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The Approximation of Vertically Stacked Area Sources with a Single Area Source in AERMOD

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Abstract

AERMOD, the Environmental Protection Agency's regulatory air dispersion model, models point, area and volume sources. Due to the numerical integration involved, computation times for area sources are significantly longer than those for point and volume sources. EDMS, the Emissions and Dispersion Modeling System, employs AERMOD in the modeling of airport dispersion. As a means of reducing AERMOD run times, and therefore EDMS run times as well, the combining of vertically stacked area sources is considered and demonstrated in this paper. Errors in concentrations resulting from such approximations are also analyzed.

Introduction

The Federal Aviation Administration's required environmental airport model, the Emissions and Dispersion Modeling System (EDMS), models both the emissions and dispersion from sources typically found at an airport. Dispersion in EDMS is modeled using the Environmental Protection Agency's (EPA) AERMOD, the regulatory air dispersion model. AERMOD's dispersion algorithms are not incorporated directly into EDMS. Instead, EDMS generates input files for AERMOD. This allows updates to EPA's models to be incorporated with little change, if any, to EDMS.

Because AERMOD had previously never been used to model aviation sources, the AMS/EPA Regulatory Model Improvement Committee (AERMIC) was consulted. They recommended modeling aircraft with a series of area sources, as opposed to a string of volume sources. Modeling dispersion from area sources requires numerical integration in two dimensions. Since this demands lengthier computation times than modeling with other source types, it is in the modeler's interests to eliminate and combine area sources whenever possible. To compound the problem, aircraft on runways accelerate. Consequently, area sources representing runways must be cut into short sections such that any aircraft's speed differential from the beginning to the end along any section is minimized while keeping the total number of individual sources manageable. For its solution, EDMS divides runways into sections fifty meters in length; hence, a single runway can require over sixty sources.

Intuitively, a fleet of aircraft of various types using a given physical runway should be modeled as several vertically stacked runways, each with a release height and initial vertical sigma for every aircraft type. Because area sources model only one release height and one initial sigma, a different area source must be used for each portion of the fleet that shares the same parameters. Although this solution is accurate, the multiplication of area sources to represent the same runway location only exacerbates the run time problem. To reduce computation time and keep the total number of sources manageable, stacked area sources may be combined into a single area source whose parameters best represent the fleet at a particular physical location. This compromise, however, will sacrifice some accuracy, as discussed in the error analysis to follow.

The initial conditions of vertical pollutant dispersion from an area source are modeled in AERMOD by a Gaussian distribution with mean at the release height and standard deviation equal to a given sigma-z_0. The effective dispersion from vertically stacked area sources is generally not Gaussian but rather the irregular or "lumpy" weighted sums of Gaussian distributions. See figures 1 and 2 for a graphic comparison. Statisticians may balk at this superposition, however the component Gaussian distributions from individual area sources do not represent probabilities but rather the distribution of physical particles in space. Because a Gaussian distribution is completely defined by its mean and standard deviation, for simplicity, we assume that the best Gaussian approximation for an irregular distribution is one with equal mean and standard deviation. In this case, the mean and standard deviation of each stacked area source is given, and therefore the mean and standard deviation of the sum distribution can be analytically computed as shown by the following derivation.

Figure 1. Plot of a Gaussian Distribution

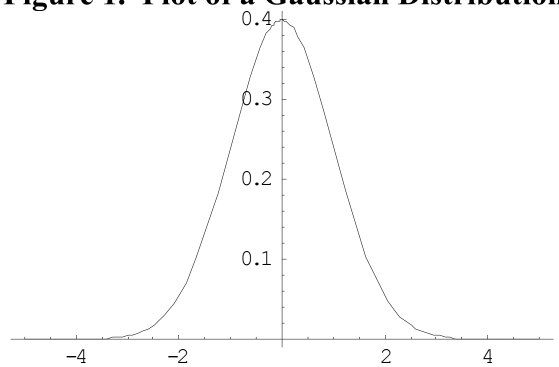
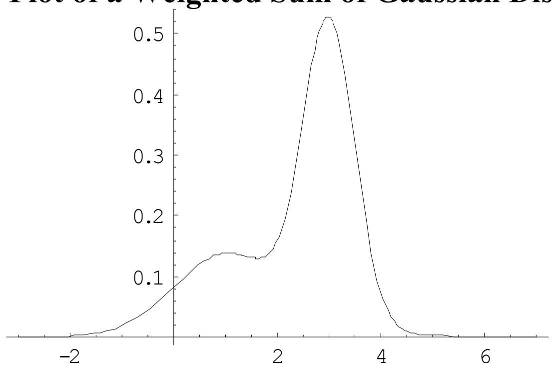


