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# Radio Market Structure and Music Diversity

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# **RADIO MARKET STRUCTURE AND MUSIC DIVERSITY**

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Executive Summary: We develop a unique measure of product diversity, and apply it to R&R playlists to examine changes in diversity following the 1996 Telecommunications Act. This represents the first attempt to measure diversity in radio markets using actual songs. We find modest changes in diversity, with a slight decrease in the diversity of songs within the same R&R format across local markets, and a slight increase in the diversity of songs within the same R&R format within each local market.

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## **Introduction**

With the wave of radio mergers following the 1996 Telecommunications Act, the radio market has captured more attention from observers and policymakers. Specifically, observers and policymakers wish to understand the consequences of consolidation in radio markets. In this piece, we employ unique data and a novel measure of product diversity to study the relationship between concentration and diversity in broadcast radio. Unlike past research, which employs formats as a proxy for diversity (Berry and Waldfogel, 2001), we employ R&R data on actual songs aired by radio stations to directly measure diversity. In addition, we employ a unique measure of product diversity, which allows us to directly estimate concentration's effect on diversity. So far, our results are tentative, and we can make no definite statement regarding the relationship between concentration and diversity.

The 1996 Act made two major regulatory changes in the radio market. First, the 1996 Act increased the amount of radio stations that a single radio owner could possess in any given locality. Under the 1996 Act, a single owner can own up to 8 radio stations in a market with 45 or more commercial radio stations, 7 radio stations in a market with 30-44 radio stations, 6 radio stations in a market with 15-29 radio stations, and five stations in markets with less than 15 radio stations. Second, the 1996 Act eliminated all caps on national ownership, replacing the old cap of 20 FM stations and 20 AM stations. We combine R&R playlists with data on ownership of radio stations and concentration in various radio markets to assess the impact of this important regulatory change on product diversity in the radio broadcasting market.

## **Literature Review**

Theoretical treatments of product diversity on media markets start with Steiner (1952). Steiner averred that a single monopolist would maximize product diversity and economic welfare in a broadcast market, because the monopolist would want to capture every single viewer, and would therefore not duplicate programming. Social welfare is therefore higher under monopoly, because more viewers receive their preferred programming.

Researchers have pointed out that Steiner's finding relies on strict assumptions about preferences. For instance, if viewers may have lesser preferred substitutes in programming (Beebe, 1977), then Steiner's finding may not hold. More importantly, Steiner ignores the issue of pricing in media markets. If viewers pay for programming, either explicitly by direct payment or implicitly by sitting through advertising (Becker and Murphy, 1993; Gabszewicz, Laussel, and Sonnac, 1999; Anderson and Coate, 2000), then Steiner's conclusions may no longer hold. As Gabszewicz, Laussel, and Sonnac (1999) and Anderson and Coate (2000) point out, if programs of the same type are perfect substitutes to viewers, then competition will now actually maximize product diversity, as competitors air different types of programming to avoid competing on price. Anderson and Coate (2000) demonstrate that Steiner's conclusions may hold only when programs of the same type are imperfect substitutes.

As theory itself does not reveal a clear relationship between concentration and diversity, researchers employ empirical methods to gauge this relationship. Notably, Berry and Waldfogel (2001) attempt to measure the effect of concentration on product diversity in radio markets between 1993 and 1997. This allows Berry and Waldfogel to

use the 1996 Act as their source of exogenous variation, thereby allaying potential concerns about the endogeneity of market structure. Berry and Waldfogel use the number of unique radio formats (using the Duncan classification system) as their proxy for product diversity. Berry and Waldfogel find that reductions in the number of owners led to increases in the number of formats, making the case that increased concentration increases product diversity in radio markets, which is consistent with Schmalensee's theory of spatial preemption (1978).

### **Our Contribution**

As Berry and Waldfogel point out, formats are merely taxonomic methods of roughly classifying various radio stations, and certain formats are more similar than others. Therefore, the change in the number of formats may not be an accurate measure of the change in product diversity. If the number of formats does not change, but the diversity of songs across formats increases by 5%, using formats as the measure of diversity would inaccurately lead a researcher to conclude that radio product diversity did not change when in fact it increased by 5%. The ideal method of measuring product diversity would use radio stations' comprehensive playlists and would measure concentration based on the relationship between the number of unique songs played and the number of total songs played.

