried and compacted in geologic time. Thus drilling engineers have no techniques at present to continuously estimate the horizontal stress. However, Dr. Louis J. Thompson's papers give very practical results to predict K_o. His results show that K_o depends upon grain distribution, grain shape, porosity, etc. These parameters can be measured continuously from cuttings. In addition, one may use porosity, Young's modulus and Poisson's ratio from logs. All this information, if properly analyzed, will allow prediction of K_o, or at least predict it if the K_o estimated from a leak-off test at a certain depth is added.

The drilling engineers has little time to analyze the data, even given a reliable technique to estimate K_o. It is true that he must get necessary information during drilling for the first exploratory well. In this case, he may not have sufficient time to analyze the data, except by reducing drilling rate. However, for the second well in the same area, he may properly design drilling fluid density and casing depths, to prevent blow-outs. In addition, he may plan proper kick control procedure.

Thus, we believe that Dr. Thompson's experiments are both scientifically and practically useful. We strongly support this research. However, there is a problem in the use of effective stress. This short report is intended as a contribution to that research effort.

GENERAL STATEMENT ON EFFECTIVE STRESS CONCEPTS

A lot of literature is available to explain the concept of effective stress ever since Terzaghi first introduced it. Some authors have tried to explain it from a theoretical basis while others have used experimental data. The theoretical effective stress concepts proposed so far are actually consistent with each other, even though some authors have tried to prove that other effective stress concepts were wrong. The differences are apparent rather than real; the differences are in definitions, not in consistency of the theories. It is true that there are some differences in basic assumptions used. However, beginning with those same assumptions, it is possible to derive identical constitutive stress-strain equations, the apparent differences being in fact only a matter of definitions of effective stress.

Although theoretical developments on this subject are consistent with one another, there are lots of discrepancies among experimental results, particularly in publications that appeared before 1970. One problem in experimental work is the maintenance of uniform pore pressure, another is experimental accuracy. One might think that if rock permeability is high (say 100 md.) then pore pressure would become uniform instantly, no matter the pore fluid. This is true in most sandstones, but there are exceptions, e.g., sandstones in the geopressured-geothermal zone along the Gulf Coast; dolomites, and limestones. These rocks have vuggy main capillaries and have a relatively high percentage of micropores. Proper fluid or special procedures must be used to achieve uniform pressure in the micropores. Otherwise, one may easily destroy these micropores or measure extraneous strain.

Uniformity of pore pressure is extremely difficult to achieve if the permeability is less than 1 md. Most clays have low permeability, so that water injection into the pores results in deformation of the bulk clay, rather than individual grains. Special procedures and special combinations of pore fluids (for example, slightly greater than irreducible pore water saturation plus air, with air as displacing fluid) to achieve uniform pressure.

Other discrepancies among experiments involve measurement errors and experimental procedures. To prove the effective stress concept experimentally, a strain measurement device with high accuracy and a proper loading device to measure exact load on the porous sample is needed (friction is one of the main sources of error). Porous material displays hysteresis during loading and unloading. The non-linearity of stress-strain behavior sometimes masks the effect of effective stress. Elimination of all experimental error mentioned above is very difficult in certain types of porous material, in which materials it is difficult to check whether the effective stress concept holds. In addition, one can easily find igneous rocks, or construct artificial rocks, which do not fit the ideas of effective stress. Consequently effective stress concepts cannot always be applied. However, for the case of interest here - sedimentary rocks and soils - the effective stress law is appropriate. The sedimentation process and natural environment in which it occurs leads to a porous material to which the effective stress law can be applied.

EFFECTIVE STRESS LAWS FREQUENTLY USED

The effective stress law in a general form can be expressed as a decomposition of total stress as follows:

$$\sigma_{ij} = (\sigma_{ij} + S_{ij}) - S_{ij} \tag{1}$$

where σ_{ij} = Total stress ($+ \rightarrow$ tension, $- \rightarrow$ compression)

σ_{ij}^{e} = Effective stress = $(\sigma_{ij} + S_{ij})$

S_{ij} = Stress component affecting strain due to pore fluid

It is inconvenient to use the equation in this form, and previous workers have simplified it. A simpler form of the above equation is:

$$\sigma_{ij} = [\sigma_{ij} + f(p)\delta_{ij}] - f(p)\delta_{ij} \tag{2}$$

where $f(p)$ = represents a factor which is a function of pore pressure and structure of the porous material.

Equation (2) is very useful when $f(p)$ can be determined. Since most of the grains of sedimentary rocks and soils are anisotropic, S_{ij} should have some directional properties. However, since orientation of grains is generally random with respect to crystal axis, we can express S_{ij} in the form $f(p)\delta_{ij}$, if the rock is assumed to be homogeneous. Although such an assumption may be applicable to sedimentary rocks, for macroscopically anisotropic rocks, such as rock consisting of thin layers of homogeneous bedding planes with different elasticity properties, S_{ij} can not be strictly represented by $f(p)\delta_{ij}$. Each layer has a different matrix compressibility, and the average strain due to pore pressure may be different along and across the bedding planes.

With the exception of such special cases, most rocks or clay grains deform approximately linearly with increasing pore pressure. Hence, Eq. (2) is frequently simplified as follows:

$$\sigma_{ij} = \overbrace{[\sigma_{ij} + \theta P\delta_{ij}]}^{\sigma_{ij}^{e}} - \theta P\delta_{ij} \tag{3}$$

where $\phi \leq \theta \leq 1$ (4)

Equations (3) and (4) are perhaps confusing, since θ can be taken continuously between ϕ and 1, depending upon the particular rock. Hence, the following form is preferred.

$$\sigma_{ij} = \overbrace{[\sigma_{ij} + (\theta + \Delta\theta)P\delta_{ij}]}^{\sigma_{ij}^{e}} - (\theta + \Delta\theta)P\delta_{ij} \tag{5}$$

where θ = Specific value depending upon the definition of effective stress

$\Delta\theta$ = Small correction factor satisfying $|\Delta\theta/\theta| \simeq 0$

The value of θ depends upon definition, rather than discrepancy between theories regarding effective stress. The following definitions for θ are commonly used in the literature.

<u>Effective Stress For Stress-Strain Relations With Superposition Rule</u>

For this effective stress definition, θ=1. This definition is very useful in deriving constitutive stress-strain relationships using the superposition rule.

Effective Stress Acting on Average Mineral Area

For this effective stress definition, $\theta = \phi$, where ϕ is the porosity. The value θ should be replaced by effective surface porosity area for clay where the boundaries between grain and pore fluid is not clear. This definition is very useful when independent treatment is necessary between grain and pore fluid. For wave propagation through porous material with pore fluid, both solid phase and fluid phase should be handled independently, considering interaction between each other. Also, where boundary conditions can be imposed separately between the solid and fluid portions, this definition for θ is useful.

Effective Stress Is The Only Factor Controlling Total Strain

In this case, $\theta = 1 - \frac{B_i}{B}$, where B_i and B are compressibilities of the rock grains and rock bulk, respectively. This definition is useful for linear stress-strain relations, because total strain ϵ_{ij} can be expressed completely by $\sigma_{ij}{}^{e}$, such as $\epsilon_{ij}(\sigma_{ij}{}^{e})$. However, most sedimentary porous materials exhibit significant non-linearity, hence this definition has limited utility.

Effective Stress For Failure Theorem

In this application, $\theta = \eta$ where $\phi \leq \eta \leq 1$. For sedimentary rocks and soils, η is very close to 1. For most igneous rocks, η is close to 1, but rocks may be found where η is substantially smaller than 1. Artificial rock can be constructed for which $\eta \simeq \phi$. In an extreme case, one could even construct an artificial rock with $\eta > 1$.

THEORIES AND EXAMPLES SUPPORTING EACH EFFECTIVE STRESS CONCEPT

In this section, each effective stress concept and examples pertinent thereto are given. Although the definitions and derivations are different, the constitutive stress-strain relations expressed by these effective stresses are all consistent. Furthermore, none of them are contradictory to equations of continuum mechanics.