Figure 2. Plot of a Weighted Sum of Gaussian Distributions



Derivation

A distribution is said to be normalized if the total area under its curve (i.e. the integral over all real numbers) is equal to one. All Gaussian distributions are both normalized and continuous. Therefore weighted finite sums of Gaussian distributions are also normalized and continuous, provided the weights sum to one. Continuity is required to perform integration over all real numbers upon them.

The mean, μ, of a normalized continuous distribution, $f(x)$, such as a Gaussian distribution, can be extracted via the integral

$$\mu = \langle x \rangle = \int_{-\infty}^{\infty} f(x)\, x dx \ , \qquad (1)$$

where the angular brackets denote expectation value. Here, the expectation value of x is the mean. The variance (i.e. the square of the standard deviation, σ) is given by

$$\sigma^2 = \left\langle (x-\mu)^2 \right\rangle = \int_{-\infty}^{\infty} f(x)(x-\mu)^2 dx \ , \qquad (2)$$

where μ is the known mean.

A composite normalized continuous distribution may be constructed by a weighted sum of component normalized continuous distributions. Let

$$f(x) = \sum_{i=1}^{N} w_i f_i(x), \qquad (3)$$

where $f_i(x)$ is a normalized continuous distribution with mean μ_i and standard deviation σ_i, w_i is a weighting coefficient such that

$$\sum_{i=1}^{N} w_i = 1 \qquad (4)$$

and $w_i > 0$ for all i and N is the finite number of component distributions.

The mean, μ, of $f(x)$ is computed by

$$\mu = \langle x \rangle_f = \int_{-\infty}^{\infty} \sum_{i=1}^{N} w_i f_i(x) x dx = \sum_{i=1}^{N} w_i \int_{-\infty}^{\infty} f_i(x) x dx = \sum_{i=1}^{N} w_i \langle x \rangle_{f_i} = \sum_{i=1}^{N} w_i \mu_i . \qquad (5)$$

Therefore, the mean of the composite distribution is equal to the weighted sum of the component means. Similarly, the variance, σ^2, of $f(x)$ is

$$\sigma^2 = \left\langle (x-\mu)^2 \right\rangle_f = \sum_{i=1}^{N} w_i \left\langle (x-\mu)^2 \right\rangle_{f_i} . \qquad (6)$$

To complete this derivation, the expectation value of $(x\text{-}\mu)^2$ for each component distribution should be rewritten in terms of the component variance and mean as

$$\begin{aligned}
\left\langle (x-\mu)^2 \right\rangle_{f_i} &= \left\langle x^2 - 2x\mu + \mu^2 \right\rangle_{f_i} \\
&= \left\langle x^2 + (-2x\mu_i + 2x\mu_i) - 2x\mu + \mu^2 + (\mu_i{}^2 - \mu_i{}^2) \right\rangle_{f_i} \\
&= \left\langle (x-\mu_i)^2 + (2x\mu_i) - 2x\mu + \mu^2 + (-\mu_i{}^2) \right\rangle_{f_i} \\
&= \left\langle (x-\mu_i)^2 \right\rangle_{f_i} + \left\langle 2x\mu_i \right\rangle_{f_i} - \left\langle 2x\mu \right\rangle_{f_i} + \left\langle \mu^2 \right\rangle_{f_i} - \left\langle \mu_i{}^2 \right\rangle_{f_i} \\
&= \sigma_i{}^2 + 2\mu_i \langle x \rangle_{f_i} - 2\mu \langle x \rangle_{f_i} + \mu^2 - \mu_i{}^2 \\
&= \sigma_i{}^2 + 2\mu_i{}^2 - 2\mu\mu_i + \mu^2 - \mu_i{}^2 \\
&= \sigma_i{}^2 + \mu_i{}^2 - 2\mu\mu_i + \mu^2.
\end{aligned} \tag{7}$$