We do not have comprehensive playlists, but we do obtain from R&R magazine the top songs played by a large sample of radio stations in March, 1996 and March, 2001. Thus, we can measure product diversity more directly, based on songs themselves, rather than format classification. Thus, we can create new measures of radio product diversity based on samples of actual songs played by radio stations. In doing so, we do forgo one

advantage of Berry and Waldfogel's study. For the most part, our data is drawn from radio stations in the top tier markets. Therefore, our sample's variation across markets is not as driven by differences in FCC rules, which raises the possibility of market structure endogeneity. In addition, our sample draws from a possibly non-representative sample within these large markets. R&R only lists stations that meet a minimum rating requirement, meaning that stations with very few listeners are not listed by R&R. In addition, R&R only lists stations that play new songs, so stations that specialize in older songs, like Oldies stations or Classical stations, are not included in our sample of R&R playlists.

Our approach allows us to create and exploit a new measure of product diversity; the distance measure. As one may imagine, measuring diversity in a market is a daunting task. However, as Alexander (1997) points out, if we can break down products into a bundle of characteristics and simply treat the existence or non-existence of each characteristic as a binary variable, then we can create a measure of product diversity. In this case, songs are individual product characteristics, and we can create a measure of diversity between any two stations by comparing the songs played on each station. Below, we detail the creation and application of the distance measure.

### **The Distance Measure of Diversity**

We compare the similarity of  $m$  different product characteristics (in this case songs) across  $n$  different producers (in this case radio stations). In this case, we compare 10 songs across 2 stations. So,  $m = 10$  and  $n = 2$ . Under this condition, we can define a distance function that embodies a measure of diversity. This distance function will

conveniently equal the number of unique songs played by each station. Therefore, when comparing two stations and the diversity of their top ten songs, if there is no duplication of songs, the distance function = 10, if the two stations duplicate one song, the distance function = 9, etc. Below, we demonstrate the creation of the distance function:

In the present case, we are interested in a subset of a set of many characteristics. Let this finite vector of dimensionality (the number of characteristics) be equal to  $m$ , and let  $n$  be the number of objects in the system. We now define the following.

First, two objects,  $X(x_1, x_2, \dots, x_m)$  and  $X'(x_1', x_2', \dots, x_m')$  are *identical* to each other provided all  $x_n = x_n'$ , where  $0 < n \leq m$ . Second, an object  $X(x_1, x_2, \dots, x_m)$  is *unique* if there is no other object  $X'(x_1', x_2', \dots, x_m')$  such that all  $x_n = x_n'$ , where  $0 < n \leq m$ . Third, two objects are *divergent* from each other provided  $x_n \neq x_n'$  for all  $0 < n \leq m$ . Fourth, two objects  $X(x_1, x_2, \dots, x_m)$  and  $X'(x_1', x_2', \dots, x_m')$  are *analogues* provided some  $x_n \neq x_n'$  and some  $x_j = x_j'$ , where  $0 < n, j \leq m$ . Finally, two objects,  $X(x_1, x_2, \dots, x_m)$  and  $X'(x_1', x_2', \dots, x_m')$  have the  $n^{\text{th}}$  *degree of analogy* to each other, provided there are exactly  $n$  characteristics such that  $x_n = x_n'$ . The greater (*lesser*) the degree of analogy, the more similar (*dissimilar*) the objects.

**Proposition 1:** Suppose that object  $A$  has the  $i^{\text{th}}$  degree of analogy with object  $B$  and the  $j^{\text{th}}$  degree of analogy with object  $C$ , where  $0 < i, j \leq m$ . Then, the degree of analogy  $k$  between  $B$  and  $C$  is such that  $0 \leq k \leq m - |i - j|$ . **Proof in technical appendix.**

We extend our measure to include binary systems. In this system, each characteristic takes the value of 0 or 1, indicating the absence or presence of a characteristic. We define the degree of divergence between objects  $X_i$  and  $X_j$  as:

$$d_{ij} = \sum_{k=1}^m |x_{ik} - x_{jk}|$$

The degree of analogy can be computed as:

$$a_{ij} = m - d_{ij}$$

allowing us to calculate the nominal analogous diversity of the system as:

$$nd = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m |x_{ik} - x_{jk}| / 2 = \sum_{i=1}^n \sum_{j=1}^n d_{ij} / 2.$$

The higher the nominal analogous diversity ( $nd$ ) of the system, the more diverse is the system.