Effective Stress For Stress-Strain Relations With Superposition Rule

Figure 1 illustrates this concept. Assume two identical porous materials having a certain stress-strain property at atmospheric pressure. If one of them is submerged in the ocean, as it sinks slowly to the bottom it shrinks slightly, due to hydrostatic pressure in the interpore material. Because this neutral stress is acting uniformly both upon pores and mineral matrix (if there are no isolated pores in the body) the interpore material shrinks proportionally to pore space and the shape of the pore space remains constant; porosity also remains constant even though each pore becomes slightly smaller. Suppose an additional stress, ΔF, is applied to the two porous samples one on the surface and one in the ocean. Since the deformation character of rock bulk mainly depends upon porosity, pore shape, and size distribution, the strains induced by the additional stress, ΔF, are approximately equal for the two rock specimens. This additional stress corresponds to $\sigma^e_{ij} = \sigma_{ij} + p\delta_{ij}$ where the neutral stress, $p\delta_{ij}$, acting equally through pore and mineral matrix is subtracted. This intuitive explanation can be replaced by the following mathematical expression. Any system of the total stress σ_{ij} acting simultaneously with a pore pressure, p, can be divided into two parts:

$$\sigma_{ij} = \underbrace{(\sigma_{ij} + p\delta_{ij})}_{\sigma_{ij}^{e}} - p\delta_{ij} \tag{6}$$

The strain corresponding to the neutral stress, $p\delta_{ij}$, is characterized by the elastic coefficient of interpore materials, consisting of interpore matter and non-interconnected cracks. Since nonlinearity caused by the interpore material is trivial, one may neglect the non-linear terms and

$$\varepsilon''_{ij} = -\frac{1 - 2\nu_i}{E_i} p\delta_{ij} \tag{7}$$

where $\frac{1 - 2\nu_i}{E_i}$ is the compressibility of the interpore material.

The strain corresponding to the stress system σ^{e}_{ij} is

$$\varepsilon'_{ij} = \frac{1 + \nu}{E} \bar{\sigma}^{e}_{ij} - \frac{\nu}{E} \delta_{ij} \bar{\sigma}^{e}_{kk} + \varepsilon^{N}_{ij} (\bar{\sigma}^{e}_{ij}) + \varepsilon^{P}_{ij} (\bar{\sigma}^{e}_{ij}) \tag{8}$$

for time independent material, where

E, ν = Elastic constants for bulk rock

ε^{N}_{ij} = Initial non-linear strain

ε^{P}_{ij} = Final non-linear strain

These initial and final non-linear strains are illustrated in Figure 2. Thus, the total stress-strain relation is given by

$$\varepsilon_{ij} = \frac{1 + \nu}{E} \bar{\sigma}^{e}_{ij} - \frac{\nu}{E} \delta_{ij} \bar{\sigma}^{e}_{kk} - \frac{1 - 2\nu_i}{E_i} p\delta_{ij} + \varepsilon^{N}_{ij} + \varepsilon^{P}_{ij} \tag{9}$$

Equation (9) is approximately applicable if the porous material has the following characteristics: (1) there are few isolated pores in the interpore material, (2) the interpore material does not include highly compressible minerals (whose effect would be similar to isolated void pores), and (3) the interpore material is macroscopically homogeneous.

Effective Stress Acting Upon Average Mineral Area

This idea is very often used to construct equations of motion for solid and fluid phases independently, rather than consider them as a whole body. When it is not necessary to isolate solid and fluid phases, total force satisfies the equation of equilibrium (Note: many previous workers have unnecessarily complicated the procedure of finding the solution to a problem in which both solid and fluid phases can be treated as a whole body). However, it is essential to handle both phases separately when the solid motion and the fluid motion should be described independently. For these kind of problems, it is necessary to introduce the average stress in the mineral matrix and average fluid pressure acting on the pore space as follows (See Figure 3).

$$\sigma = \hat{\sigma} A_c + pA_w \qquad (10)$$

where σ = Total normal stress

$\hat{\sigma}$ = Mineral normal stress

A_c, A_w = Ratio of the area of the mineral and of the water respectively, to the total area

In Equation (10), A_w, is the area of pore fluid; it is equal to surface porosity, ϕ, for porous material with solid skeleton. However, it is necessary to introduce effective surface porosity for those materials (like clay) where the phase boundary between grain and pore fluid is not clear.

The important fact is that although the concept of mineral stress (Eq. 10) is very useful to describe the equation of motion, care must be taken in applying it to constitutive stress-strain relations and failure theorems, because the mineral stress is not directly related to total strain or failure theories of a porous body, except in certain special problems. Figure 4 illustrates this problem intuitively. Assume the same average mineral stress to be applied to both rocks. For rock (B), pressure P is applied on the boundary of the rock and throughout the pores. If the grains do not include lots of isolated pores, the stress distribution in each grain is approximately hydrostatic. Assume that for rock (A), force $F = PA_c$ (per unit area) is applied on the boundaries of the rock but the pressure in the pores is zero. In this case, the average mineral stress is still hydrostatic, but the stress distribution in each grain is very complicated. Most of past experiments have shown that the total strains in both cases are quite different and the compressibilities of rock bulk and rock mineral matrix, respectively, must be introduced to each case. There are, however, certain special synthetic rocks where it is possible to describe the stress-strain relations using only the average mineral matrix stress. As will be explained later in Figure 7, if the distribution of stress in the rock matrix remains constant no matter how the load is applied to the porous material, then the resultant total strain can be expressed using only the average mineral stress.

In order to overcome the above problem Biot introduced strain energy, V, to describe a constitutive stress-strain relation with respect to mineral stress as follows:

$$V = \tfrac{1}{2}(\sigma_x^e \varepsilon_x + \sigma_y^e \varepsilon_y + \sigma_z^e \varepsilon_z + \tau_{yz}^e \varepsilon_{yz} + \tau_{zx}^e \varepsilon_{zx} + \tau_{xy}^e \varepsilon_{xy} + \bar{\varepsilon}\bar{\sigma}) \tag{11}$$

where $\bar{\sigma} = -\phi P$

$\bar{\varepsilon}$ = deformation of pore fluid

In this equation average stress $\sigma_{ij}{}^e$ acting upon the mineral part is used, where the total stress is decomposed into the following terms:

$$\sigma_{ij} = \overbrace{(\sigma_{ij} + \phi p \delta_{ij})}^{\sigma_{ij}^e} - \phi p \delta_{ij} \tag{12}$$

Since this approach is very popular, details of the derivation are not given here. Using

$$\partial V/\partial \varepsilon_{xx} = \sigma_{xx},\ \partial V/\partial \varepsilon_{xy} = \sigma_{xy},\ \text{etc.,}$$
$$\partial V/\partial \bar{\varepsilon} = \bar{\sigma}.$$

Biot obtains

$$\begin{aligned}
\sigma_{xx}^e &= 2N\varepsilon_{xx} + Ae + \frac{Q}{R}(\bar{\sigma} - Qe) \\
\sigma_{yy}^e &= 2N\varepsilon_{yy} + Ae + \frac{Q}{R}(\bar{\sigma} - Qe) \\
\sigma_{zz}^e &= 2N\varepsilon_{zz} + Ae + \frac{Q}{R}(\bar{\sigma} - Qe) \\
\sigma_{yz}^e &= N\varepsilon_{yz},\ \sigma_{zx}^e = N\varepsilon_{zx},\ \sigma_{xy}^e = N\varepsilon_{xy}
\end{aligned} \tag{13}$$

The important fact in Equation (13) is that the total strain is not only a function of mineral stress $\sigma_{ij}{}^e$ but also a function of pore pressure. The physical interpretation of coefficients N, A, Q, and R shows that Equation (13) is consistent with Equation (9) without ε_{ij}^{N} and ε_{ij}^{P} although its appearance is different because of the difference in the definition of effective stress.

Several assumptions are used in Biot's work. One may, therefore, question whether these assumptions could be applied to porous matter or not. In particular, the existence of strain energy, V, is doubtful if one considers that the rocks deform with friction between individual grains and with minute crack propagation, which dissipates energy. However, the strain energy concept can be used only when the coefficients relating to pore pressure are to be determined. Since porous materials deform uniformly with an increment in pore pressure, the slippage, friction, or crack propagation may be very small, in which case dissipation of energy may be neglected. Thus, Biot's approach may be reasonable if the coefficients related to pore fluid are determined.