By substituting equation 7 back into equation 6,

$$\begin{aligned}
\sigma^2 &= \sum_{i=1}^{N} w_i (\sigma_i{}^2 + \mu_i{}^2 - 2\mu\mu_i + \mu^2) \\
&= \sum_{i=1}^{N} w_i (\sigma_i{}^2 + \mu_i{}^2) + \sum_{i=1}^{N} w_i (-2\mu\mu_i) + \sum_{i=1}^{N} w_i \mu^2 \\
&= \sum_{i=1}^{N} w_i (\sigma_i{}^2 + \mu_i{}^2) - 2\mu \sum_{i=1}^{N} w_i \mu_i + \mu^2 \sum_{i=1}^{N} w_i \\
&= \sum_{i=1}^{N} w_i (\sigma_i{}^2 + \mu_i{}^2) - 2\mu(\mu) + \mu^2(1) \\
&= \sum_{i=1}^{N} w_i (\sigma_i{}^2 + \mu_i{}^2) - \mu^2.
\end{aligned} \tag{8}$$

By equations 5 and 8 above, the mean and standard deviation of the approximating Gaussian distribution can be computed given the weights, w_i. However, theses weighting coefficients could be determined by a number of methods.

A simple scheme, which was implemented in EDMS 4.0, is to set each weight equal to the fraction of total airport LTOs that the respective aircraft contributes and use this to determine the dispersion parameters for all runway, queue and taxiway area sources. This calculation is shown below in equation 9.

$$w_i = \frac{LTO_i}{\sum_{k=1}^{N} LTO_k}, \tag{9}$$

where LTO_i is the annual number landing-takeoff operation cycles for aircraft i. Because emission rates vary for each aircraft, operating mode and pollutant, more sophisticated schemes could increase accuracy.

For each aircraft operation (takeoff, taxi, or landing) in a given area source, consider the product of annual time spent within the source boundaries and the aircraft emission rate in such an operating mode. The sum of each of these products for a given aircraft is the total emissions that the aircraft contributes to the area source. Setting each aircraft's weighting factor equal to the fraction of the fleet emissions that it contributes is a more accurate schema. If only a specific pollutant is to be modeled, then only the emission rates for the specific pollutant should be used. Otherwise if any pollutant is equally likely to be modeled, then the aircraft emission rates for all pollutants may be summed together into an unspeciated emission rate. For runways, w_i is given by

$$w_i = \frac{L_i t_{i,landing} r_{i,idle} + TO_i t_{i,takeoff} r_{i,takeoff}}{\sum_{k=1}^{N} (L_k t_{k,landing} r_{k,idle} + TO_k t_{k,takeoff} r_{k,takeoff})}, \qquad (10)$$

where L_i and TO_i are the annual number of landings and takeoffs, respectively, for aircraft i, $t_{i,action}$ is the time aircraft i spends in the runway source performing operation *action*, and $r_{i,mode}$ is the total aircraft emission rate for aircraft i under mode *mode*.

Implementation

Consider a fleet of three aircraft types: an Airbus 320-200 using two CFM56-5A1 engines, a Boeing 737-300 with two CFM56-3-B1 engines and a Cessna 172 Skyhawk employing a single Textron Lycoming O-320. Assume the Airbus makes 1000 LTOs, the Boeing 500 LTOs and the Cessna 100 LTOs annually on the first 50-meter length of runway to be modeled. The times-in-mode are derived from the associated default flight profiles in EDMS, which are based on the methodology presented in the Society of Automotive Engineers (SAE) Aerospace Information Report (AIR) 1845. The emission rates are derived from the ICAO databank and AP-42.

