

# QCD effects in B-decays: Lecture 2

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# Outline of these lectures

## 1. Heavy quark physics

- Heavy-quark spin and flavor symmetry
  - Spectroscopic implications
- Heavy Quark Effective Theory
  - $V_{cb}$  from exclusive semileptonic decay

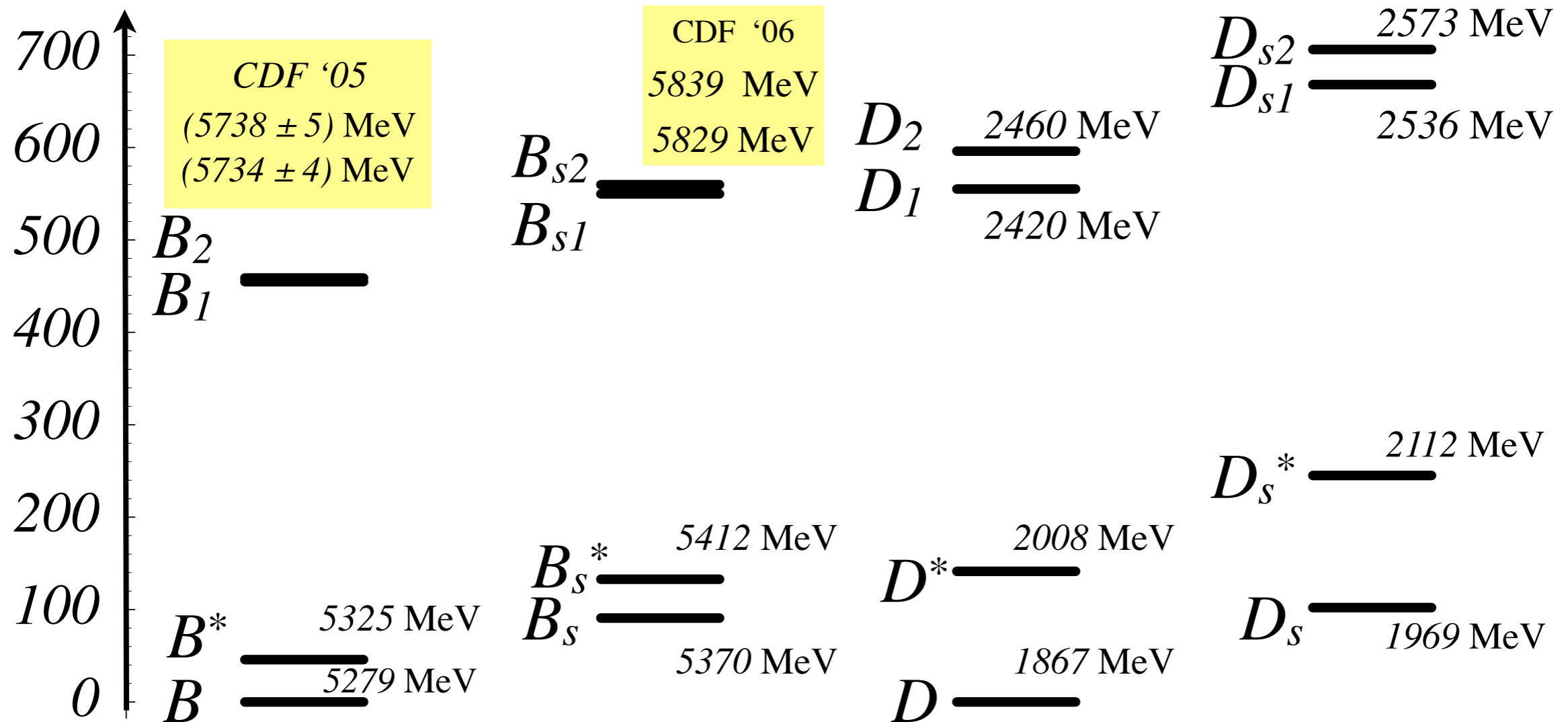
## 2. Inclusive B-decays

- Operator Product Expansion
- Determination of  $V_{ub}$ ,  $V_{cb}$  from semileptonic decays
- Radiative decays: test of FCNC interactions
- Heavy hadron lifetimes

## 3. Exclusive radiative and hadronic B-decays

- Factorization, Soft Collinear Effective Theory

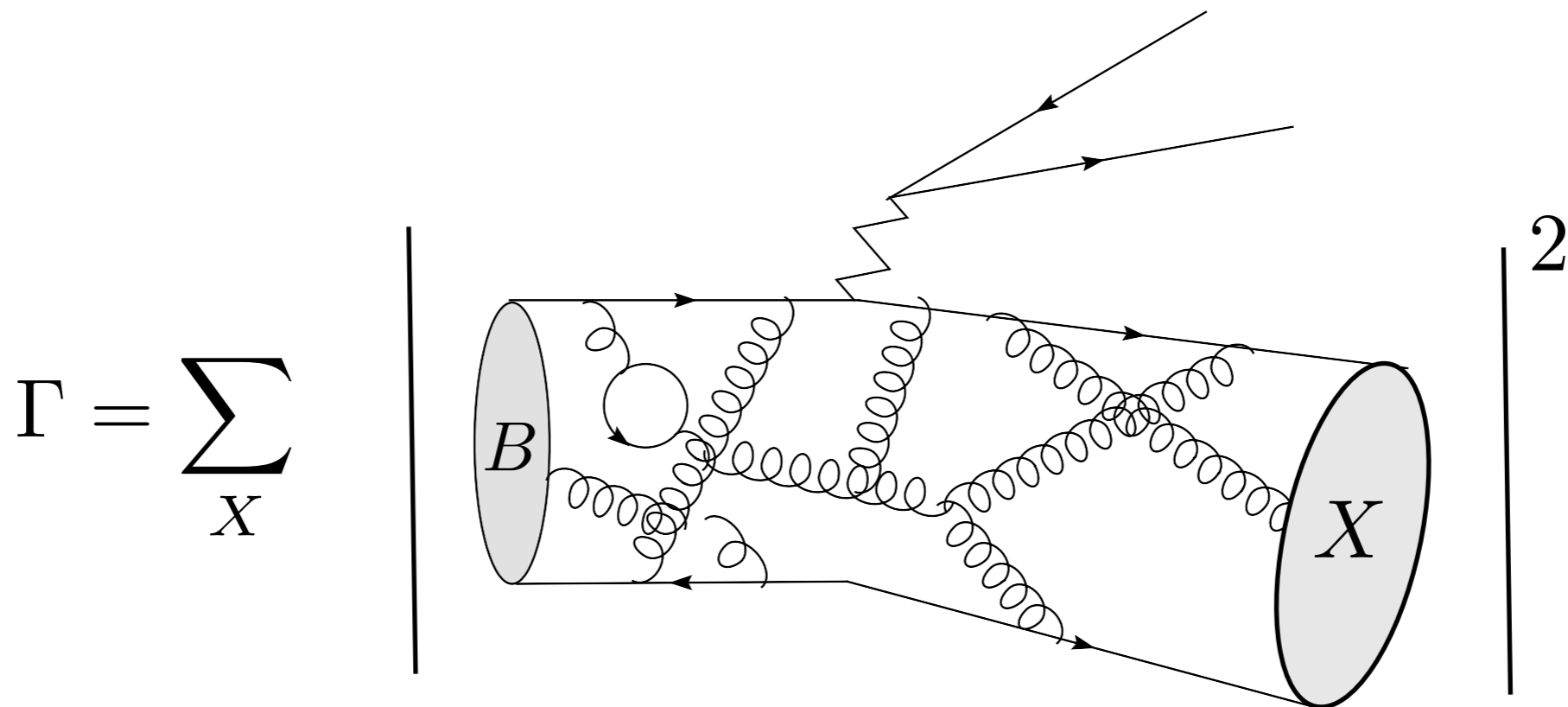
# Heavy-light meson spectrum



- To leading power, bottom and charm spectra are simply shifted by constant amount  $m_b - m_c = 3.4 \text{ GeV}$ .
  - $M_{B_1} - M_B = (455 \pm 4) \text{ MeV}$ ,  $M_{D_1} - M_D = (555 \pm 1) \text{ MeV}$
- “Spin doublets” almost degenerate:
  - e.g.  $M_{B^*} - M_B = 46 \text{ MeV}$