<u>Effective Stress Is The Only Factor Controlling Total Strain</u>

It is often convenient to rewrite Equation (13) as follows:

$$\varepsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij}^{e} - \frac{\nu}{E} \delta_{ij} \sigma_{kk}^{e} \tag{14}$$

It is possible to determine σ_{ij}^{e} for porous material with linear properties if the strain energy concept proposed by Biot is applied. This approach has been used by several authors (Amos Nur, S. K. Garg, ...). The final result is given by:

$$\sigma_{ij}^{e} = \sigma_{ij} + (1 - \frac{B_i}{B}) p \delta_{ij} \tag{15}$$

where B_i and B are the compressibilities of mineral and rock bulk, respectively.

This concept is very useful to analyze experimental data since strain is expressed only in terms of σ^e_{ij}. However, its use is very limited. The reason is that most granular materials are very non-linear and one may easily conduct erroneous experiments or produce useless data if this effective stress concept definition is used. Although the constitutive equations appear different, they are actually equivalent.

Effective Stress For Failure Theorem

In developing an effective stress concept for use in failure theory, one must apply an appropriate failure criterion. In general, failure criteria are expressed in terms of stress, strain, or energy. In the explanation below, failure criteria in terms of stress are used. When hydrostatic stress is applied to a non-permeable material, the rock strength is affected in two different ways. Figure 5 illustrates these differences intuitively. Let a certain material having tensile strength σ_t at atmospheric pressure, be submerged into the ocean so that hydrostatic pressure is applied upon the surface of the sample. If additional stress, ΔF, is applied to the porous material on the bottom of the ocean, two extreme effects of hydrostatic pressure are observed. If additional stress ΔF is applied to the sample without void space or defects inside, (B in Figure 5), this material fails if additional stress $\Delta F = \sigma_t$. This is so because, for solid materials failure theorems proposed by von Mises, Tresca, or distortion energy theorem can be applied and hydrostatic stress should be subtracted. However, if the material has lots of void space (B' in Figure 5) and the pressure in this void space is zero, this material does not fail even if $\Delta F = \sigma_t$ is applied. In some extreme cases, one must apply additional stress ($\sigma_t + p$) until it fails.

Most materials have properties lying between A and A' in Figure 5. However, as far as the grains of natural rock are concerned, they are closer to type A rather than type A'. In this section for effective stress concepts for failure, the porous material is assumed to have two types of mineral matrix represented by B and B' in Figure 5. Assume rocks A and B in Figure 6 have grains and cementing material whose character is represented by the material A and A' in Figure 5, respectively. Because the mineral matrix is non-porous for case A in Figure 6, the hydrostatic stress should be subtracted from the failure criterion no matter how large the contact area between grains. Hence, the following effective stress should be applied to failure criterion of the bulk of the porous body:

$$\sigma_{ij}^{e} = \sigma_{ij} + \eta p \delta_{ij} \tag{16}$$

where

σ_{ij} = Total stress (compression: negative)

η = Effective stress constant

η = 1 in this case

Most sedimentary rocks have their character represented by A in Figure 6, however, there are certain types of rocks whose mineral-texture is more closely represented by B. For this type of rock, the effective constant is not necessarily close to 1. However, it will be shown that it should be close to 1 for most sedimentary porous rocks.

In order to demonstrate this, start with counter examples where $\eta < 1$ and $\eta > 1$. Figure 7 shows an artificial rock with $\eta \approx \phi$. The mineral matrix consists of impermeable material with voids in it. Since the shape of each matrix is rectangular, there exists no stress concentration in it. Hence, rock fails in tension when the effective stress acting on the mineral matrix equals the strength of the mineral matrix. Hence $\eta = \phi$.

Figure 8 shows an example where $\eta > 1$. There exists this type of natural sedimentary rock which is weakened by stress concentration due to pore pressure. Most of the previous extension tests have shown that, with pore pressure, rocks get weakened, and fail in compression stress state even during extension tests. However, these results may be partially due to stress concentrations induced by uneven loading rather than heterogeneity of the rock. The results in our laboratory experiments showed that if uneven loading is eliminated, rock is strengthened slightly with pore pressure due to void space or defects existing in the rock matrix, if the pore pressure is less than a certain amount. Of course, the Griffith theory states that if effective confining pressure exceeds a certain amount, rock fails in compression, i.e., $\sigma_{ij} + p\delta_{ij} < 0$, even for extension tests.

All existing rocks do not have an idealistic pore structure given by Figure 9. Most sedimentary rocks consist of grains and pores, where pore shape is very irregular and sharp corners exist around the pores (A in Figure 9). When applying a failure theorem to these rocks, one either uses the weakest plane failure theorem (B in Figure 9) or Griffith theorem (C in Figure 9). The previous model is based upon the fact that when a granular material fails, the failure surface develops along the weakest plane with smallest grain contact area. In this case, the strengthening effect of pore pressure upon this smallest area is negligible even if the porous material has mineral matrix with void space (B in Figure 6). Hence, in the effective stress $\sigma^e_{ij} = \sigma_{ij} + \eta p\delta_{ij}$, η should be close to 1. However, rocks that are subjected to a leaching process do not have small contact area because they do not consist of spherical grains with small grain to grain contact area.

For these rocks, one may apply modified Griffith theorem with pore pressure applied to elliptical flat cracks (C in Figure 9).

In order to avoid unnecessary complication, one assumes small elliptical cracks with pressure, P, on inner elliptical surface and outer surface. If one applies total stress, σ, and extends the rock, the stress induced at the end of the major axis of the ellipse is given by

$$\sigma_\eta = (\sigma + p)[2(a/b) + 1] - p \tag{17}$$

If the porous material consists of mineral matrix without void space (A in Figure 6), it is obvious that the effective stress concept $\sigma^e_{ij} = \sigma_{ij} + p\delta_{ij}$ holds for any a/b. However, if the mineral matrix has void space (B in Figure 6), the rock bulk does not fail until $\sigma_\eta = \sigma_t$ is satisfied, where σ_t is matrix strength. If $p = 0$, we have

$$\sigma_t = S_o[2(\tfrac{a}{b}) + 1] \tag{18}$$

where S_o is the strength of rock bulk at atmospheric pressure, Hence,

$$S_o = \sigma_t/[2(\tfrac{a}{b}) + 1] \tag{19}$$

For pore pressure p, the failure condition gives

$$(S + p)[2(\tfrac{a}{b}) + 1] - p = \sigma_t \tag{20}$$

where S is the rock strength at pore pressure p.

Thus,

$$S + p = \frac{\sigma_t + p}{2(\frac{a}{b}) + 1} = S_o + p\,\frac{S_o}{\sigma_t} \tag{21}$$

$$\text{or} \qquad S + p\left(1 - \frac{S_o}{\sigma_t}\right) = S_o$$

$$\text{or} \qquad S + \eta p = S_o \qquad (21)$$

where

$$\eta = 1 - \frac{S_o}{\sigma_t}$$

Since $\frac{S_o}{\sigma_t} \rightarrow 0$ as $\frac{a}{b} \rightarrow 0$, one can conclude that η is close to 1. Comparing the models given by Figure 7 and Figure 10, one finds that the value η varies from ϕ to 1, depending upon the stress concentration around the pore surface. As explained in this report, porous material more frequently has a mineral matrix with small amounts of void space (A in Figure 6) but as a special case may have mineral matrix with a significant amount of void space (B in Figure 6). Hence, even if stress concentration around the pore is small, the rock bulk fails according to the effective stress law with $\eta = 1$.

CONCLUSIONS

1. There are three definitions of the effective stress concept as applied to porous material with pore fluid. These are $\sigma^e_{ij} = \sigma_{ij} + (\theta + \Delta\theta)p\delta_{ij}$ where: $\theta = 1$, for effective stress for stress-strain relations with superposition rule; $\theta = \phi$ for effective stress acting on average mineral area (ϕ is porosity for solid rock and should be replaced by effective porosity for clay or soil); and $\theta = 1$ for failure criteria. In addition, there are several other effective stress definitions. However, constitutive stress-strain relations based upon these definitions are idential and not contradictory to each other.

2. The correction term, $\Delta\theta$, in the effective stress law is not necessarily zero. However, some porous materials have trivial $\Delta\theta$ unless there exists significant amounts of isolated cracks or compressible materials in the mineral matrix, or the mineral matrix is macroscopically heterogeneous, which causes local stress concentrations.

Natural processes of sedimentation work in favor of satisfying the above conditions, and most sedimental soil and clay have trivial $\Delta\theta$. However, one may find non-trivial $\Delta\theta$ in volcanic rocks and artificial rocks.