Compute the total Airbus emissions using the numerator in equation 10.
(1000 takeoffs/year)(2 engines)*(2.459 secs/takeoff)**
(0.9459 g of CO/s + 0.24172 g of HC/s + 25.8546 g of NOx/s + 1.051 g of SOx/s)
+ (1000 landings/year)(2 engines)*(0.712 secs/landing)**
(1.7794 g of CO/s + 0.14154 g of HC/s + 0.4044 g of NOx/s + 0.1011 g of SOx/s)
= 141.6 Kg/year

Similarly, compute the total Boeing emissions.
(500 takeoffs/year)(2 engines)*(2.409 secs/takeoff)**
(0.8514 g of CO/s + 0.03784 g of HC/s + 16.7442 g of NOx/s + 0.946 g of SOx/s)
+ (500 landings/year)(2 engines)*(0.709 secs/landing)**
(3.9216 g of CO/s + 0.25992 g of HC/s + 0.4446 g of NOx/s + 0.114 g of SOx/s)
= 48.12 Kg/year

Compute the total Cessna emissions.
(100 takeoffs/year)(2.509 secs/takeoff)**
(12.096 g of CO/s + 0.1322 g of HC/s + 0.02453 g of NOx/s + 0.0012 g of SOx/s)
+ (100 landings/year)(1.799 secs/landing)**
(1.285 g of CO/s + 0.0439 g of HC/s + 0.00062 g of NOx/s + 0.00013 g of SOx/s)
= 33.13 Kg/year

Sum the total emissions for all aircraft. This is the denominator in equation 10.
141.6 Kg/year + 48.12 Kg/year + 33.13 Kg/year = 222.88 Kg/year

By equation 10, determine the weight for each aircraft type.
For the Airbuses, *141.6 / 222.88 = 0.6354.*
For the Boeings, *48.12 / 222.88 = 0.2159.*
For the Cessnas, *33.13 / 222.88 = 0.1487.*

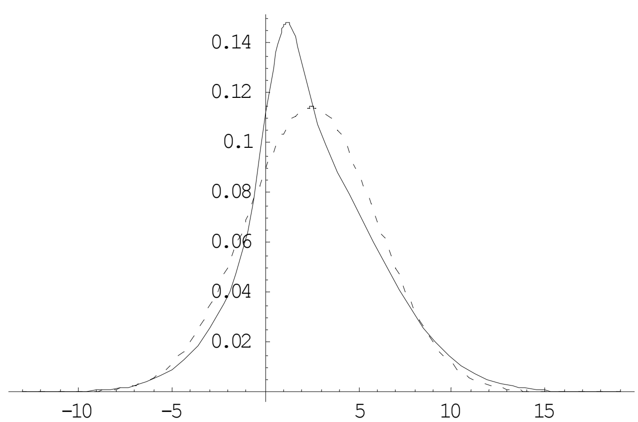
Hypothetically, suppose the Airbus 320-200 has a release height of 3 meters and an initial vertical sigma of 4 meters, the Boeing 737-300 a release height of 2 meters with an initial vertical sigma of 2.5 meters, and the Cessna 172 a release height and initial vertical sigma both equal to 1 meter.

By equation 5, compute the mean release height.
$(0.6354)(3\ m) + (0.2159)(2\ m) + (0.1487)(1\ m) = 2.4867\ meters$

By equation 8, compute the composite variance.
$(.6354)[(4m)^2 + (3m)^2]$
$+ (0.2159)[(2.5m)^2+(2m)^2]$
$+ (0.1487)[(1m)^2+(1m)^2]$
$- (2.4867m)^2$
$\cong 12.2117\ m^2.$

The composite standard deviation is the positive square root of the variance and is therefore about *3.49 meters*. Figure 1 is a plot of the composite distribution with its Gaussian approximation superimposed with a dashed line.

