Recording Medium Properties and Capacity Bounds

J. A. O'Sullivan, D. G. Porter, R. S. Indeck and M. W. Muller
Department of Electrical Engineering, Washington University, St. Louis, MO USA 63130-4899

Abstract— **The effect of such medium properties as magnetic unit size, film thickness, and strength of anisotropy, dipolar, and exchange interactions on the storage capacity limit of a thin-film recording medium is investigated. Relationships are dervied for media with different properties that have equivalent capacity. The physically realizable range of material properties and their effect on the capacity bound are explored. Results on edge effects for capacity bound computations are presented. The results indicate that media for which anisotropy is the dominant source of magnetic energy have the greatest potential for information storage.**

I. Introduction

Recent measurements have shown that medium noise is largely repeatable, suggesting that it is generated by a permanent magnetic microstructure of the recording medium.[1], [2] These measurements motivate a model of the recording medium comprising fixed magnetic structures. We have devised such a model.[3] It has many features of other micromagnetic medium tessellation models,[4], [5], [6] but requires less computation. We model a variety of media by varying parameters which represent the physical properties of recording media. These parameters include tile size, film thickness, and strength of anisotropy, dipolar, and exchange interactions. A tile represents a single domain microstructural entity, and may correspond to a single grain, a part of a grain, or a cluster of grains.

Bounds on storage capacity of magnetic recording systems have been computed based on models of their physical operation.[7], [8], [9] We have presented a method to compute upper bounds on the capacity of an ensemble of media and a single medium, irrespective of the read and write transducers.[10] The bound on the capacity of a single recording medium serves as a bound on the capacity of any recording system using that medium for storage. The inclusion of read/write transducers will reduce this capacity. The capacity bound is based on an argument that only stable states of the model may be used to store information. Thus, an estimate of the number of stable states of the model provides a fundamental upper limit to the capacity of information storage possible for a given medium. The capacity bound is computed in units of information bits per single domain magnetic tile. This capacity can be transformed into the more usual capacity measure of bits per unit of medium area based on the tile size.

II. Equivalent Media

Table I displays the parameters of the medium model. The interactions of the tiles which make up the medium depend on these parameters. Through this dependence, the parameters determine the medium storage capacity.

Symbol	*Description*	*Units*
K_e	Exchange Strength	H/m
δ	Film Thickness	m
d	Tile Spacing	m
K_a	Anisotropy Energy Density	J/m^3
M	Magnetization Magnitude	A/m

TABLE I
Medium Model Parameters

The energy of a tile is determined by the normalized effective magnetic field, $\mathbf{h}$, at the tile. The field is normalized to the anisotropy field, as in Prutton,[11]

$$\mathbf{h} = \mathbf{H}/H_a, \text{where } H_a = 2K_a/M. \tag{1}$$

The magnetic field, $\mathbf{H}$, is the sum of two component effective fields, due to exchange and dipolar interactions respectively, in dimensional and normalized form,

$$\mathbf{H} = \mathbf{H}_e + \mathbf{H}_d; \quad \mathbf{h} = \mathbf{h}_e + \mathbf{h}_d. \tag{2}$$

The exchange field, $\mathbf{H}_e$, is generated by the neighboring tiles. Using i to index the set of neighbor tiles, and $\mathbf{M}_i$ to denote the corresponding magnetizations, we may write

$$\mathbf{H}_e = \sum_i K_e \mathbf{M}_i. \tag{3}$$

The dipolar field, $\mathbf{H}_d$, is generated by the magnetic dipole moments of all other tiles in the medium. These dipole moments are the product of the tiles' magnetizations with their volumes. Using similar notation, we may write

$$\mathbf{H}_d = \sum_i \mathbf{A}_i (d^2 \delta \mathbf{M}_i), \tag{4}$$

where $\mathbf{A}_i$ represents the appropriate linear operation to generate the magnetostatic field at the tile under examination due to the ith tile in the sum. The dependence of $\mathbf{A}_i$ on the parameters from Table I is

$$\mathbf{A}_i \propto 1/d^3. \tag{5}$$

This proportion represents the inverse relationship of field magnitude with the cube of the distance between the source dipole and the test point.