3. Some of past experiments show discrepancies. However, one may obtain better results in favor of effective stress concepts if: 1. the pore pressure is uniform, where the pores includes both main capillaries and micropores, 2. load is uniform, 3. proper experimental procedures are used to handle non-linearity, hysteresis, and the difference between loading and unloading character, and 4. errors due to apparatus friction and strain measurement are minimized.

ANALYSIS OF DATA USING EFFECTIVE STRESS CONCEPTS

Since Dr. Thompson's experimental data to obtain the water area A_w are based upon the following equation

$$A_w = \Delta P_c / (P_p - P_{p_o}) \tag{22}$$

by setting axial strain to be zero, the convenient constitutive stress-strain relation to use is given by

$$\varepsilon_{ij} = \frac{1+\nu}{E} \sigma^e_{ij} - \frac{\nu}{E} \delta_{ij} \sigma^e_{kk} \tag{23}$$

where

$$\sigma^e_{ij} = \sigma_{ij} + (1 - \frac{B_i}{B}) p \delta_{ij} \tag{24}$$

In this equation, σ^e_{ij} is effective stress, B_i and B are compressibilities of mineral matrix and rock bulk, respectively, ε_{ij} is total strain and p is pore pressure. Comparing with the experiment in Dr. Thompson's paper one obtains

$$A_w = 1 - \frac{B_i}{B} \tag{25}$$

where A_w = water area

Tables 1 and 2 show the results for incremental A_{w_i} and compressibility ratio $\frac{B_i}{B}$. Variation of A_{w_i} shows the difficulty of monitoring radial strain, friction and pore pressure. However, there is nothing wrong with the experiment; it simply is extremely difficult to conduct.

As far as sand compaction is concerned, $\frac{B_i}{B}$ is fairly constant and reasonable. The tendency of decreasing $\frac{B_i}{B}$ appears contrary to our experience since the formation bulk modulus is normally smaller at higher effective stresses ($\sigma_v^e = \sigma_v P$) in the loading process. However, since the measurement of A_w is done in the unloading process, i.e., for decreasing effective stress, the results may be acceptable. This is because total bulk volume consisting of sand particles tends not to rebound properly (or completely) without cementing material between them during unloading. Hence, the value B tends to increase during unloading, until the effective stress reaches a certain level where sand particles interfere with each other without sliding.

The results given in Table II (Marine clay) are difficult to interpret. From common sense, $\frac{B_i}{B}$ should increase as effective stress increases and porosity decreases. One may consider the following reasons to explain the results:

1. Since the phase boundary between solid and water changes with compaction by dewatering, the value B_i decreases with increase in effective stress.

2. As explained in the case of unconsolidated sand, bulk volume does not rebound properly during unloading.

3. Since the permeability is extremely small, incrementally added pore pressure tends to deform rock bulk rather than mineral matrix. Although final pore pressure is constant throughout the pores, the non-uniform pore pressure at the instant of pore pressure increment causes residual strain, especially when the skeleton of the porous material is weak (when effective stress is small) and permeability is small.

Although application of the constitutive equation given by effective stress $\sigma_{ij}^{e} = \sigma_{ij} + (1 - \frac{B_i}{B})p\delta_{ij}$ makes it easy to interpret deformation physically, it complicates rigorous development of constitutive equations and experimental procedures. For compaction problems, a better approach would be to apply effective stress for stress-strain relations with superposition. As stated earlier, this concept can be applied to clays, soils, and rocks unless the mineral matrix includes a lot of isolated porosity. After decomposition of total stress $\sigma_{ij} = (\sigma_{ij} + p\delta_{ij}) - p\delta_{ij}$, the clay or soil matrix part approximately deforms linearly with $- p\delta_{ij}$ and non-linearly with the effective stress $\sigma_{ij}^{e} = \sigma_{ij} + p\delta_{ij}$. Hence, the total strain is given by

$$\varepsilon_{ij} = \varepsilon_{ij}(\sigma_{ij}^{e}) + \beta p\delta_{ij} \qquad (26)$$

where ε_{ij} = a function of σ_{ij}^{e} only.

The difficulty involved for compaction problems is that β is not a constant, unlike rocks or soils without compaction in progress. β and coefficients ε_{ij} are functions of degree of compaction, that is, they may be expressed by porosity, effective stress, effective contact area, ..., whatever related to the degree of compaction. However, there is nothing wrong in the application of effective stress ($\sigma_{ij}^{e} = \sigma_{ij} + p\delta_{ij}$); it is very useful to isolate the effect of pore pressure for compaction problems, since this effective stress definition does not include rock parameters.

TABLE 1 - Horizontal Stress Measurements with Pore Pressures Induced for Sample No. OS-3

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	(A_{wi})	A_w	Correlation Coefficient	β_i/β
407	38.5	0	182	.378	0.713	0.98751	.287
		45	199	.977			
		90	242	.705			
		134	273	.689			
		179	304				
1385	38.1	0	645	.622	0.723	0.99928	.277
		90	701	.775			
		179	770	.777			
		269	840	.667			
		359	900	.730			
		448	965				
2688	37.6	0	1274	.620	0.723	0.99893	.277
		179	1385	.778			
		359	1525	.737			
		538	1657	.721			
		717	1786				
5295	36.6	0	2584	.748	0.784	0.99974	.216
		179	2718	.794			
		359	2861	.793			
		538	3003	.749			
		717	3137	.843			
		896	3288				

TABLE 1 - (Continued)

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	A_w	Correlation Coefficient
10,183	34.5	0	5744	0.749	0.99911
		179	5875		
		359	6011		
		538	6145		
		717	6306		
		896	6412		
		1075	6546		
		1255	6680		

TABLE 2 - Horizontal Stress Measurements with Pore Pressures Induced for Sample No. MC-II-U

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	(A_{wi})	A_w	Correlation Coefficient	Bi/B
407	67.6	0	107	).322	0.591	0.96233	.409
		90	136	).795			
		134	171	).711			
		179	203	).778			
		224	238				
1385	55.0	0	620	).637	0.644	0.99995	.356
		179	734	).650			
		359	851	).631			
		538	964	).665			
		717	1083				
2688	48.5	0	1264	).682	0.728	0.99963	.272
		359	1509	).701			
		717	1760	).751			
		1075	2029	).752			
		1434	2299	).735			
		1792	2562				

TABLE 2 - (Continued)

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi (A_{wi})	A_w	Correlation Coefficient	β_i/β
5295	42.0	0	2440	0.740	0.99821	.260
		359	2659			
		717	2921			
		1075	3240			
		1434	3538			
		1792	3787			
		2151	4024			
		2509	4273			
		2878	4520			
10,183	35.6	0	4942	0.716	0.99947	.284
		359	5206			
		717	5473			
		1075	5704			
		1434	5960			
		1792	6239			