Figure 3. Plot of a Distribution and its Gaussian Approximation



To implement this approximation in EDMS 4.0, the main AREMOD input file (.INP file) must be manually edited. AERMOD input files generated by EDMS contain a key that matches the user-supplied names of sources to the eight-character names used by AERMOD. If this example is applicable to the first section of runway 9, use the key to find the AERMOD name, `RW09X001`. Each AERMOD source has a corresponding

`SRCPARAM` line for the source parameters. The `SRCPARAM` line for `RW09X001` might originally look something like this:

```
SRCPARAM RW09X001 0.0     1.83     20.00     50.00     90.00     3.00
```

The second number is the above ground release height in meters. The last number is the vertical plume size or sigma-z_0 in meters. After altering this line for this example, it should look like the following line. The values have been rounded to the nearest centimeter.

```
SRCPARAM RW09X001 0.0     2.49     20.00     50.00     90.00     3.49
```

For runways, this process is usually repeated for each runway section. Because each aircraft is accelerating and thus spends a different amount of time in each of the fifty-meter sections of a runway, and because the weighting scheme depends upon these times, the aircraft weights generally change for each section. If the weights change, the composite distribution and its Gaussian approximation also change. The release height and initial sigma of each runway section will vary.

Error Analysis

To discover the worst possible impacts to concentrations by performing this approximation, an error analysis was performed on concentrations from area sources in AERMOD using the full year of 1996 weather at Corpus Christi International Airport (CRP). Concentrations were measured with a polar receptor network centered on and completely encircling the sources with a vector every 10 degrees and distances ranging from 100 to 10,000 meters. For each weather hour and radial distance, only the peak concentration was recorded, as it would clearly be significant and would exclude most of the insignificant zero or trace receptor readings from the analysis. The receptors were placed at a flagpole height of 1.8 meters. The area sources were 10 meter by 10 meter squares.

The overall error, E, between two continuous distributions, f and g, was measured by the integral

$$E^2 = \int_{-\infty}^{\infty} (f - g)^2 \, dx \,. \qquad (11)$$

Through exhaustive search, it was determined that the greatest error between a single approximating Gaussian and the weighted sum of Gaussians it approximates occurs when there are only two Gaussians in the weighted sum. This can be explained intuitively as when more Gaussians are added to a weighted sum, the more the sum resembles a single Gaussian.

The release heights and initial vertical sigmas of the component Gaussians were limited to the real world value range of one to six meters. No widely used aircraft is known to have an effective release height greater than six meters or less than 1 meter. Preliminary information on the scale of the sigmas was unavailable at the time of the analysis and was believed to also be within the same range as the release height. The exhaustive search also revealed that the greatest error between a weighted sum of two Gaussians and its approximating Gaussian occurs when differences between the means, μ_1 and μ_2, and standard deviations, σ_1 and σ_2, of the Gaussians in the sum are maximized and a large majority of the weight is applied to the component with the smaller standard deviation. This method was used to develop test cases I and II.

Case I attempts to maximizes error with $\mu_1 = 1m, \sigma_1 = 6m, \mu_2 = 6m, \sigma_2 = 1m, w_1 = 0.167$ and $w_2 = 0.833$. The approximating Gaussian has a mean, μ, equal to $(0.167)(1m) + (0.833)(6m) = 5.165m$, and a standard deviation, σ, equal to

$$\sqrt{(0.167)[(1m)^2+(6m)^2]+(0.833)[(6m)^2+(1m)^2]-(5.165m)^2} = \sqrt{37m^2-(5.165m)^2} \cong 3.21m.$$

Case II is identical to case I except that the values of μ_1 and μ_2 were swapped. The approximating Gaussian of case II is also the same except for having a mean of $1.835m$.

One might think that the differences between the concentrations from such a weighted sum and its Gaussian approximation would be maximized, but the relationship is not linear. Intuitively, the weighted sum of Gaussians most incongruous to its approximating Gaussian should be the sum of two equally weighted Gaussians with maximally separated means and minimal standard deviations. This was also tested as case III; although, its error value from equation 11 was less than that obtained at the maxima found by exhaustion.

In case III, $\mu_1 = 1m, \sigma_1 = 1m, \mu_2 = 6m, \sigma_2 = 1m,$ and $w_1 = w_2 = 0.5$. Therefore, $\mu = 3.5m$ and $\sigma = \sqrt{(0.5)[(1m)^2+(1m)^2]+(0.5)[(6m)^2+(1m)^2]-(3.5m)^2} \cong 2.69m$ in the approximating Gaussian. Table 1 summarizes the three cases.