# Operator Product Expansion and Inclusive Weak Decays

# Inclusive decays



- Inclusive decays rates are much less sensitive to hadronization effects than exclusive decays.
- Scale hierarchy

$$\frac{1}{m_b} \ll \frac{1}{\Lambda_{QCD}}$$

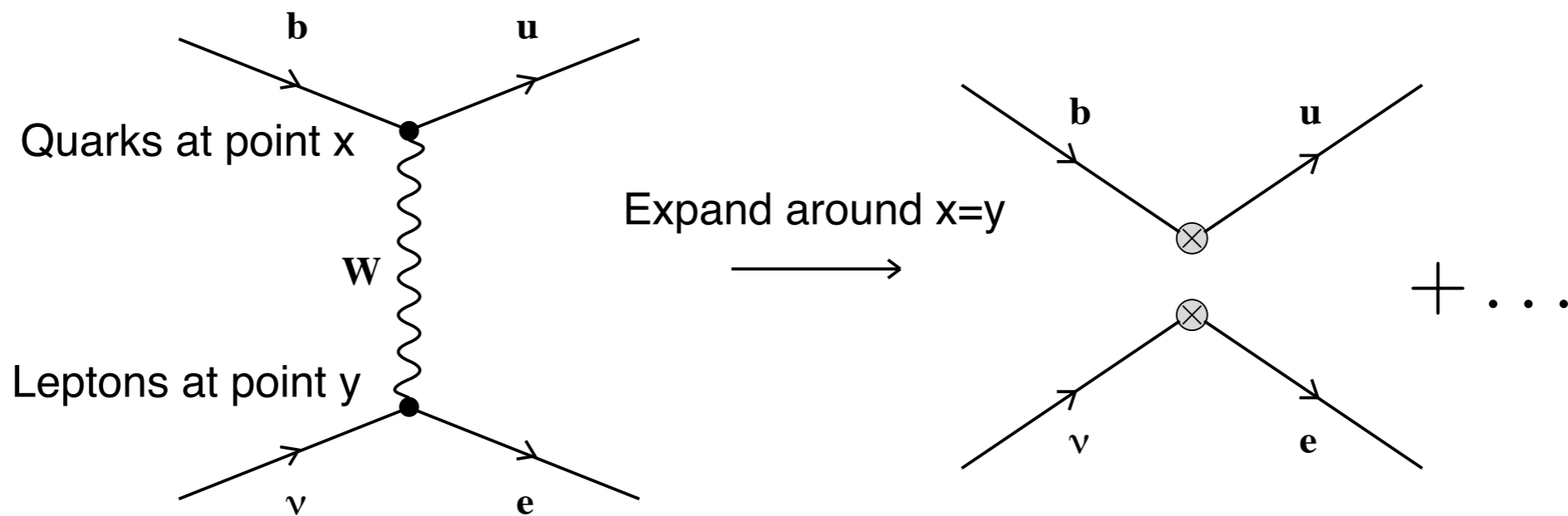
b-quark decay                      hadronic effects

# Inclusive $b$ -decays

- Important class of decays, since rate can be calculated perturbatively  $m_Q \rightarrow \infty$
- Semileptonic decay  $B \rightarrow X_c l \nu$ 
  - Most precise determination of  $|V_{cb}|$ ,  $m_b$  and  $m_c$ .
- Semileptonic decay  $B \rightarrow X_u l \nu$ 
  - Most precise determination of  $|V_{ub}|$
- Radiative decays  $B \rightarrow X_s \gamma$ ,  $B \rightarrow X_s l^+ l^-$ 
  - Sensitive probe of FCNC interactions
- Lifetime:  $B \rightarrow X$

# Operator Product Expansion

- Used this tool before, when integrating out heavy particles in the construction of the effective weak Hamiltonian.



- At low energies  $W$  is highly virtual. Propagates only very short distance. Expand around  $x=y$ .
- In momentum space this translates into expansion of  $W$ -propagator

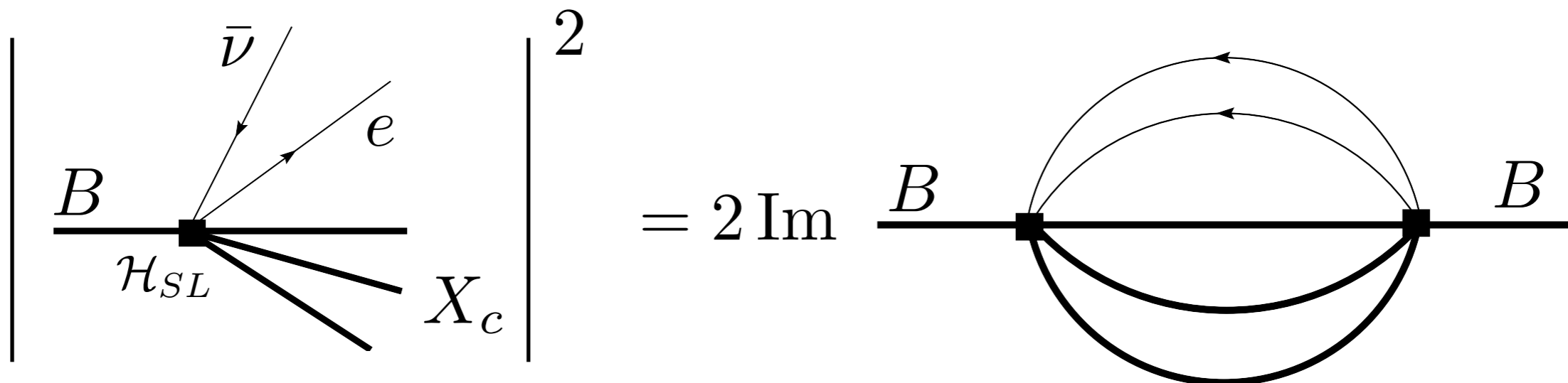
$$\frac{1}{p^2 - m_W^2} = \frac{1}{-m_W^2} + \dots$$

# Optical theorem

- To apply the same technique to the inclusive B-decay, first use the optical theorem

$$\begin{aligned} \Gamma(B \rightarrow X_c e \bar{\nu}) &= \frac{1}{2M_B} \sum_X (2\pi)^4 \delta^4(p_B - p_{X_c} - p_e - p_\nu) |\langle X_c e \bar{\nu} | \mathcal{H}_{SL} | B \rangle|^2 \\ &= \frac{1}{2M_B} 2 \operatorname{Im} \langle B | i \int d^4x T \left\{ \mathcal{H}_{SL}^\dagger(x), \mathcal{H}_{SL}(0) \right\} | B \rangle \end{aligned}$$

forward-scattering amplitude





# Operator product expansion

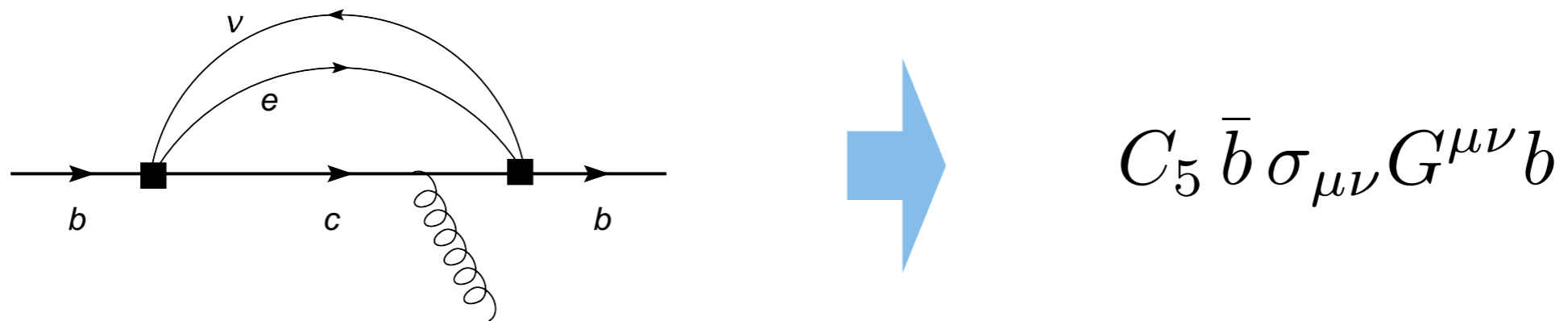
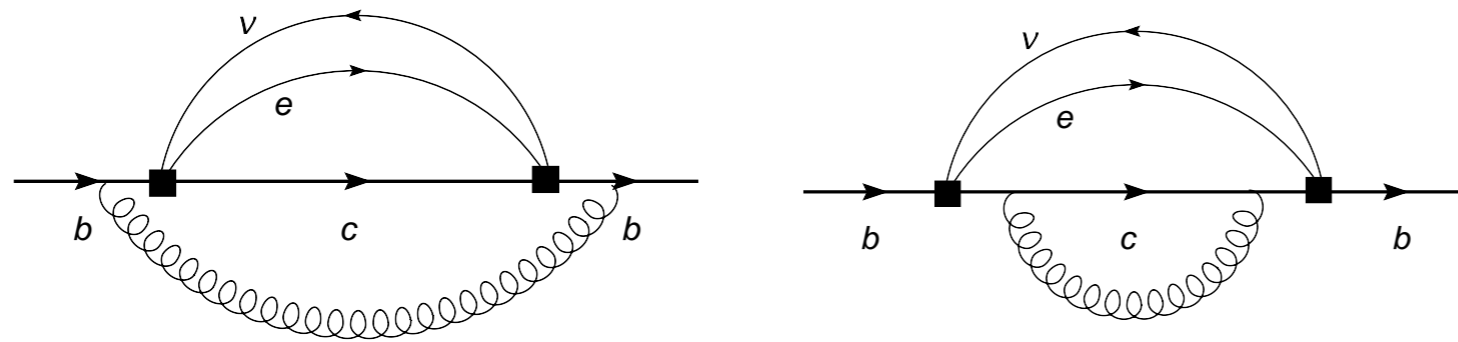
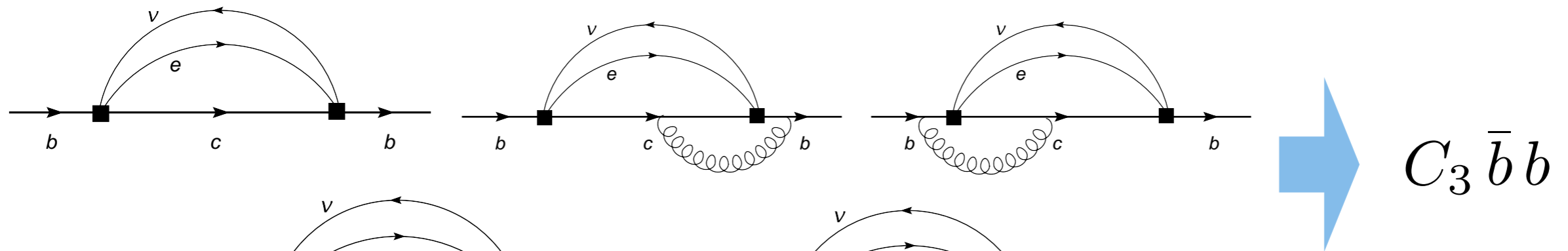
- Expand the product of operators into local ones

$$i \int d^4x T \left\{ \mathcal{H}_{\text{SL}}^\dagger(x), \mathcal{H}_{\text{SL}}(0) \right\} = C_3 \bar{b} b + \frac{C_5}{m_b^2} \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b + \dots$$