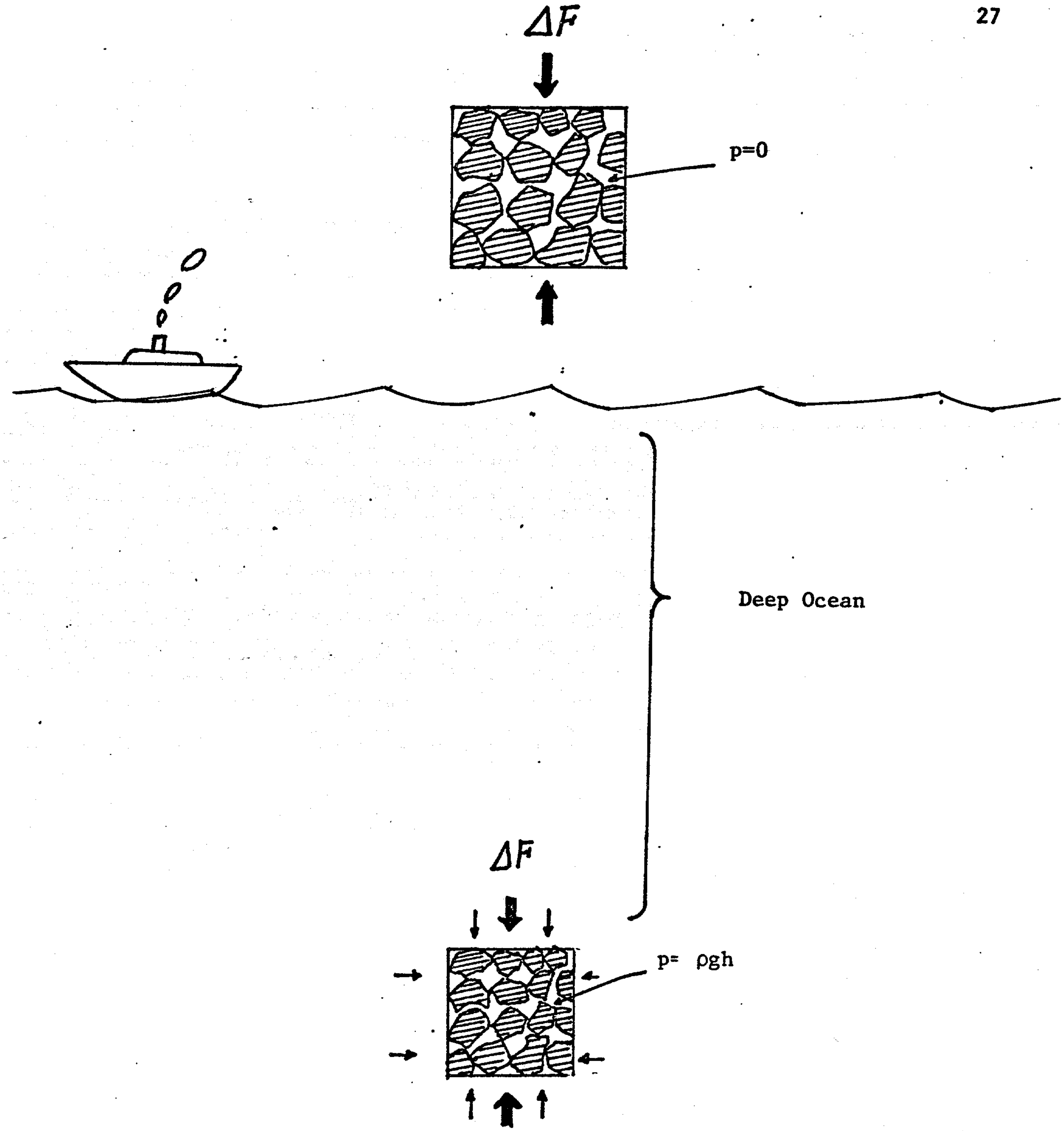


FIGURE 1 INTUITIVE ILLUSTRATION OF EFFECTIVE STRESS FOR STRESS-STRAIN RELATIONS WITH SUPERPOSITION

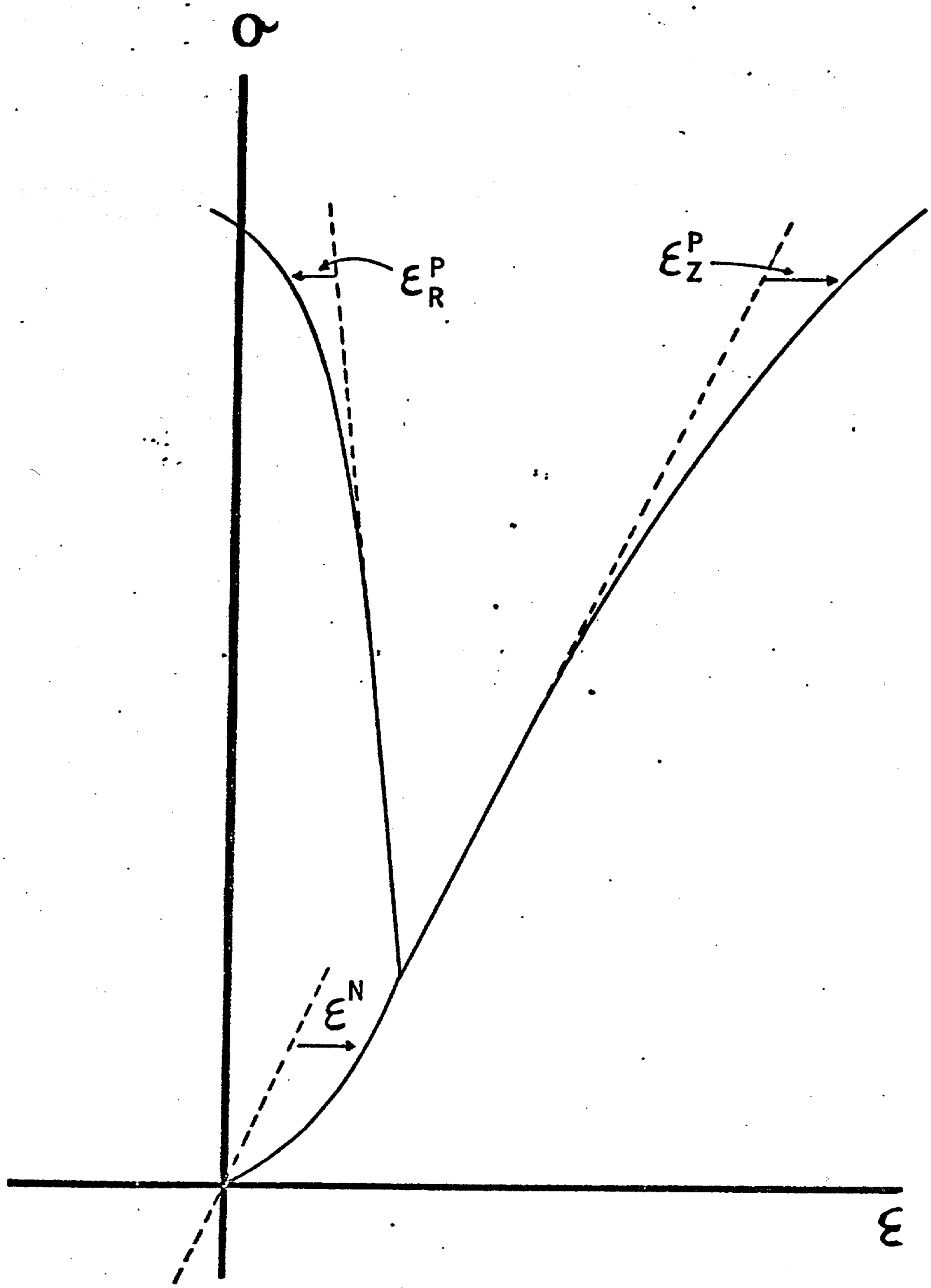


FIGURE 2 SCHEMATIC DIAGRAM FOR INITIAL AND FINAL PORTIONS OF NON-LINEARITY

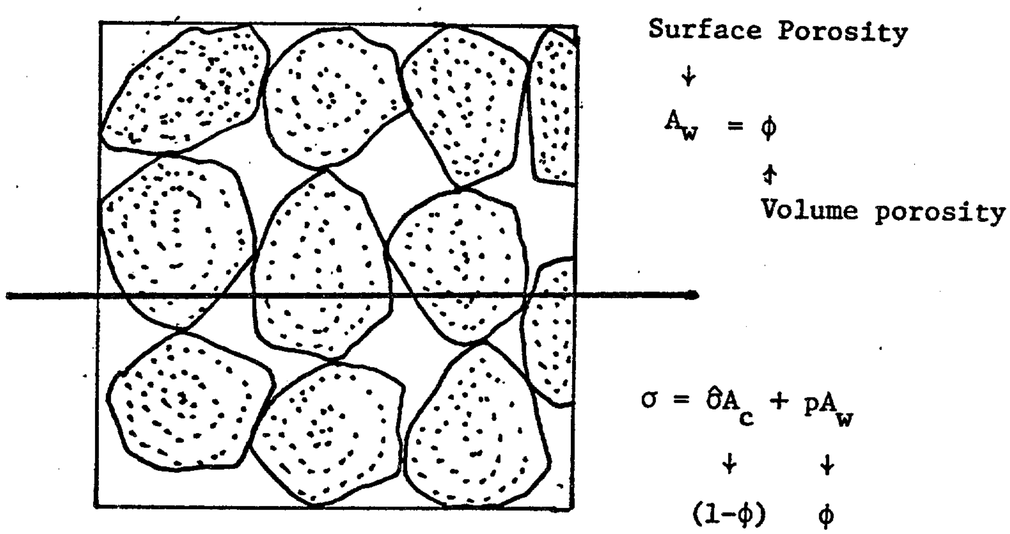