Equations (1-5) imply that the field components vary according to[12]

$$\mathbf{h}_e \propto K_e M^2/K_a; \quad \mathbf{h}_d \propto \delta M^2/dK_a. \tag{6}$$

Symbol	*Low*	*High*
K_e	10^{-10}	10^{-7}
δ	3×10^{-8}	2×10^{-7}
d	10^{-8}	10^{-7}
K_a	10^4	10^5
M	5×10^5	2×10^6

TABLE II
PHYSICAL PARAMETER RANGES

When the model parameters are varied, the total magnetic field at a tile is unchanged if both proportionality factors of Eq. (6) are unchanged.

If we represent a medium by a 5-tuple of its parameters, and use the symbol $\equiv$ to indicate that two media generate the same normalized field values, we conclude for any positive values of the medium parameters

$$(K_e, \delta, d, K_a, M) \equiv (K_e M^2/K_a, \delta M^2/dK_a, 1, 1, 1). \quad (7)$$

Thus, two parameters suffice to describe the influence of all five parameters on the behavior of the medium model. We may represent a medium by a 5-tuple, $(K_e', \delta', 1, 1, 1)$, or equivalently by the pair (K_e', δ'). The parameters (K_e', δ') in our model are similar to the parameters h_e, h_m in the numerical simulation work of Zhu and Bertram.[13]

Our method for computing capacity bounds depends on the normalized field values. According to the model, a state of the medium is stable if each tile in the medium has a magnetization stable against reversal. That is, the normalized field at each tile does not force the tile's magnetization to reverse direction. This reversal test does not consider thermal effects. Effectively, the model represents a medium at absolute zero. Furthermore, rotations of magnetization off the easy axis are not permitted. Reversals of direction along the easy axis are the only variations of tile magnetization accounted for by this model. The easy axes of the tiles are independently generated from a uniform distribution. The results depend on the choice of distribution for the easy axes; other distributions have been investigated.[2], [14] This model constrains each tile to two magnetic states. If a medium has N stable states, the information storage capacity of that medium is bounded above by $\log_2 N$.[10] The constraint of two states per tile implies an absolute maximum possible storage capacity (when all states of a medium are stable) of 1 bit per tile.

Thus, we conclude that two media which produce the same proportionality factors in (6) share the same capacity bound in bits per tile. Varying the tile size yields different capacities in bits per unit area among what we call equivalent media.

III. PARAMETER AND CAPACITY VALUES

Table II displays ranges of parameter values which include physically realizable media. When these parameter ranges are transformed into the two-variable parameter space (K_e', δ'), a bounding rectangle may be drawn around

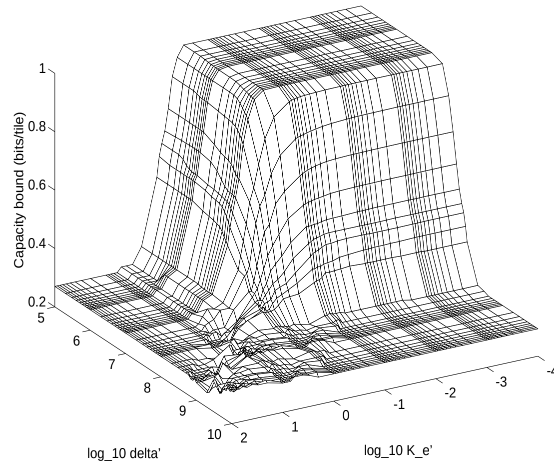

Fig. 1. Capacity bound as a function of parameters K_e' and δ'

the set of physically realizable parameters, yielding transformed parameter ranges:

$$10^{-4} \le K_e' \le 10^2; \quad 10^5 \le \delta' \le 10^{10}. \quad (8)$$

To estimate the variation of the capacity bound over these parameter ranges, the capacity bound was computed for a set of grid points within the parameter ranges. The largest square medium patch for which a direct count of stable states may be used to exactly compute the capacity bound in reasonable time using our simulations and computing resources is a 5×5 medium. Figure II displays the capacity bounds computed for a single 4×4 medium in bits per tile as a function of the base 10 logarithms of its transformed parameters K_e' and δ'. The use of a slightly smaller patch allows a large number of simulations to be run. A plot of the average of the capacity bounds for 50 consecutive media has a similar shape. Note by comparison of the definitions of K_e' and δ' with Eq. (6) that the parameters K_e' and δ' correspond to the relative magnitudes of the exchange and dipolar field components.

Larger patches require statistical analysis to estimate the number of stable states, N. As the patch size increases, N decreases as a fraction of the total number of states. Estimating N by a Monte Carlo simulation corresponds to estimating the probability of a stable state assuming a uniform distribution on possible states. This event grows rarer as the patch size increases, and Monte Carlo simulation fails to estimate the probability of such a rare event in reasonable time. More powerful computational methods are needed.