No dim. 4 operators:  $\bar{b} i \not{D} b = m_b \bar{b} b$

- To evaluate the coefficients  $C_3$  and  $C_5$ , we can use arbitrary external states
  - Use quark and gluon states and calculate the coefficients in perturbation theory!
- Then use HQET to evaluate the B-meson matrix elements of the operators  $\bar{b} b$  and  $\bar{b} \sigma_{\mu\nu} G^{\mu\nu} b$ 
  - Will be given by HQET parameters  $\lambda_1, \lambda_2$ , etc.
- Q: Is the expansion well behaved? Are the higher order terms really suppressed by  $1/m_b^2$ ?

# Feynman Diagrams



# Matrix elements

- To calculate the matrix elements, we use HQET.

$$\frac{1}{2M_B} \langle B | \bar{b} b | B \rangle = 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + \dots$$

No  $1/m_b$   
corrections!

$$\frac{1}{m_b^2} \frac{1}{2M_B} \langle B | \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle = \frac{6\lambda_2}{m_b^2} + \dots$$

$$\lambda_2 = \frac{1}{4} (M_{B^*}^2 - M_B^2) = 0.12 \text{GeV}^2$$

# Result for the rate

$$\Gamma(B \rightarrow X_c e \bar{\nu}) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left\{ \left( 1 + \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \right) \left[ f(\rho) + \frac{\alpha_s}{\pi} g(\rho) \right] - \frac{6\lambda_2}{m_b^2} (1 - \rho)^4 + \dots \right\}$$

$$f(\rho) = 1 - 8\rho - 12\rho^2 \log \rho + 8\rho^3 - \rho^4, \quad \rho = \frac{m_c^2}{m_b^2}$$

$g(\rho)$  = "lengthy, known expression"

- Leading term in limit  $m_Q \rightarrow \infty$  is the free  $b$ -quark decay ("naive parton model")!
- Hadronization effects are suppressed as  $1/m_b^2$ . Reduce the rate by  $\approx 4\%$
- Values of  $m_b$ ,  $m_c$  and  $\lambda_1$ ?
- Strictly speaking expansion is  $1/(m_b - m_c) \approx 1/m_b$

# Side remark: $b$ -quark mass

- We calculated in terms of the  $b$ -quark pole mass, i.e. the location of the pole in the heavy quark propagator.
  - Well defined in perturbation theory, but
  - does not make sense non-perturbatively because of confinement.
- The lack of a non-perturbative definition shows up via large higher-order perturbative corrections.
  - Upon relating the pole to the  $\overline{\text{MS}}$  quark mass, one finds a badly divergent PT series.
  - Can resum this series, but prescription is not unique (“renormalon ambiguity”).

# $b$ -quark mass definition

- Bad perturbative behavior also shows up in the decay rate, if it is expressed in the pole mass.
- Eliminate pole mass in favor quark mass! Many mass schemes in the literature:
  - $\overline{\text{MS}}$  mass (not suited for HQET)
  - Kinetic mass (Uraltsev)
  - $\overline{\text{Y(1S)}}$  mass (Hoang, Ligeti, Manohar)
  - Potential subtracted (Beneke)
  - Shape-function (Bosch, Lange, Neubert, Paz)
- Also, better definition for  $\lambda_1, \lambda_2$  are available in the kinetic and shape-function scheme.
  - The corresponding parameters are denoted by  $\mu_\pi^2 (\equiv -\lambda_1)$  and  $\mu_G^2 (\equiv \lambda_2)$

# Moments

- Calculate moments of the decay spectrum (with exp. cuts).

- Leptonic moments

$$L_n = \frac{1}{\Gamma} \int dE_e (E_e)^n \frac{d\Gamma}{dE_e}$$

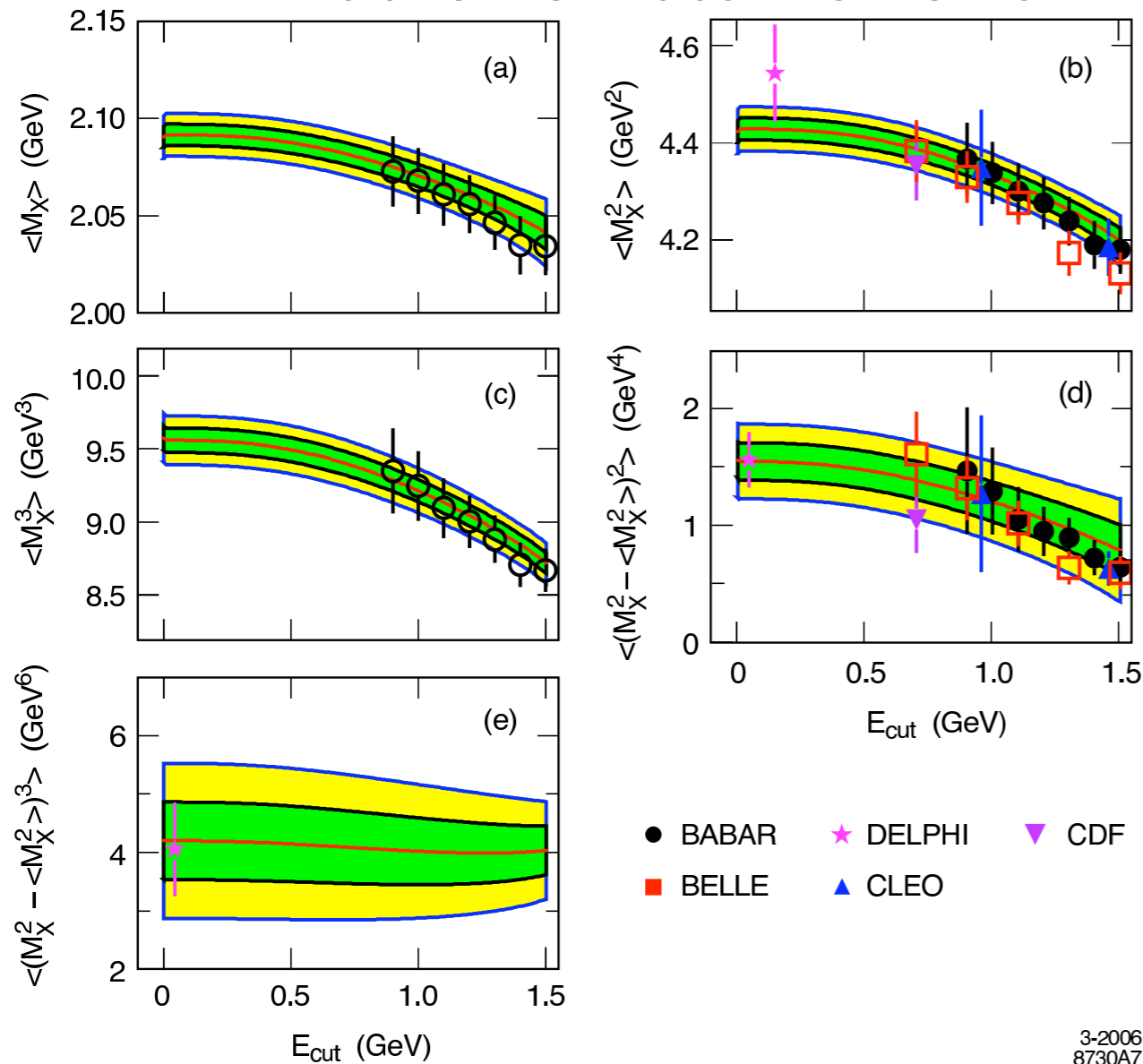
- Hadronic moments

$$H_{ij} = \frac{1}{\Gamma} \int dM_X^2 dE_X (M_X^2)^i (E_X)^j \frac{d\Gamma}{dM_X^2 dE_X}$$