FIGURE 3 EFFECTIVE STRESS ACTING UPON AVERAGE MINERAL AREA

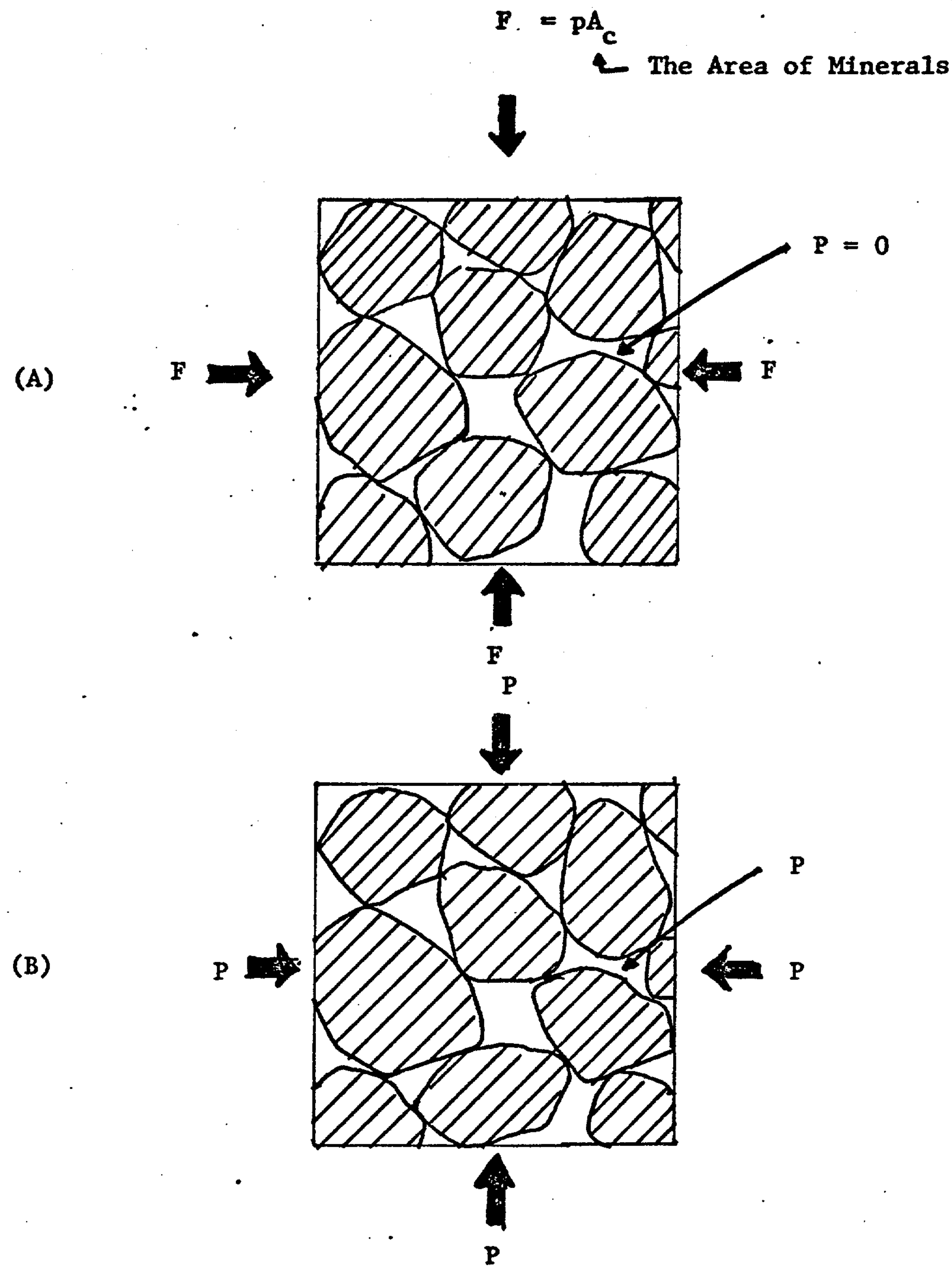


FIGURE 4 BOTH SAMPLES HAVE THE SAME AVERAGE MINERAL STRESS, HOWEVER, TOTAL STRAINS MAY BE DIFFERENT DUE TO THE DIFFERENCE OF STRESS DISTRIBUTION IN THE GRAINS

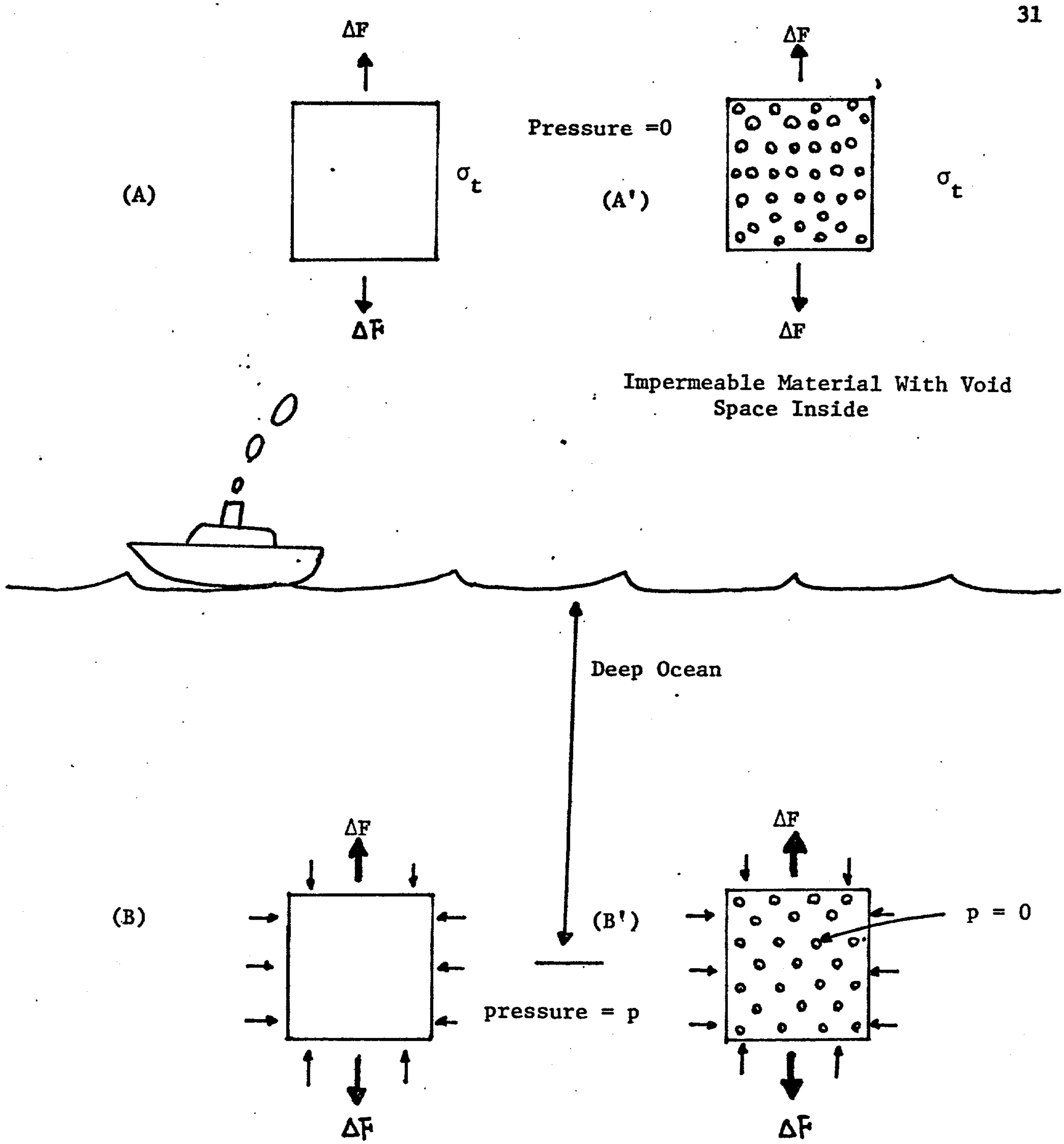
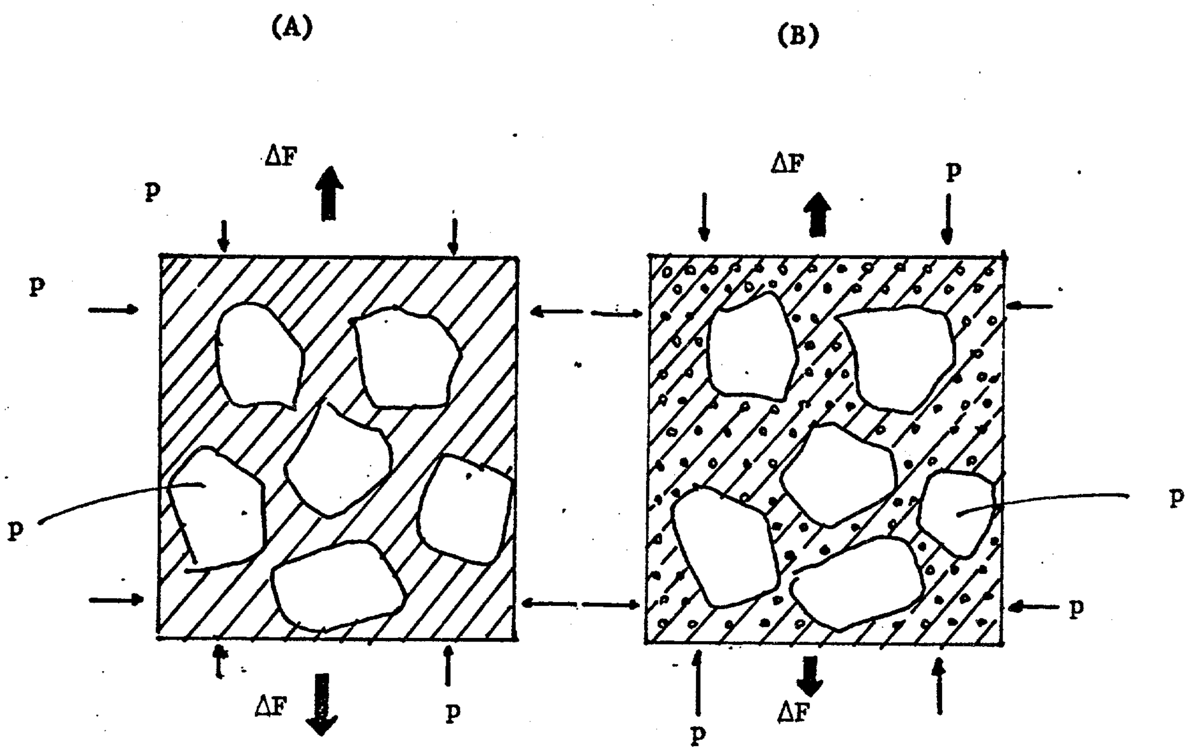