Table 1. Values of the statistical variables in each case.

Variable	Case I	Case II	Case III
w_1	0.167	0.167	0.5
μ_1	1 m	6 m	1 m
σ_1	6 m	6 m	1 m
w_2	0.833	0.833	0.5
μ_2	6 m	1 m	6 m
σ_2	1 m	1 m	1 m
μ	5.17 m	1.84 m	3.5 m
σ	3.21 m	3.21 m	2.69 m

Relative error between the peak concentrations from the approximating and approximated area sources was categorized into the following bins: "less than -50%" (more than a factor of 2 underprediction), "-10% to 10%" (on target prediction), "greater than 100%" (more than a factor of 2 overprediction) and bins for every interval of 10% in between. Tables 2, 3 and 4 show the bin totals out of 8,784 hours (the number of hours in a leap year) at each measured distance in the polar network for cases I, II and III, respectively.

In addition to the 1996 weather at CRP, Case III was also conducted with 1992 weather at Washington National Airport (DCA). However, this did not cause any error

Table 2. Case I error bin counts at each measured distance.

Distance (meters)	<-50 %	[-50%, -40%)	[-40%, -30%)	[-30%, -20%)	[-20%, -10%)	[-10%, 10%]	(10%, 20%]	(20%, 30%]	(30%, 40%]	(40%, 50%]	(50%, 60%]	(60%, 70%]	(70%, 80%]	(80%, 90%]	(90%, 100%]	>100 %
100	0	0	0	0	0	**7701**	684	59	170	47	1	0	1	0	3	118
200	0	0	0	0	0	**8508**	147	9	2	6	9	20	32	13	14	24
300	0	0	0	0	0	**8583**	83	11	31	31	19	12	13	1	0	0
400	0	0	0	0	0	**8617**	61	44	25	23	14	0	0	0	0	0
500	0	0	0	0	0	**8647**	64	34	36	3	0	0	0	0	0	0
600	0	0	0	0	0	**8691**	41	47	5	0	0	0	0	0	0	0
700	0	0	0	0	0	**8681**	72	31	0	0	0	0	0	0	0	0
800	0	0	0	0	0	**8667**	116	1	0	0	0	0	0	0	0	0
900	0	0	0	0	0	**8681**	103	0	0	0	0	0	0	0	0	0
1000	0	0	0	0	0	**8697**	87	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	4	**8710**	70	0	0	0	0	0	0	0	0	0
3000	0	0	0	1	1	**8659**	123	0	0	0	0	0	0	0	0	0
4000	2	0	3	0	1	**8497**	274	0	0	0	0	1	0	0	0	6
5000	3	0	5	2	7	**8420**	338	0	1	0	2	0	0	0	1	5
6000	11	0	6	0	9	**8380**	374	2	0	0	0	0	0	0	0	2
7000	1	2	11	0	2	**8197**	563	6	0	0	0	0	0	0	0	2
8000	15	1	8	1	13	**8191**	547	7	1	0	0	0	0	0	0	0
9000	9	0	5	1	3	**7871**	863	22	0	0	0	0	0	0	0	10
10000	11	2	17	3	6	**8078**	629	28	0	2	0	0	0	0	0	8

Table 3. Case II error bin counts at each measured distance.