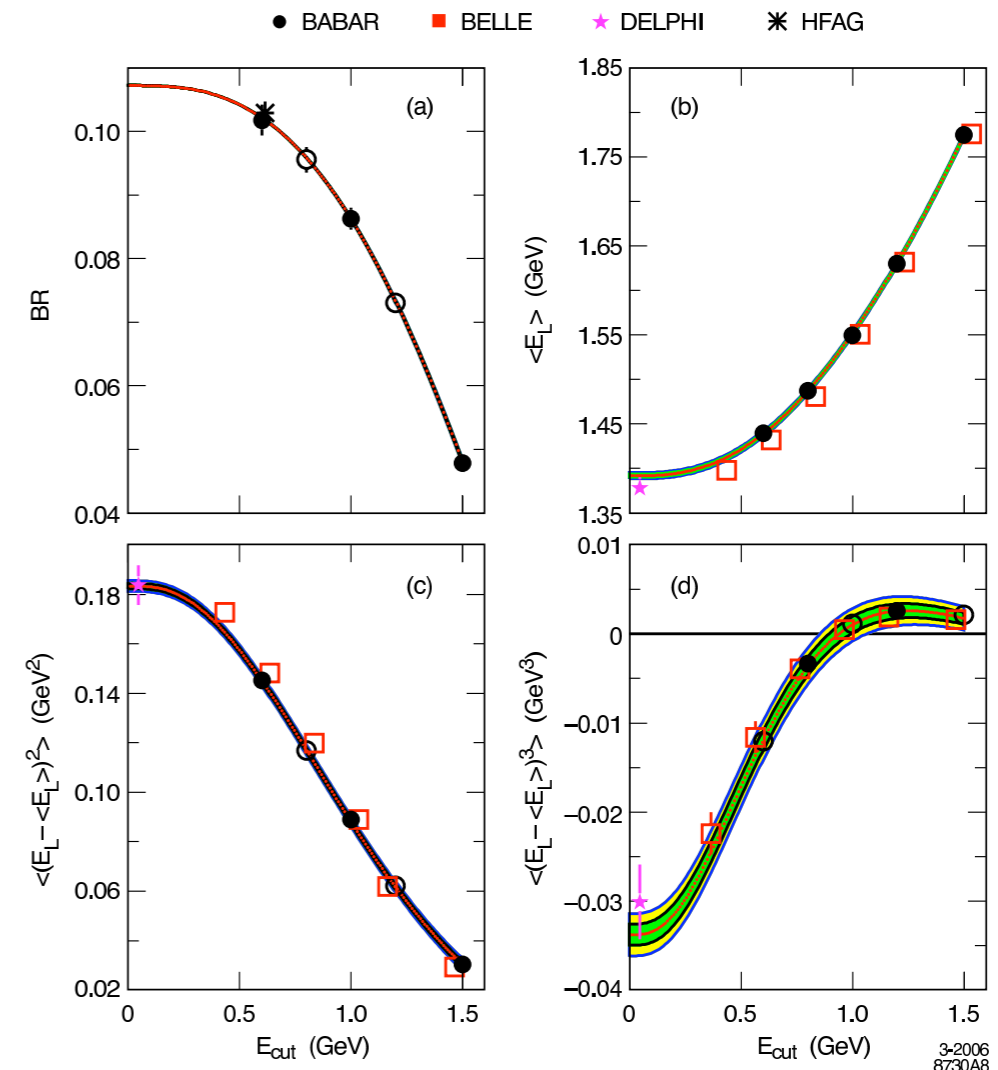
- Measurement of the moments and the rate determines  $V_{cb}$ ,  $m_b$ ,  $m_c$  and  $\lambda_1$ .

# Moment measurements and global fit

## hadronic mass moments



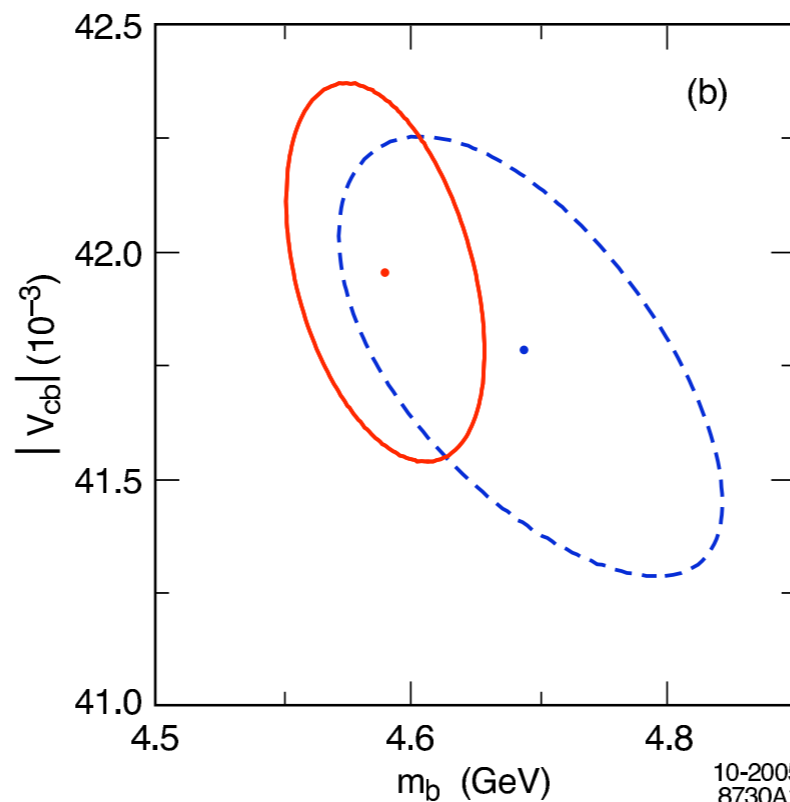
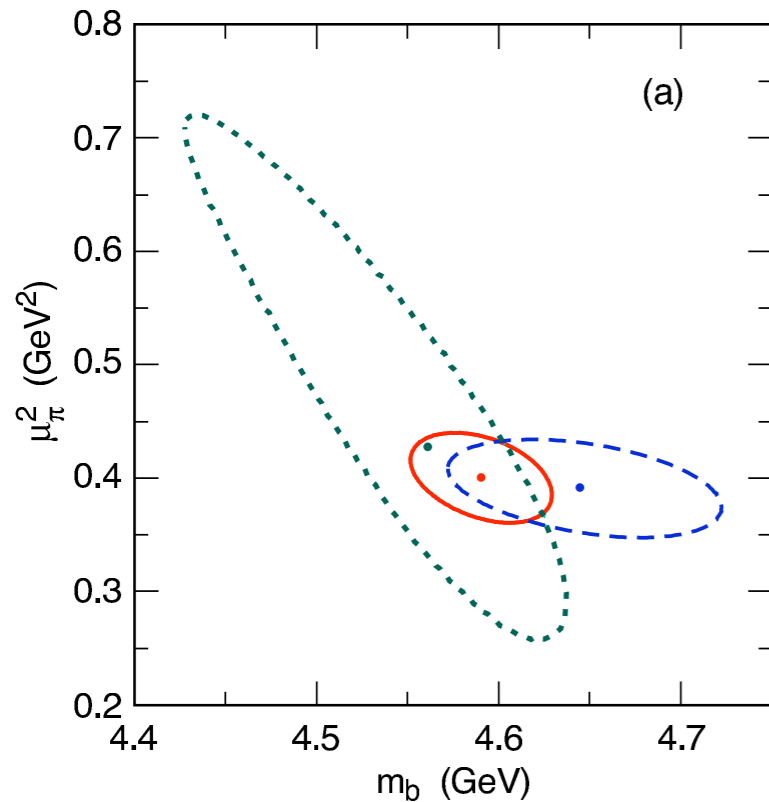
## lepton energy moments



- Red line: fit result. Green band: exp. uncertainty. Yellow band: exp. + th. uncertainty



# Fit results



Moments included:

**Solid red:** all

**Dashed blue:**  $B \rightarrow X_c e \nu$

**Dotted green:**  $B \rightarrow X_s \gamma$

$$|V_{cb}| = (41.96 \pm 0.23_{\text{exp}} \pm 0.35_{\text{HQE}} \pm 0.59_{\Gamma_{\text{SL}}}) \times 10^{-3}$$

$$m_b = 4.590 \pm 0.025_{\text{exp}} \pm 0.030_{\text{HQE}} \text{ GeV}$$

$$m_c = 1.142 \pm 0.037_{\text{exp}} \pm 0.045_{\text{HQE}} \text{ GeV}$$

$$\mu_\pi^2 = 0.401 \pm 0.019_{\text{exp}} \pm 0.035_{\text{HQE}} \text{ GeV}^2$$

results of hep-ph/0507253, see also hep-ph/0408002

**Most precise determination of 3 SM parameters in a single process!**

# News flash

- The “most precise determination” statement was true until last Tuesday:

arXiv.org > hep-ph

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All papers

## High Energy Physics – Phenomenology

hep-ph new abstracts, Tue, 13 Feb 07 01:00:10 GMT  
0702103 -- 0702123 received

hep-ph/0702103 [[abs](#), [ps](#), [pdf](#), [other](#)] :

Title: Heavy Quark Masses from Sum Rules in Four-Loop Approximation

Authors: [Johann H. Kuehn](#), [Matthias Steinhauser](#), [Christian Sturm](#)

Comments: 29 pages

New data for the total cross section  $\sigma(e^+e^- \rightarrow \text{hadrons})$  in the charm and bottom threshold region are combined with an improved theoretical analysis, which includes recent four-loop calculations, to determine the short distance  $\overline{\text{MS}}$  charm and bottom quark masses. A detailed discussion of the theoretical and experimental uncertainties is presented. The final result for the  $\overline{\text{MS}}$ -masses,  $m_c(3 \text{ GeV}) = 0.986(13) \text{ GeV}$  and  $m_b(10 \text{ GeV}) = 3.609(25) \text{ GeV}$ , can be translated into  $m_c(m_c) = 1.286(13) \text{ GeV}$  and  $m_b(m_b) = 4.164(25) \text{ GeV}$ . This analysis is consistent with but significantly more precise than a similar previous study.