**Proposition 2:** Minimum (nominal analogous) diversity,  $nd_{\min}$ , equals zero.

Maximum (nominal analogous) diversity,  $nd_{\max}$ , equals  $\frac{1}{4}m(n^2)$  when  $n$  is even, and  $\frac{1}{4}m(n^2 - 1)$  when  $n$  is odd. **Proof in technical appendix.**

Therefore, when we compare playlists of  $m$  songs between  $n = 2$  stations, maximal diversity equals  $\frac{1}{4}m(n^2) = \frac{1}{4}(m)(4) = m$ , so that maximum diversity obtains when we have two completely unique playlists, and our distance measure is equal to the number of songs played. Note that these playlists must be of equal size.



## **Data and Methodology**

We combine data on station playlists and station ownership from R&R magazine with BIA data on station-level market share and market-level concentration. Using the partial playlists displayed in R & R for March 15, 1996 and March 16, 2001, we count the total number of different songs within each format. Table 1 presents data on the number of unique songs in our sample by R&R format. In six out of twelve formats (AC, Active Rock, Adult Alternative, CHR Pop, Jazz, and Rock) the number of unique songs has remained the same or declined between these two time periods. The number of different songs played in the other six other formats (Alternative, CHR Rhythm, Country, Hot AC, Urban AC, and Urban) has increased. There is, of course, some overlap between these formats; some songs are common to a number of formats, especially similar formats, such as Rock and Active Rock. On the bottom row of Table 1, we present the total number of unique songs across all formats published in our selected R & R magazines: 1241 for 1996 and 1228 for 2001.

**Table 1: Unique Number of Songs by Formats**

Our methodology raises some issues concerning our sample. First, R&R playlists draw mainly from the largest markets. Therefore, our sample draws mainly from the top markets and does not include data from smaller markets. However, the markets from which our data draw include over 60% of all listeners. First, we assess song diversity across every R&R station-pairs in March 1996 and in March 2001. For each station pair in 1996 and 2001, we calculate the distance function, which measures the number of unique plays between any two stations. For instance, if two radio stations each play 10 songs and none of those songs are the same, then our distance function would be 10; if

<b>Format</b>	<b>Number of Stations</b>		<b>Number of Unique Songs</b>	
	<b>1996</b>	<b>2001</b>	<b>1996</b>	<b>2001</b>
<b>AC</b>	18	20	93	93
<b>Active Rock</b>	20	19	152	120
<b>Adult Alternative</b>	20	20	221	183
<b>Alternative</b>	40	39	207	221
<b>CHR Pop</b>	45	45	255	199
<b>CHR Rhy</b>	15	15	179	187
<b>Country</b>	45	30	138	166
<b>Hot AC</b>	20	20	110	140
<b>Jazz</b>	20	20	191	143
<b>Rock</b>	20	20	152	149
<b>Urban AC</b>	11	12	82	125
<b>Urban</b>	28	28	153	160
<b>All Formats</b>	302	288	1241	1228

those two stations have one song in common, then the distance function would be 9, etc. In our case, we employ only the top ten songs played by each radio station. We do this because the entropy measure requires equality in the number of songs we compare between radio stations, and radio stations have playlists of varying sizes. All radio stations, however, have playlists with at least ten songs.

Given that we have 302 stations in our sample in 1996, we generate  $\sum_{i=1}^{301} i = 45,451$  station-pair distance observations. In 2001, we have 288 stations, so we generate 41,328 station-pair distance observations.

We use the distance function to create some simple descriptive statistics. Table 2 shows the average distance between top ten lists among stations *within the same R&R format*.

**Table 2**  
**Average Distance Within Formats**

<b>Format</b>	<b>1996</b>	<b>2001</b>
Adult Contemporary	6.51	5.53
Active Rock	6.84	6.02
Adult Alternative	7.65	7.06
Alternative	7.44	6.68
CHR Pop	7.09	6.00
CHR Rhythm	7.66	6.79
Country	4.68	5.94
Hot Adult Contemporary	7.09	6.49
Jazz	8.05	6.39
Rock	7.25	6.94
Urban Adult Contemporary	6.75	7.21
Urban	5.33	6.68