One strategy to increase computational power is to implement simulations on a massively parallel architecture. Such an implementation has been developed, and it does improve computational efficiency. As an example, the massively parallel implementation can sample and evaluate states of a 32×32 medium approximately 12 times faster than a single processor implementation. At present, this improvement is not sufficient to generate reliable estimates for large patches in reasonable time, although improved sampling methods are being studied.

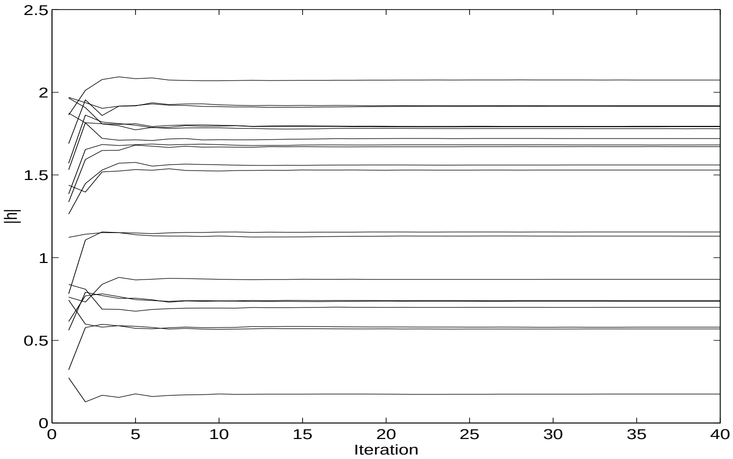

Fig. 2. Field magnitude vs. patch size

IV. Edge Effects

Although full computation of capacity bounds for large numbers of tiles is not feasible at present, we can characterize the edge effects resulting from model simulations of small numbers of tiles. We study the set of equivalent media represented by transformed medium parameters $(K_e', \delta') = (0.15, 7.8 \times 10^7)$, a medium near the middle of our parameter ranges. A simulation was run to illustrate the effect of small patch size on the magnetic fields of randomly generated media. A medium was constructed in steps, beginning with a 3×3 medium, and adding a new square ring of tiles at each step (making a 5×5, 7×7, etc.). The magnetic field of the center tile was examined at each step. This simulation was run several times to collect results for several randomly generated media.

Figure III plots the magnitude of the magnetic field versus the step in the construction for several runs of the simulation. Similar plots were generated for the magnetic field direction. The plots indicate that after about eight steps, the magnetic field changes relatively little. This indicates that the field at a tile at the center of a randomly generated 17×17 medium serves as a good representation for the field at a tile in the interior of a larger randomly generated medium. Similar experiments may be run with other choices of medium parameters.

For these parameters, it appears that a 8-tile-wide border around a small patch will produce small patch capacity bounds with reduced dependence on the patch size. For example, we randomly generate and evaluate the stability of the center 5×5 collection of tiles within a randomly generated 25×25 media. The capacity bound estimated for an ensemble of isolated 5×5 media based on 10^6 samples is 0.44 ± 0.02 bits per tile. The capacity bound estimated for an ensemble of 5×5 cores in a 25×25 medium based on 10^6 samples is 0.30 ± 0.05 bits per tile. As expected, the latter capacity is smaller due to the increase in the effective field magnitudes in the core due to the effect of the outer ring. The large difference between the two motivates further study of larger media.

V. Conclusion and Future Work

We have investigated the dependence of the fundamental limit of storage capacity of a medium on its physical properties. Our analysis has shown that there are media with different parameter values which have the same storage capacity. Media may be grouped into equivalence classes by this analysis, and each class may be represented by a parameter pair (K_e', δ'). Large values of K_a lead to small values of both K_e' and δ', which correspond to a capacity bound of 1 bit per tile, the largest possible capacity consistent with the medium model used. This indicates that media for which anisotropy is the dominant source of magnetic energy have the greatest potential for information storage. The value of K_a necessary to achieve such a medium given other medium properties can be computed using Eq. (7) and Fig. II.

The computed capacity bounds do not decrease monotonically with large values of K_e' and δ' as might be expected on physical grounds, but reach non-zero plateaus. This effect may be an artifact caused by the constraint precluding magnetization rotation; it will be removed in future work.

VI. Acknowledgment

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