# Future improvements of the moment analysis

- To go to next higher level in theoretical precision, we'll need
  - Tree-level OPE to  $1/m_b^3$ . Already included.
    - Has recently even been calculated up to  $1/m_b^4$ , hep-ph/0611168.
  - Perturbative corrections to the leading power corrections, terms
$$\alpha_s(m_b) \frac{\mu_\pi^2}{2m_b} \quad \alpha_s(m_b) \frac{\mu_G^2}{2m_b}$$
    - doable, but nontrivial 1-loop calculation
  - Two-loop corrections to the leading power rate
    - Possible with new numerical techniques. Muon decay has been calculated, hep-ph/0505069 (same kinematics, but QED instead of QCD corrections)

# Quark hadron duality

- Are there pieces that we are missing when calculating the rate using the OPE?
- It is often stated that the OPE calculation “assumes quark hadron duality”, since we calculated the coefficients  $C_3$  and  $C_5$  with quarks instead of hadrons.
- More precisely, we have expanded in the rate  $1/m_b$ ,  $\alpha_s(m_b)$  and  $1/m_W^2$ . Upon expanding, we loose non-analytic terms, such as

pert. expansion

$$e^{-1/\alpha_s},$$

OPE integrating out W

$$e^{-a^2/m_W^2},$$

W cannot go on-shell  
(Euclidean OPE)

OPE for incl. B-decay

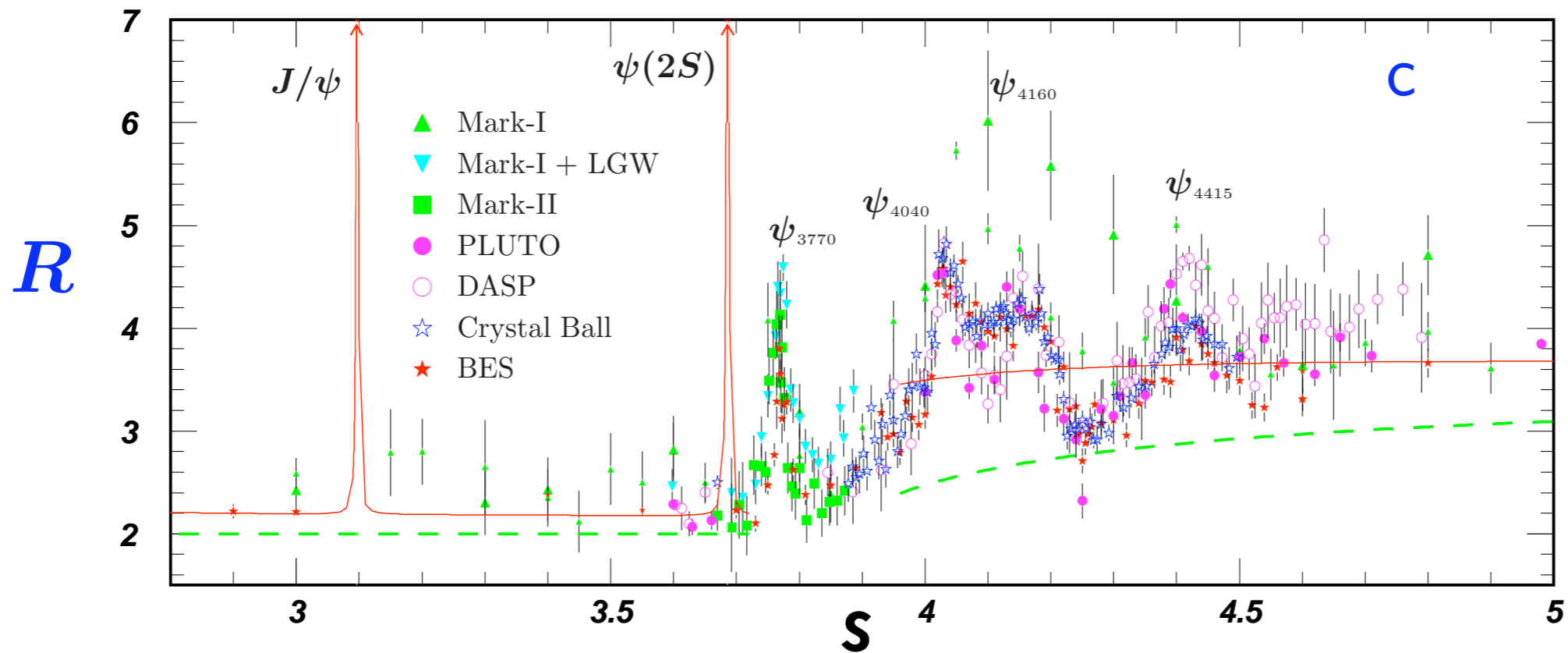
$$\frac{1}{m_b^n} \sin\left(\frac{m_b}{b}\right)$$

quark, gluons can be on shell  
(Minkowskian OPE)

- Models give  $n=8$  suppression compared to leading order in SL decay, see hep-ph/0009131. Hopefully, these effects are tiny.

“duality violation”

# Example of oscillatory behavior



$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons}, s)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-, s)}$$

- Oscillatory behavior is not captured by OPE calculation.
- In inclusive quantities, the oscillations average out.

# Heavy hadron lifetimes

and the  $\Lambda_b$  (ex-)puzzle

# Heavy hadron lifetimes

- Same OPE technique can also be used to calculate the hadron lifetimes  $\tau=1/\Gamma$

$$\begin{aligned}\Gamma(H_b) = \Gamma(H_b \rightarrow X) &= \frac{1}{2M_B} \sum_X (2\pi)^4 \delta^4(p_B - p_X) |\langle X | \mathcal{H}_{\Delta B=1} | B \rangle|^2 \\ &= \frac{1}{2M_B} 2 \text{Im} \langle B | i \int d^4x T \{ \mathcal{H}_{\Delta B=1}(x), \mathcal{H}_{\Delta B=1}(0) \} | B \rangle\end{aligned}$$

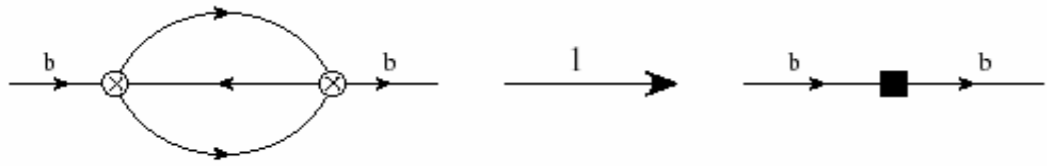
- Complete  $\Delta B=1$  eff. Hamiltonian:

$$\begin{aligned}\mathcal{H}_{\Delta B=1} &= \frac{4G_F}{\sqrt{2}} V_{cb} \left\{ c_1(m_b) \left[ \bar{d}'_L \gamma_\mu u_L \bar{c}_L \gamma^\mu b_L + \bar{s}'_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L \right] \right. \\ &\quad + c_2(m_b) \left[ \bar{c}_L \gamma_\mu u_L \bar{d}'_L \gamma^\mu b_L + \bar{c}_L \gamma_\mu c_L \bar{s}'_L \gamma^\mu b_L \right] \\ &\quad \left. + \sum_{\ell=e,\mu,\tau} \bar{\ell}_L \gamma_\mu \nu_\ell \bar{c}_L \gamma^\mu b_L \right\} + \text{h.c.},\end{aligned}$$