The first row in Table 2 shows that, on average, Adult Contemporary top 10 radio lists differ by 6.51 songs in March 15, 1996 and 5.53 times in March 16, 2001. Overall, Table 2 shows that the average difference between radio station top ten lists declined in our time period for nine out of twelve formats; these include Adult Contemporary, Active Rock, Adult Alternative, Alternative, CHR Pop, CHR Rhythm, Hot Adult Contemporary, Jazz, and Rock. Average diversity for top 10 songs increased in the remaining formats: Country, Urban AC, and Urban. These changes are statistically significant with 99% confidence except for Rock and Urban; these are statistically significant with 95% confidence. The overall changes for within format song diversity are more modest; we find that the average distance across all pairs within the same format in March 1996 is 6.51, which declines approximately 2.4% to 6.35 in March 2001.<sup>2</sup>

We should note that of the three categories that showed an increase in song diversity between 1996 and 2001, two of them, Country and Urban, had the two smallest averages of distance for 1996. Thus stations in Country and Urban had relatively more uniform playlists among formats in the earlier time period for our sample. Their shift toward more diversity appears to place them closer to the others in terms of diversity, although Country, with a distance measure of 5.94, still appears to be slightly less diverse than most of the other formats. One additional point here is that the relatively large number of Country stations in our sample (and in the populations of stations) is an important factor why the overall decline in distance appears more modest than for the nine individual cases where distance declines.

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<sup>2</sup> This change is statistically significant with 99% confidence.

As we noted above, we also compare songlists between radio stations in similar formats, such as Adult Contemporary and Hot Adult Contemporary. In many cases, however, radio stations in different formats have no top 10 songs in common. For instance, there will be no top ten Jazz song played on any Country station. Thus, for many format pairs, our distance measure yields 10 (meaning no top 10 songs in common). For our purposes at the moment, we focus only on relatively similar formats and discard format pairs that are highly dissimilar.<sup>3</sup>

**Table 3**  
**Average Distance Across Selected Formats**

Formats		March 1996	March 2001
Adult Contemporary	Hot Adult Contemporary	8.21	9.06
Active Rock	Alternative	7.76	7.40
Active Rock	Rock	8.05	7.30
Adult Alternative	Alternative	8.80	9.37
Adult Alternative	CHR Pop	9.45	9.47
Adult Alternative	Hot Adult Contemporary	9.54	8.96
Alternative	CHR Pop	9.35	9.44
Alternative	Rock	8.23	8.33
CHR Pop	CHR Rhythm	8.84	8.49
CHR Pop	Hot Adult Contemporary	7.83	8.11
CHR Rhythm	Urban AC	8.21	9.41
CHR Rhythm	Urban	7.40	7.85
Urban AC	Urban	6.76	8.61

<sup>3</sup> We therefore ignore for the moment those format pairs whose average distance is equal or higher than 9.5.

Table 3 presents the results of comparing radio stations across similar formats. Seven of the thirteen cases displayed show statistically significant increases in the average difference (more diversity) between top ten playlists. These format pairs include: Adult Contemporary / Hot Adult Contemporary, Adult Alternative / Alternative, Alternative / CHR Pop, Alternative / Rock, CHR Pop / Hot Adult Contemporary, CHR Rhythm / Urban AC, CHR Rhythm / Urban, and Urban AC / Urban. Two cases, Adult Alternative / CHR Pop and Alternative / Rock, show no statistically significant change. The remaining four format pairs show statistically significant reductions in song diversity, suggesting that playlists in Active Rock / Alternative, Active Rock / Rock, Adult Alternative / Hot Adult Contemporary, and CHR Pop / CHR Rhythm are sharing more songs in common now than in 1996.

In general, the differences between these averages for similar but different format pairs show changes of less magnitude than those displayed in Table 2. Of course, we remind the reader that we've excluded all format pairs from our analysis here that have average differences greater than 9.5. Together, however, Tables 2 and 3 suggest that diversity may have declined between stations of the same format, while rising somewhat for stations across different formats. When we compute the average measure of diversity for our entire sample, we find that the overall diversity has increased very slightly, from 9.26 to 9.32, only 0.74%.<sup>4</sup> Song diversity looks very stable overall, with some significant changes occurring within formats and across some format pairs.

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<sup>4</sup> Here we include all stations pairs, even those with categories that have no overlapping songs. This has a strong effect of raising the average and lowering the difference between the time periods.