Distance (meters)	<-50 %	[-50%, -40%)	[-40%, -30%)	[-30%, -20%)	[-20%, -10%)	[-10%, 10%]	(10%, 20%]	(20%, 30%]	(30%, 40%]	(40%, 50%]	(50%, 60%]	(60%, 70%]	(70%, 80%]	(80%, 90%]	(90%, 100%]	>100 %
100	0	0	0	0	2038	**6494**	97	29	4	0	0	0	0	0	0	122
200	0	0	0	0	467	**8120**	46	24	5	0	1	1	1	1	1	117
300	0	0	0	0	165	**8393**	38	39	18	9	4	0	0	3	4	111
400	0	0	0	0	142	**8417**	60	32	9	6	1	4	2	8	11	92
500	0	0	0	0	90	**8423**	95	35	19	8	5	8	6	20	42	33
600	0	0	0	0	67	**8433**	86	51	31	13	4	12	29	29	29	0
700	0	0	0	0	64	**8413**	109	54	29	24	25	24	25	17	0	0
800	0	0	0	0	71	**8359**	132	71	72	40	31	8	0	0	0	0
900	0	0	4	6	79	**8290**	187	96	77	45	0	0	0	0	0	0
1000	0	6	4	0	83	**8277**	211	94	84	25	0	0	0	0	0	0
2000	1	8	3	3	160	**7921**	331	185	142	30	0	0	0	0	0	0
3000	16	3	5	6	245	**7498**	539	229	230	13	0	0	0	0	0	0
4000	26	5	13	8	308	**7149**	577	424	252	22	0	0	0	0	0	0
5000	41	4	5	25	339	**6786**	691	605	249	39	0	0	0	0	0	0
6000	42	6	11	22	432	**6411**	635	903	245	77	0	0	0	0	0	0
7000	57	7	17	11	492	**6107**	711	948	342	92	0	0	0	0	0	0
8000	69	19	25	17	522	**5845**	720	1090	343	133	1	0	0	0	0	0
9000	78	12	25	22	585	**5537**	675	1283	431	133	3	0	0	0	0	0
10000	95	29	21	6	607	**5361**	646	1362	471	183	3	0	0	0	0	0

Table 4. Case III error bin counts at each measured distance.

Distance (meters)	<-50 %	[-50%, -40%)	[-40%, -30%)	[-30%, -20%)	[-20%, -10%)	[-10%, 10%]	(10%, 20%]	(20%, 30%]	(30%, 40%]	(40%, 50%]	(50%, 60%]	(60%, 70%]	(70%, 80%]	(80%, 90%]	(90%, 100%]	>100 %
100	0	0	0	0	2	**8414**	47	88	73	29	7	2	0	0	0	122
200	0	0	0	0	203	**8337**	65	41	12	4	1	1	1	1	1	117
300	0	0	0	0	196	**8341**	49	38	28	10	4	0	2	5	5	106
400	0	0	0	0	355	**8157**	80	37	29	9	5	5	6	15	51	35
500	0	0	0	0	634	**7869**	89	51	28	2	7	34	43	27	0	0
600	0	0	0	0	874	**7628**	86	64	22	18	35	49	8	0	0	0
700	0	0	0	0	1039	**7429**	114	64	52	27	49	10	0	0	0	0
800	0	0	0	0	1189	**7265**	137	75	59	47	12	0	0	0	0	0
900	0	0	0	0	1282	**7125**	194	66	91	26	0	0	0	0	0	0
1000	0	0	0	0	1391	**6994**	196	110	93	0	0	0	0	0	0	0
2000	0	0	0	10	1945	**6192**	342	218	77	0	0	0	0	0	0	0
3000	2	1	4	9	2294	**5596**	438	403	37	0	0	0	0	0	0	0
4000	11	0	0	16	2360	**5288**	588	440	81	0	0	0	0	0	0	0
5000	14	0	4	23	2238	**5048**	802	507	147	0	1	0	0	0	0	0
6000	25	0	9	19	2078	**4899**	970	641	143	0	0	0	0	0	0	0
7000	16	2	12	18	1961	**4788**	1111	699	177	0	0	0	0	0	0	0
8000	25	4	10	23	1938	**4642**	1084	830	225	0	2	0	0	1	0	0
9000	28	9	27	21	1778	**4463**	1274	903	278	3	0	0	0	0	0	0
10000	27	4	16	53	1747	**4279**	1381	1011	258	6	1	0	1	0	0	0

bin total at any receptor distance to differ by more than 456 or 5.2% of 8,784. Hence, the data presented in the following tables are likely to slightly vary for different airports.