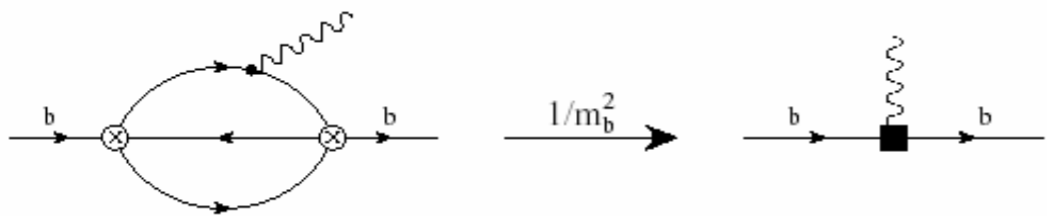
hadronic decays

semileptonic decays

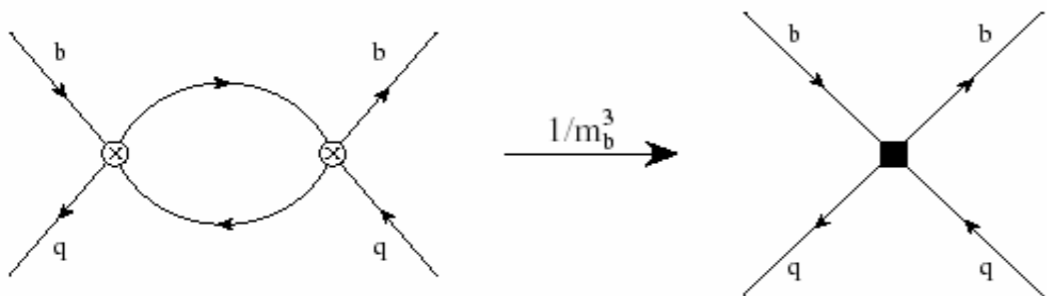
# OPE



$$\bar{b} b$$



$$\bar{b} \sigma_{\mu\nu} G^{\mu\nu} b$$



$$\bar{b} \Gamma_i q \bar{q} \Gamma_i b$$

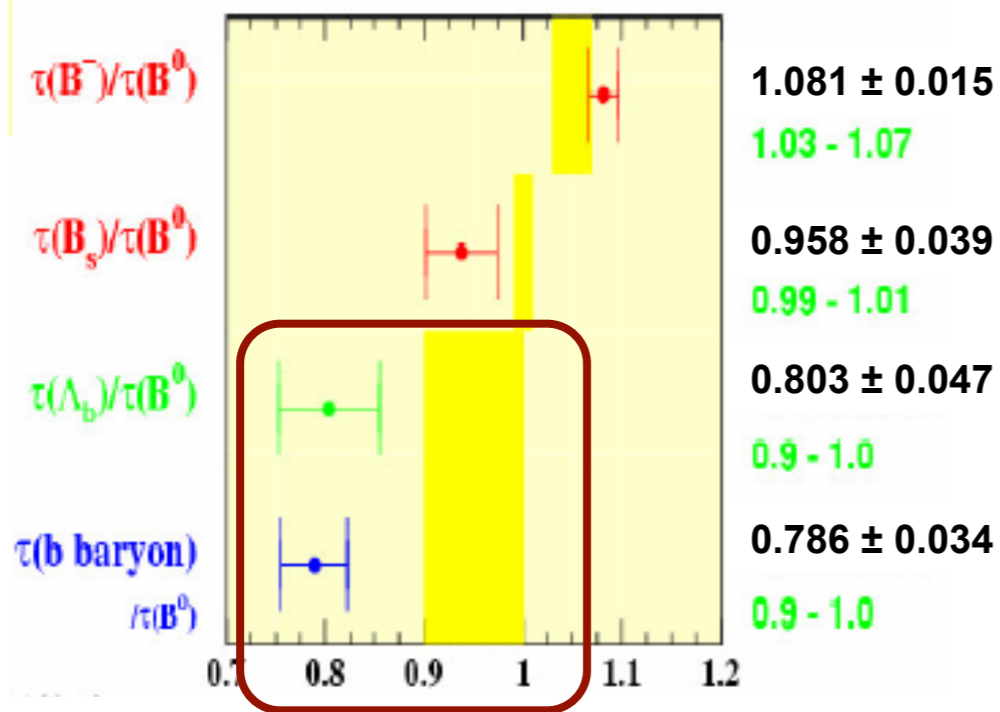
Dependence on light quarks!  
Induces lifetime differences.  
Evaluate matrix elements with  
LQCD.

- Result for the rate has the same structure as we had before.
- Only small lifetime differences  $\sim 1\text{-}2\%$  to  $O(\Lambda^2/m_b^2)$ . Arise because  $\lambda_1, \lambda_2$  are slightly different for different hadrons.
- Dominant contribution to lifetime *differences* from the four-quark operators suppressed by  $(\Lambda/m_b)^3$ . Enhanced by a large numerical prefactor  $4\pi^2$ , they are  $O(5\text{-}10\%)$ .



# The $\Lambda_b$ ex-puzzle

Situation a few years ago



Before HFAG 2004

Latest numbers:

Theory, hep-ph/0612176	Experiment
$\left[ \frac{\tau(B^+)}{\tau(B_d^0)} \right]_{\text{NLO}} = 1.063 \pm 0.027$	$\left[ \frac{\tau(B^+)}{\tau(B_d^0)} \right] = 1.071 \pm 0.009$
$\frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01$	$\frac{\tau(B_s)}{\tau(B_d)} = 0.957 \pm 0.027$
hep-ph/9906031	
$\frac{\tau(\Lambda_b)}{\tau(B_d)} = \begin{cases} 0.91(1) & \text{for } am_\pi = 0.74(4) \\ 0.93(1) & \text{for } am_\pi = 0.52(3) \end{cases}$	
$a^{-1} = 1.1 \text{ GeV}$	

- New CDF result, hep-ex/0609021

$$\frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 1.041 \pm 0.057 \text{ (stat. + syst.)}$$

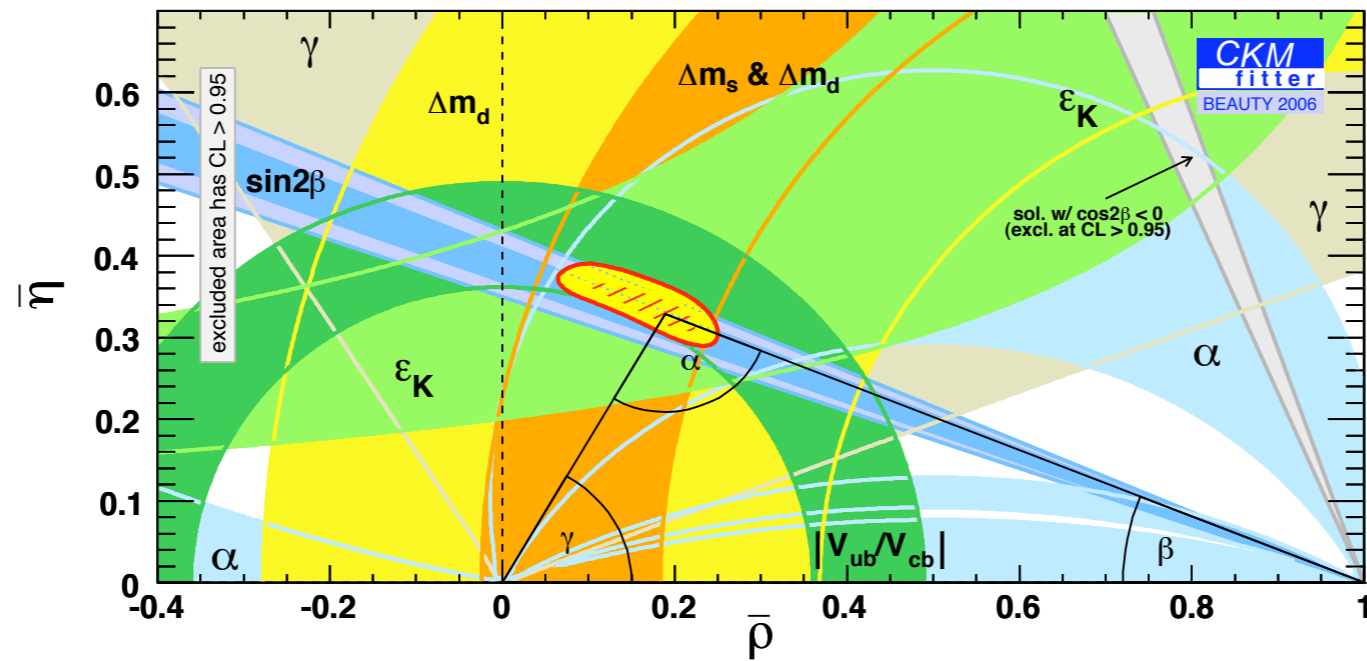
DZero:  $\frac{\tau(\Lambda_b^0)}{\tau(B^0)} = 0.87_{-0.14}^{+0.17} \text{ (stat)} \pm 0.03 \text{ (syst)}$ ,

3.2 $\sigma$  higher than world average, with comparable precision!

$$B \rightarrow X_u e \nu$$

Experimental cuts, shape function and the extraction of  $V_{ub}$

# $|V_{ub}|$



- Interesting tension between  $|V_{ub}|$  and  $\sin(2\beta)$  measurements
  - $\sin(2\beta)$ : loop process in SM
    - sensitive to new physics
  - $|V_{ub}|$ : tree level weak decay
    - insensitive to new physics,
    - but extraction is sensitive to QCD effects!