We use the distance function in a panel regression to determine the causes of these changes in diversity. In particular, we examine the role that market structure may have played, with particular focus on the consolidation of ownership that resulted from changes in ownership rules under the Telecom Act in 1996. In addition, we study the extent to which diversity reductions within formats reduce the alternatives available to listeners.

To answer these questions we estimate a linear model of the change in distance between radio pairs using our R&R data. To do this, we select only those portions of our data where each radio station appears in both our selected time periods, giving us 174 stations. We then compute the percent change of distance across our time period for all possible pairs of these 174 stations.

We regress the percent change in distance (between top ten playlists of station pairs) on a set of variables indicating the status of format, market, and ownership between station pairs.<sup>5</sup> Our set of variables includes the following: an indication of whether or not both stations are within the same format (Same format); whether both stations broadcast within the same market or city (Same city); whether both stations share the same owner (Same owner); whether both stations share the same format and broadcast in the same city (Same format and city); whether both stations share the same format and owner (Same format and owner); whether both stations share same owner and broadcast in the same market (Same owner and city); whether either station has changed ownership over the time period (Change in ownership); whether there has been a change in ownership for

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<sup>5</sup> We set each variable to 1 or 0 to indicate whether or not a particular condition is satisfied. For example, the variable “Same format” is set to 1 if both stations in a pair have the same R&R format; otherwise it is set to 0.

either station within a pair of stations sharing the same format (Change in ownership for pair in same format); whether two stations switch from separate to common ownership (Change from different to common ownership); and, finally, whether two stations within the same format switch from separate to common ownership (Change from different to common ownership within the same format). Table 4 summarizes our results.

**Table 4**  
**OLS Regression on Percent Change in Distance Between Radio Playlists, 1996-2001**



We find that the variable indicating a station pair within the same format is

<b>Coefficients</b>	<b>Value</b>	<b>t Value</b>
Intercept	2.12**	4.499
Same format	-4.16**	-2.905
Same city	0.84	0.713
Same owner	0.33	0.368
Same format and city	11.48*	1.931
Same format and owner	-4.46	-1.607
Same owner and city	0.43	0.155
Change in ownership	-1.04*	0.043
Change in ownership for pair in same format	11.46**	7.195
Change from different to common ownership	-0.18	-0.175
Change from different to common ownership within the same format	3.38	1.034
Degrees of Freedom		14179
R-Squared		0.009

significance with 99% confidence. The value on the coefficient suggests that song diversity between two top ten lists has declined between March 1996 and March 2001 by approximately 4%. However this does not appear to suggest that listeners in a given market have less choice when switching among stations of similar formats. The coefficient here suggests that diversity has grown significantly among stations within the same format and within the same city. Thus stations within the same R&R formats competing within the same market appear to differentiate themselves to appeal to their listeners.

The role that concentration plays appears to be less clear. The variables we have indicating concentration or changes in ownership have different signs, muddying the overall story. The variable indicating the same owner between two station within the same format has a negative sign and is significant with 90% confidence.<sup>6</sup> Further, a variable indicating a change in ownership (but not necessarily an increase in concentration) leads to a slight, but statistically significant reduction in format diversity. This variable is for change in ownership for either of the two stations in a pair, regardless of whether or not they belong to the same city or format. However, our variable indicating ownership change for station pairs within the same format is strongly significant and positive, suggesting increases in diversity. Neither variable directly related to consolidation (change from different to common ownership) is statistically significant.

## **Conclusion**

We investigate the change in diversity among playlists within the radio industry since the passage of the Telecom Act in 1996. For this purpose we introduce a straightforward measure of diversity. Overall, we found that song diversity, whether measured as the number of unique songs or the difference between top ten playlists has remained stable between March 1996 and March 2001. While playlists for stations within the same format have grown slightly more uniform across local markets, playlists for same format stations competing in the same local market have diverged, so that

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<sup>6</sup> Thus it is not quite as statistically significant as the other variables we have discussed.

listeners in local radio markets may have experienced increasing song diversity even absent an increase in R&R radio formats.

We have not yet concluded how concentration in ownership affects playlist diversity. We speculate that concentration may be partly responsible for more uniformity for stations within formats and more diversity between formats. Changes in diversity may also stem from exogenous changes in market conditions that affect all owners simultaneously. Further, changes in ownership among stations within a format have led to increases in playlist diversity. Our results at this time suggest that recent consolidation has played very little role in playlist diversity, although this might not be the case in smaller markets, which are not represented in our sample.