Tables 2, 3 and 4 indicate that error tends to increase with distance. This may seem counterintuitive, because with distance the initial dispersion conditions at the source should have a smaller impact on the concentrations at the receptors. However, recall that this examines only the peak concentration at a distance, the one nearest the plume centerline. Since the receptors at greater distances have greater spatial separation, they are more apt to not catch a reading near the plume centerline and more apt to measure the less significant concentrations.

Table 5. Error bin totals as a percentage out of 8,784 for each of the three cases broken out into two distance ranges.

	Case I		Case II		Case III	
Error Bin	100-1000m	1000-10000m	100-1000m	1000-10000m	100-1000m	1000-10000m
< -50%	0.0%	0.1%	0.0%	0.5%	0.0%	0.2%
[-50%, -40%)	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%
[-40%, -30%)	0.0%	0.1%	0.0%	0.1%	0.0%	0.1%
[-30%, -20%)	0.0%	0.0%	0.0%	0.1%	0.0%	0.2%
[-20%, -10%)	0.0%	0.1%	3.7%	4.3%	8.2%	22.5%
[-10%, 10%]	**97.3%**	**95.3%**	**92.9%**	**76.2%**	**88.3%**	**59.4%**
(10%, 20%]	1.7%	4.4%	1.2%	6.5%	1.2%	9.3%
(20%, 30%]	0.3%	0.1%	0.6%	8.1%	0.7%	6.6%
(30%, 40%]	0.3%	0.0%	0.4%	3.2%	0.6%	1.7%
(40%, 50%]	0.1%	0.0%	0.2%	0.9%	0.2%	0.0%
(50%, 60%]	0.0%	0.0%	0.1%	0.0%	0.1%	0.0%
(60%, 70%]	0.0%	0.0%	0.1%	0.0%	0.1%	0.0%
(70%, 80%]	0.1%	0.0%	0.1%	0.0%	0.1%	0.0%
(80%, 90%]	0.0%	0.0%	0.1%	0.0%	0.1%	0.0%
(90%, 100%]	0.0%	0.0%	0.1%	0.0%	0.1%	0.0%
>100%	0.2%	0.0%	0.5%	0.0%	0.4%	0.0%

Table 5 shows the error bin counts as a percentage out of the 8,784 weather hours in all three cases, averaged over two source-receptor distance ranges, 100 to 1,000 meters and 1,000 to 10,000 meters.

The error for case I is minimal. At distances of 1000 meters and less, 97.3% of the concentrations from the approximation were within 10% of the concentrations from the approximated sources. This value dips to 95.3% for measurements at 1000 meters and greater.

In case II, this statistic falls to 92.9% for the nearer distances and drops to 76.2% for the farther distances. Therefore, the approximation worked better in case I than in II. The effective release height in case I is 3.33 meters higher than it is in case II. The higher release height apparently allows greater concentrations of pollutant to travel farther and reduce the error at greater distances.

Case III, in which the plume masses were concentrated at and evenly split between the high and low extremes, fares even worse than case II, especially at distances greater than 1000 meters where only 59.4% of the errors fell between –10 and 10%. The smaller initial sigmas in case III evidently keep the plumes sufficiently concentrated such that the more distant receptors are even less likely to measure the more significant concentrations.

In all three cases, overprediction or underprediction by a factor of two or more at any given distance was insignificant.

Conclusion

The accuracy of the concentrations produced by AERMOD is generally accepted to be within a factor of two.[1] The results presented here show that the proposed approximation method maintains this level of acceptability.

The fraction of the total airport emissions inventory is attributable to aircraft might necessitate the use of stacked sources to represent an aircraft fleet. If aircraft actually contribute little to an airport's total emissions, then the modeling of aircraft sources need not be as accurate as possible and an approximation would suffice. When measured initial sigmas from aircraft become publicly available, they may lie in a more constrictive range than what is assumed here. If so, this should reduce the greatest amount of error from any approximation.

This paper's derivation may be generalized to apply to other types of coinciding sources to further reduce AERMOD run times.

[1] Seventh Conference on Air Quality Modeling, Volume I, EPA Auditorium, 401 M Street, S.W., Washington, DC, June 28, 2000