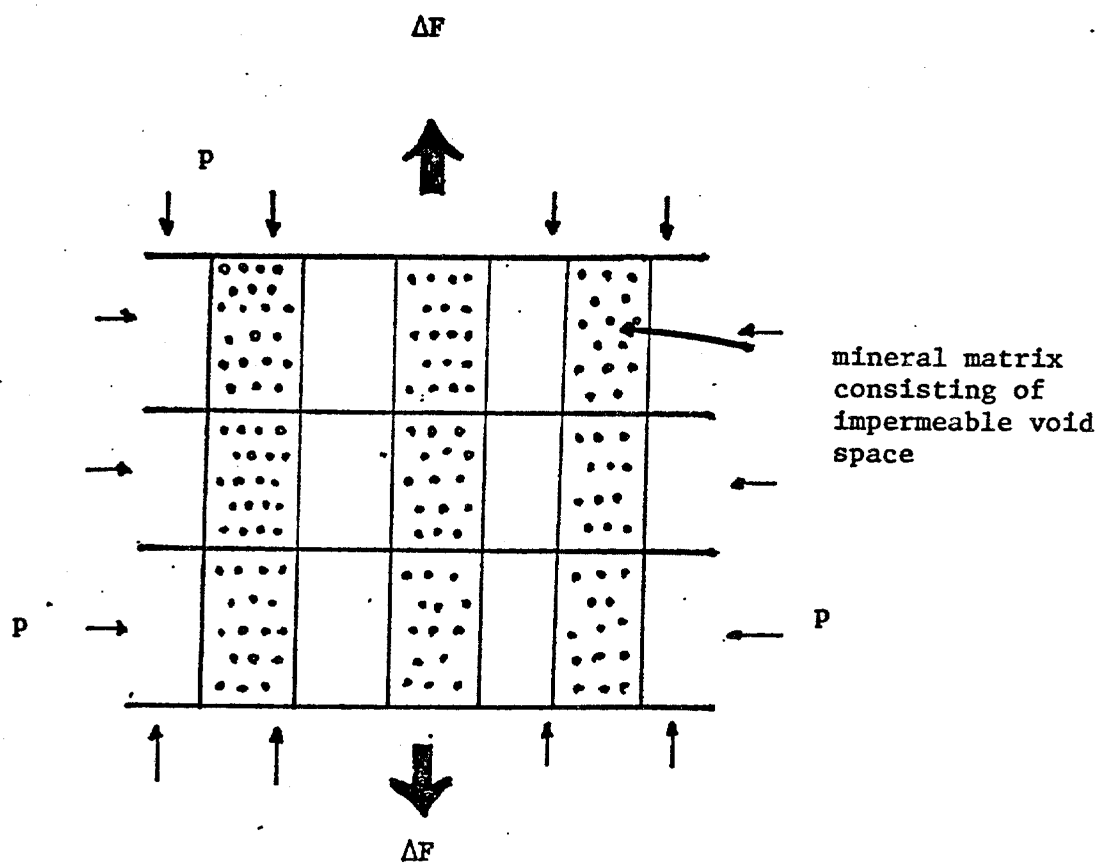
FIGURE 5 TWO EXTREME CASES ON THE EFFECT OF HYDROSTATIC STRESS UPON STRENGTH OF POROUS MATERIALS



Porous Material with Mineral Matrix without Void Space

Porous Material with Mineral Matrix with Void Space

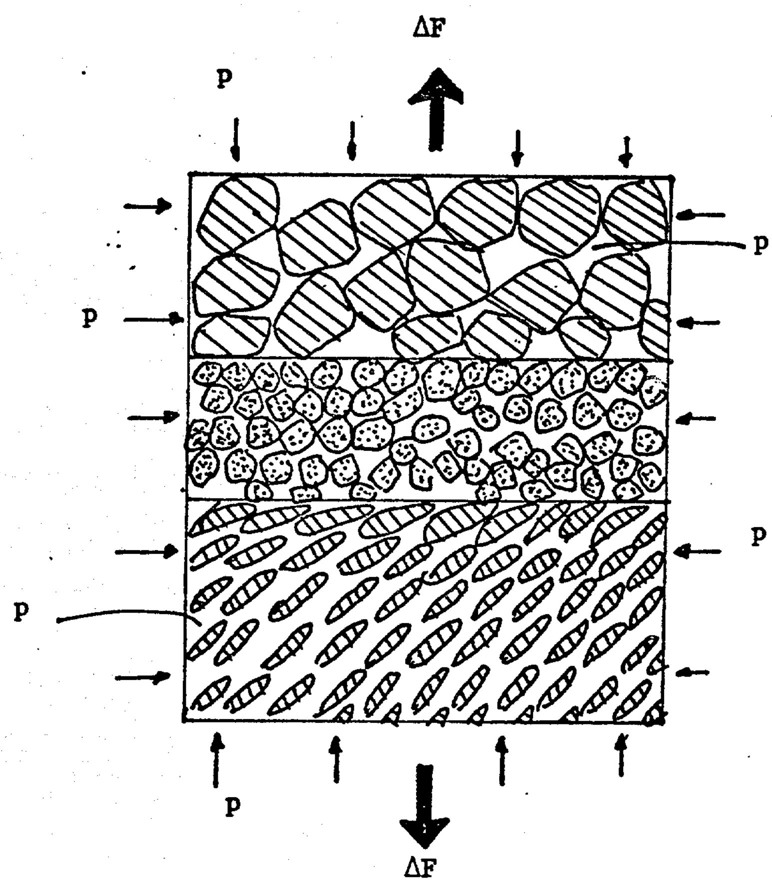
FIGURE 6 TYPICAL EXAMPLES OF POROUS MATTER WITH GRAINS AND CEMENTING MATERIAL WHOSE FAILURE PROPERTIES ARE DIFFERENTLY AFFECTED BY PORE FLUID PRESSURE



Calculation of η for extension test

1. Assume the tensile strength of mineral matrix to be σ_t.
2. Strength of rock(S) = $(1-\phi)\sigma_t$ at atmospheric pressure where ϕ is porosity.
3. With pore pressure p, the failure condition of mineral matrix is still σ_t, or it is S as a bulk.
4. Since the effective stress acting upon mineral matrix is $\sigma_{ij} + \phi p \delta_{ij}$, failure occurs if $\sigma_{ax} + \phi = S$ or $\eta = \phi$ for failure theorem.