## **Technical Appendix**

**Proof of Proposition One:** While  $0 \leq k$  is obvious, we must prove

$k \leq m - |i - j|$ . First, assume that  $i = j$ . Clearly,  $k \leq m = m - |i - j|$ . Now, let  $i \neq j$ .

Without loss of generality, we assume that  $i > j$ , and consider  $i$  characteristics such that

$a_n = b_n$ . Note that there are at most  $j$  number of characteristics such that  $a_n = b_n = c_n$ ,

and there are at most  $m - i$  characteristics such that  $a_n \neq b_n = c_n$ . Thus,

$$k \leq m - |i - j| = j + (m - i).$$

*Q.E.D.*

**Proof of Proposition Two:** When  $n = 1$ ,  $nd_{\min} = nd_{\max} = 0$ . Now, assume that

$n > 1$ . We consider two cases,  $m = 1$  and  $m > 1$ .

**Case 1:**  $m = 1$ . We have  $n$  cases  $X_i(x_i)$  which can take on a value of 0 or 1 and

$k$  objects such that  $x_i = 1$  and  $n - k$  objects such that  $x_i = 0$ .

**Lemma 1:** When the dimensionality  $m = 1$ , the nominal (analogous degree of)

diversity in the binary system is given by  $nd = k(n - k)$ , where  $k$  is the number of

objects that have the same characteristic values.

**Proof of Lemma 1:** Let  $k$  be the number of objects with characteristic values

$x_1 = x_2 = \dots = x_k = 1$ . Note that we originally designated these objects as  $X_1, X_2, \dots, X_k$ .

Thus, there are  $n - k$  number of objects  $X_{k+1}, \dots, X_n$  such that  $x_{k+1}, \dots, x_n = 0$ . Notice that

$$\sum_{j=1}^n |x_1 - x_j| = \sum_{j=1}^n |x_k - x_j| \text{ and } \sum_{j=1}^n |x_{k+1} - x_j| = \sum_{j=1}^n |x_n - x_j|. \text{ Therefore,}$$

$$nd = \sum_{i=1}^n \sum_{j=1}^n |x_i - x_j| / 2 = k(n - k). \text{ Q.E.D.}$$

As we stated in Lemma 1,  $nd = k(n - k)$ , where  $n$  is given, and  $0 \leq k \leq n$ . Noting that  $\partial nd / \partial k = n - 2k$ , the function is maximized when  $k = \frac{1}{2}n$ . Note that when  $n$  is even,  $k$  is an integer, and  $nd_{\max} = k(n - k) = \frac{1}{2}n(n - \frac{1}{2}n) = \frac{1}{4}n^2$ . If  $n$  is odd, the closet possible  $k$  to  $\frac{1}{2}n$  is  $\frac{1}{2}(n - 1)$  or  $\frac{1}{2}(n + 1)$ . In either case,  $nd = \frac{1}{2}(n + 1)\frac{1}{2}(n - 1) = \frac{1}{4}(n^2 - 1)$ . This represents the maximum (nominal) diversity of the system when  $n$  is odd. However, if  $k$  is not  $n + 1$  or  $n - 1$ , then  $k = \pm q$  where  $q$  is a natural number greater than 1, and  $nd = \frac{1}{4}(n^2 - q^2) < \frac{1}{4}(n^2 - 1)$ . Clearly,  $nd_{\min} = 0$  when  $k = n$  or  $k = 0$ .

**Case 2:**  $m > 1$ . Now we consider the case where  $m > 1$ . We begin by noting that

$$nd = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^m |x_{ik} - x_{jk}| / 2 = \sum_{i=1}^n \sum_{j=1}^n |x_{i1} - x_{j1}| / 2 + K + \sum_{i=1}^n \sum_{j=1}^n |x_{im} - x_{jm}| / 2. \text{ It is then}$$

immediately clear that  $nd \leq \frac{1}{4}n^2 + K + \frac{1}{4}n^2$  when  $n$  is even;

$nd \leq \frac{1}{4}(n^2 - 1) + K + \frac{1}{4}(n^2 - 1)$  when  $n$  is odd; and  $nd = m(\frac{1}{4}n^2)$  when  $n$  is even and

$nd = m(\frac{1}{4}(n^2 - 1))$  when  $n$  is odd.

*Q.E.D.*

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