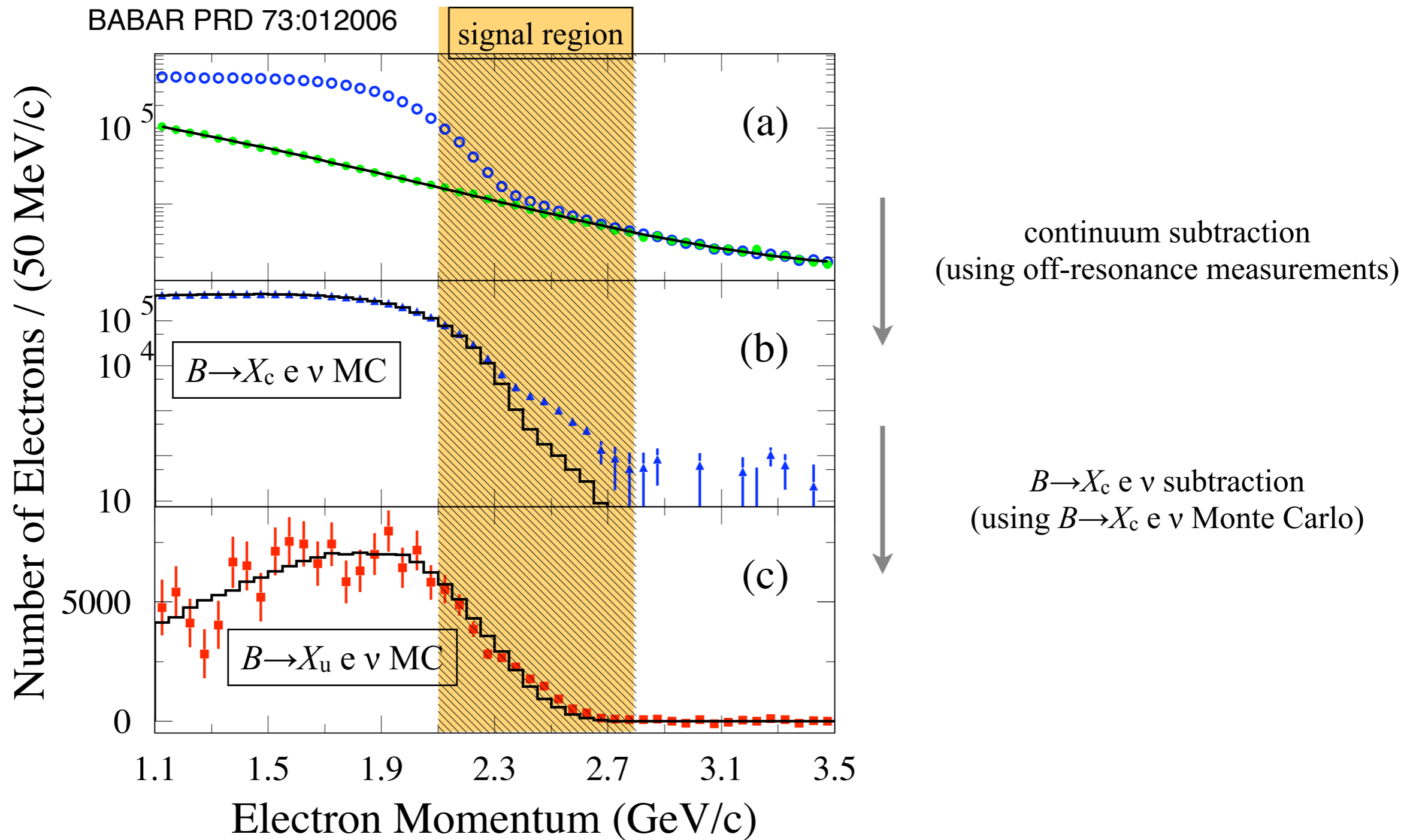
# $|V_{ub}|$

- It is trivial to obtain the  $B \rightarrow X_u e \nu$  rate from our expression for  $B \rightarrow X_c e \nu$ 
  - Set  $m_c=0$ , replace  $V_{cb} \rightarrow V_{ub}$
- However, experimentally, it is impossible to measure the total  $B \rightarrow X_u e \nu$  rate.
  - $B \rightarrow X_c e \nu$  signal is much larger

$$\left| \frac{V_{ub}}{V_{cb}} \right| \approx 0.1 \quad \frac{\Gamma_c}{\Gamma_u} = \left| \frac{V_{cb}}{V_{ub}} \right|^2 (1 - 8\rho - \rho^4 - 12\rho^2 \ln \rho + 8\rho^3) \approx 50$$

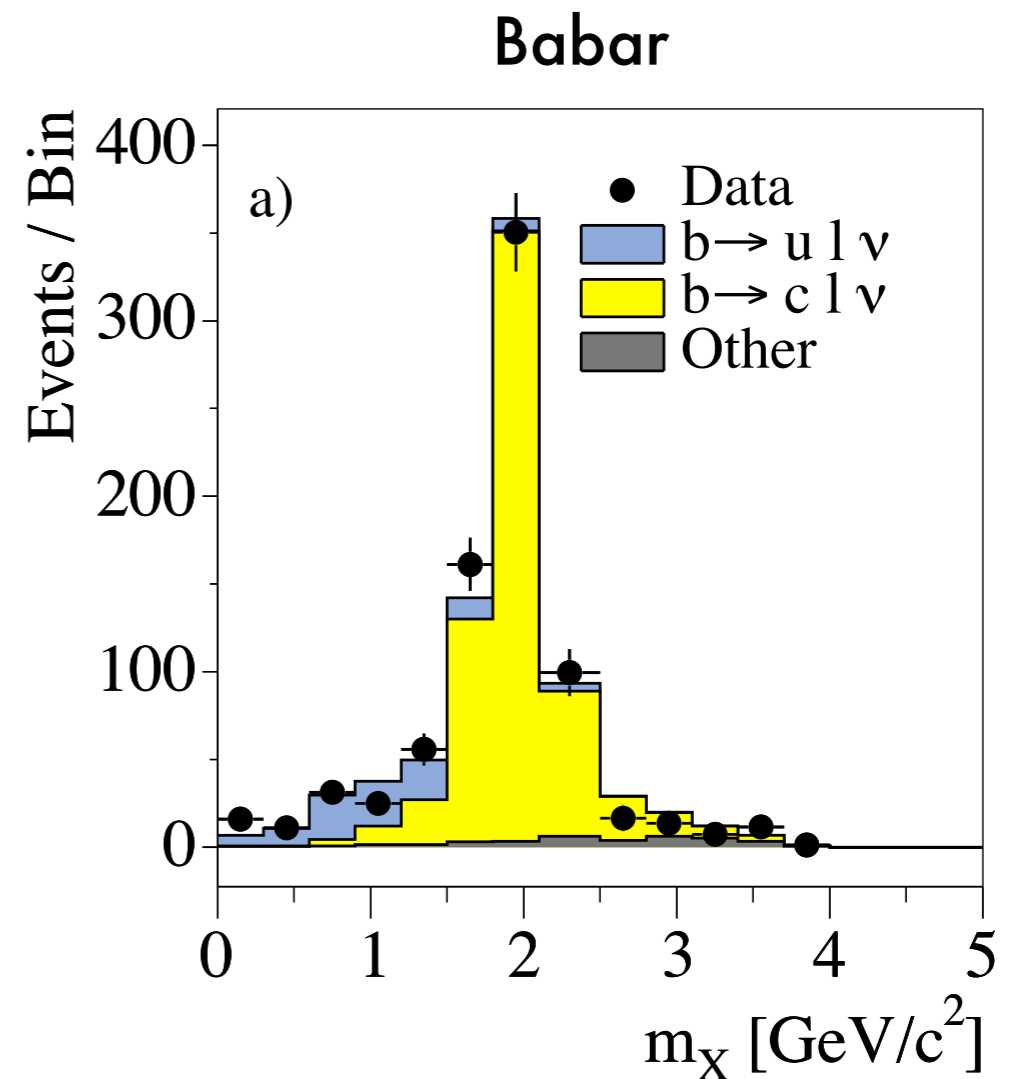
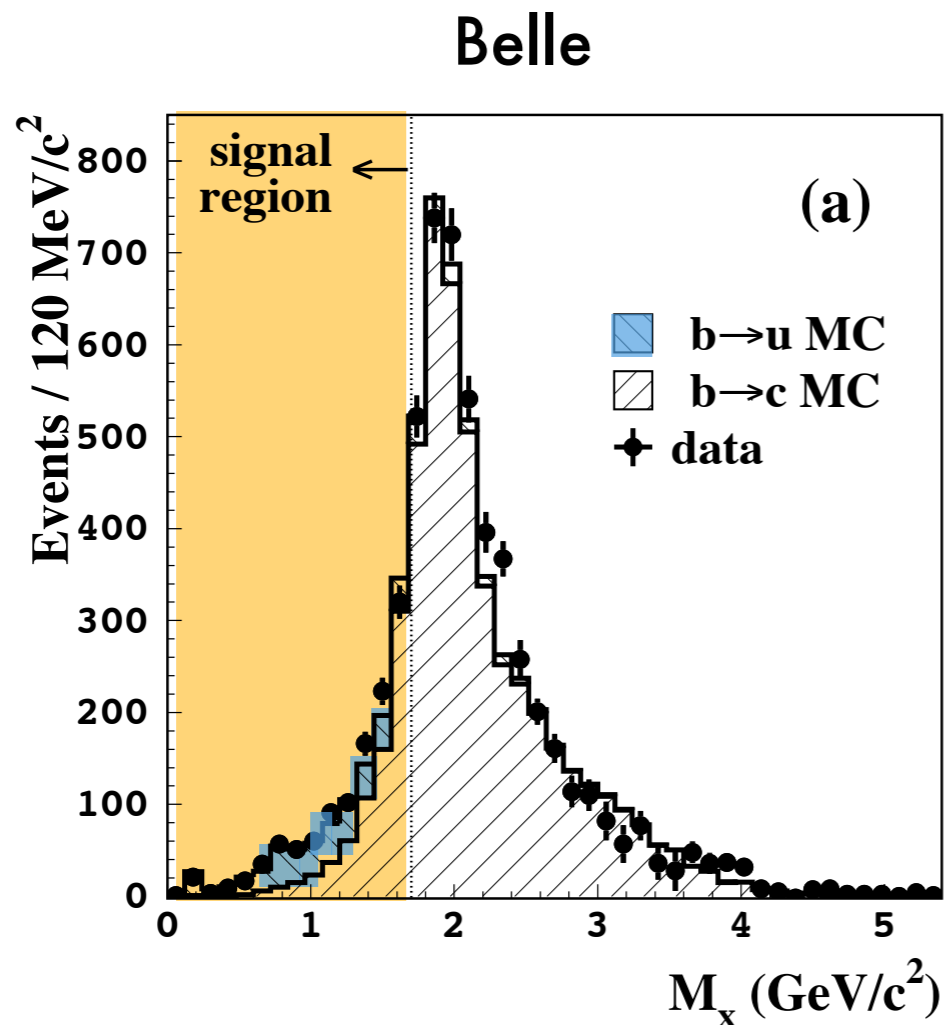
- Need kinematical cuts to eliminate  $B \rightarrow X_c e \nu$ 
  - *e.g.*  $M_X < M_D$  or  $E_e > \frac{M_B^2 - M_D^2}{2M_B}$