Fig.7 ARTIFICIAL ROCK WITH EFFECTIVE STRESS CONSTANT $\eta = \phi$



1. Assume compressibilities of mineral matrix to be different for each layer, and the tensile strength of the rock to be S at atmospheric pressure.

2. If hydrostatic pressure is applied at rock boundaries and thoroughout pores, mineral matrix of each layer deforms differently. Due to this difference, stress concentration occurs around the pore, and hence, the rock is weakened and failure occurs for

$$\sigma + p = S' << S$$

where σ is the total axial stress. Hence

$$\sigma + ((S-S')+p) = S$$

or $\sigma + \eta p = S$ where $\eta > 1$.

Fig. 8 . NATURAL ROCK WITH EFFECTIVE STRESS CONSTANT $\eta > 1$.

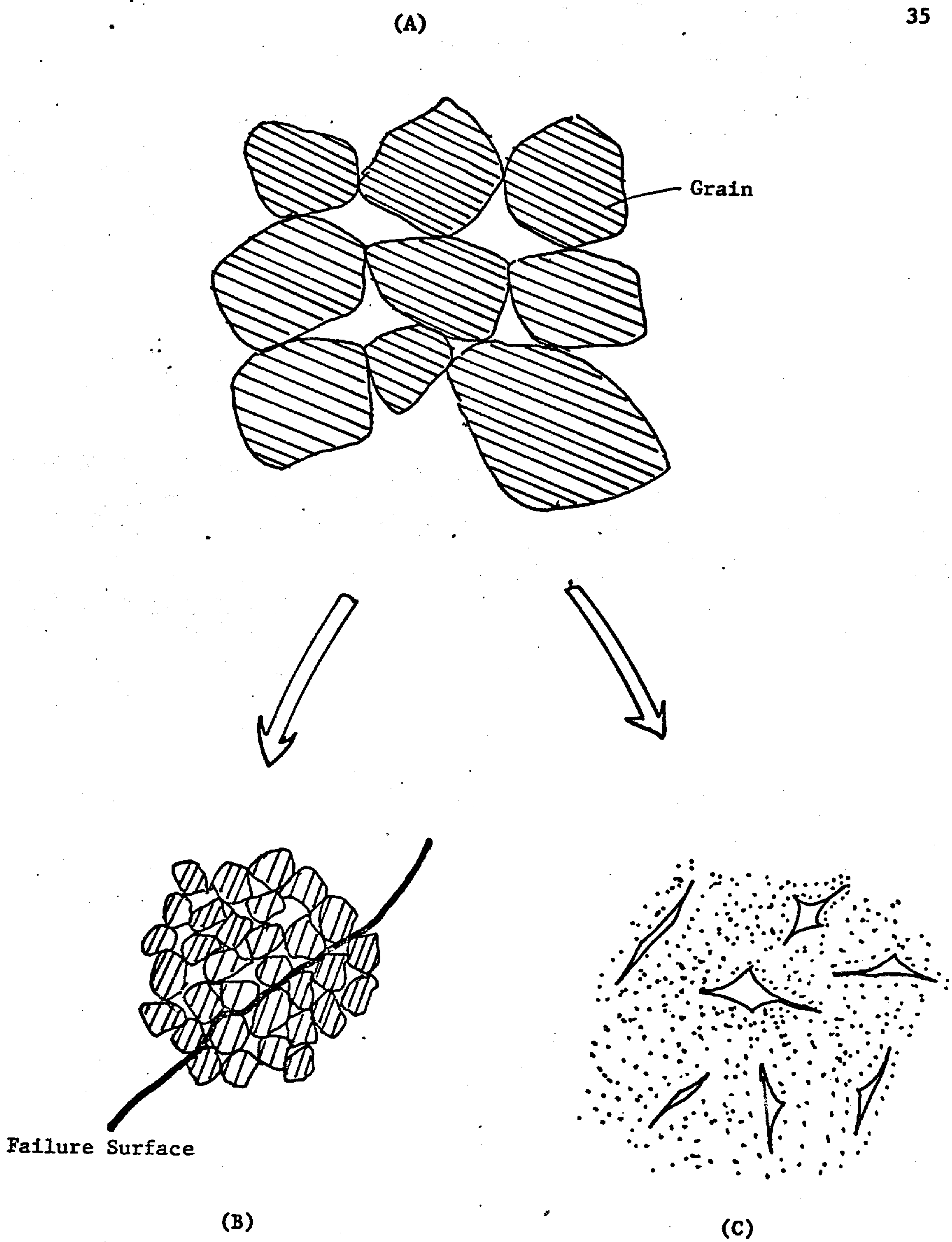


FIGURE 9 ROCK MODELS USED FOR FAILURE THEOREMS

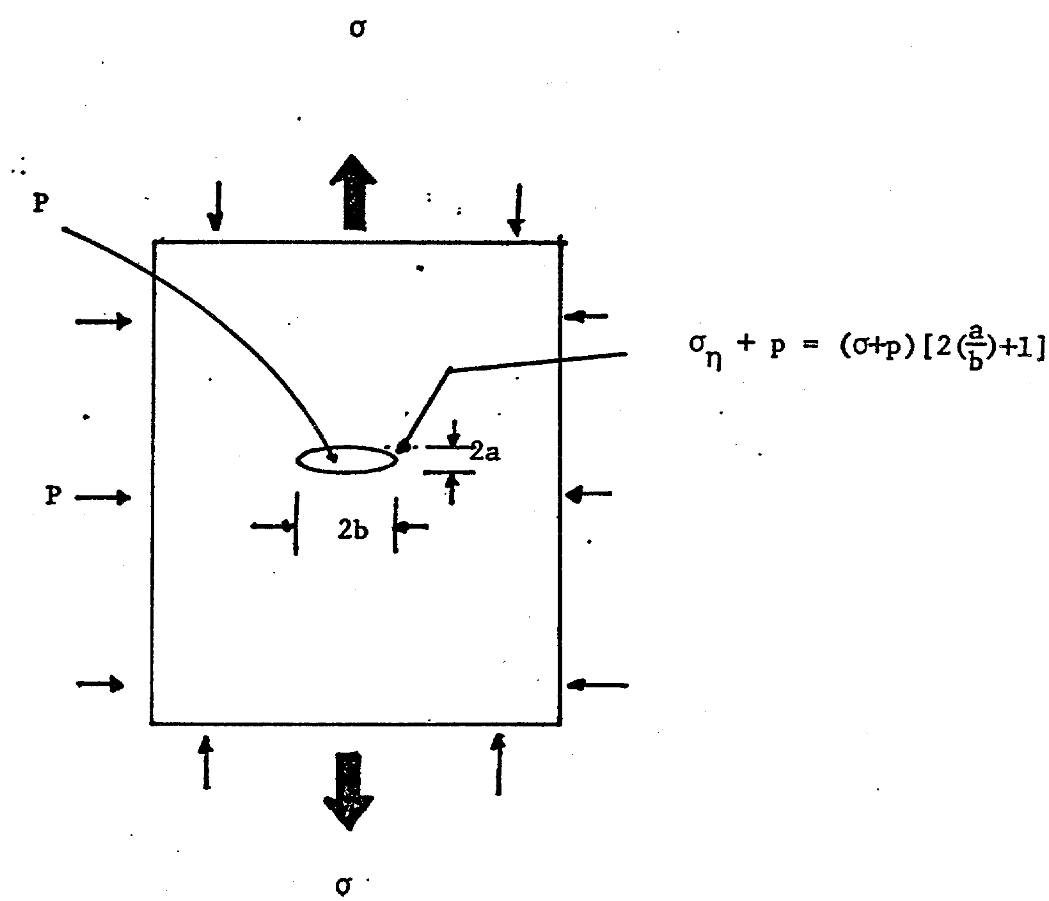


FIGURE 10 · GRIFFICE CRACK WITH PORE PRESSURE