# $E_e$ spectrum



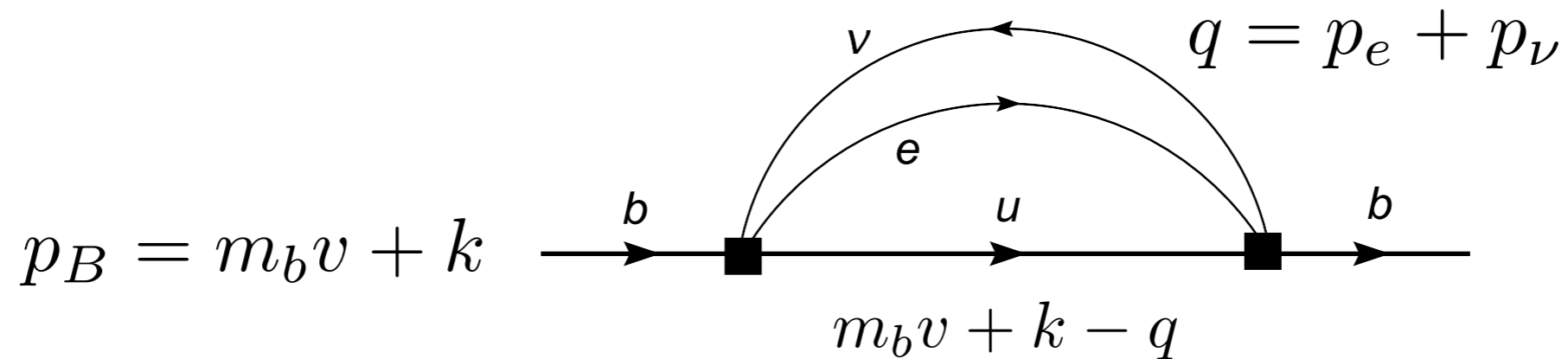
- $S/B \sim 1/15$  for  $E_e > 2\text{GeV}$ . Background subtraction challenging!

# $M_X$ spectrum



- The cuts reduce the small  $b \rightarrow u$  signal even farther!
- Are a theoretical challenge
  - Reduce available phase space enforce small  $M_X$ .
  - **OPE breaks down!** Terms  $\frac{\Lambda_{QCD} E_X}{M_X^2}$  in OPE are no longer suppressed.

# OPE with cut



- $u$ -quark propagator denominator

$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{(m_b v - q)^2 - 2(m_b v - q)k + k^2}$$

- Total rate:  $(p_X = m_b v - q)$

$$p_X^2 \sim m_b^2, \quad p_X \cdot k \sim m_b \Lambda_{\text{QCD}}, \quad k^2 \sim \Lambda_{\text{QCD}}^2$$

- After cut to eliminate  $B \rightarrow X_c e \nu$

$$p_X^2 \sim m_b \Lambda_{\text{QCD}}, \quad p_X \cdot k \sim m_b \Lambda_{\text{QCD}}, \quad k^2 \sim \Lambda_{\text{QCD}}^2$$

# Shape function

$$p_B = m_b v + k \xrightarrow{b} \text{---} \text{---} \text{---} \text{---} \xrightarrow{b} q = p_e + p_\nu$$

$$m_b v + k - q$$

- Without cut (or modest cuts)

$$\frac{1}{(p_X + k)^2} = \frac{1}{p_X^2} \left[ 1 + \frac{2 p_X \cdot k}{p_X^2} + \dots \right]$$

- Hadronic part: local operators with derivatives

$$h_v(k) h_v(k) \rightarrow h_v(x) h_v(x),$$

$$h_v(k) k_\mu h_v(k) \rightarrow h_v(x) iD_\mu h_v(x) \text{ etc.}$$

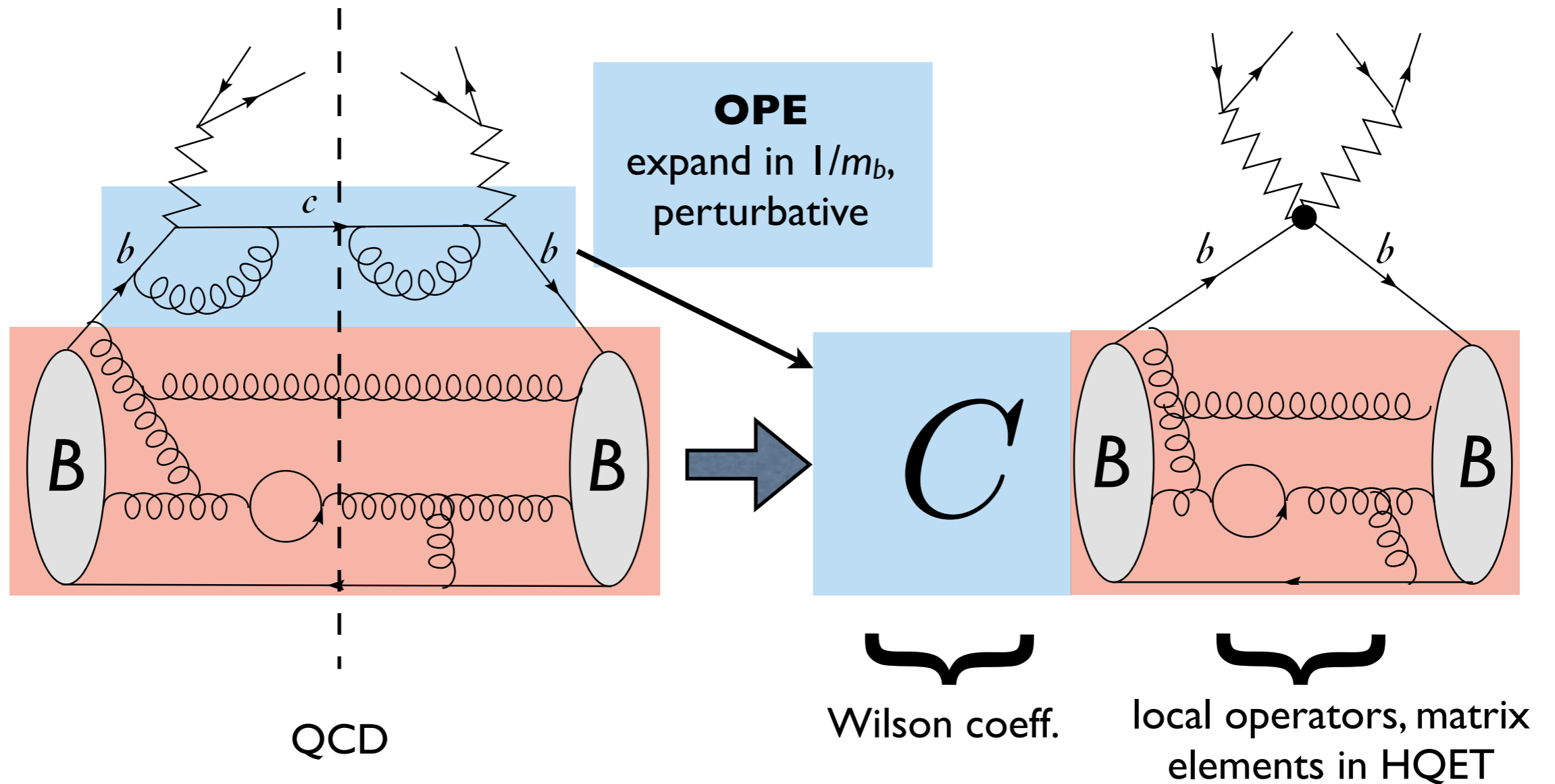
- With cut to eliminate  $B \rightarrow X_c e \nu$

$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{p_X \cdot (p_X - k)} \left[ 1 + \frac{k^2}{p_X \cdot (p_X - k)} + \dots \right]$$

Function of  $p_X \cdot k$  ! Nonlocal object in position space. Matrix element is “Shape function”

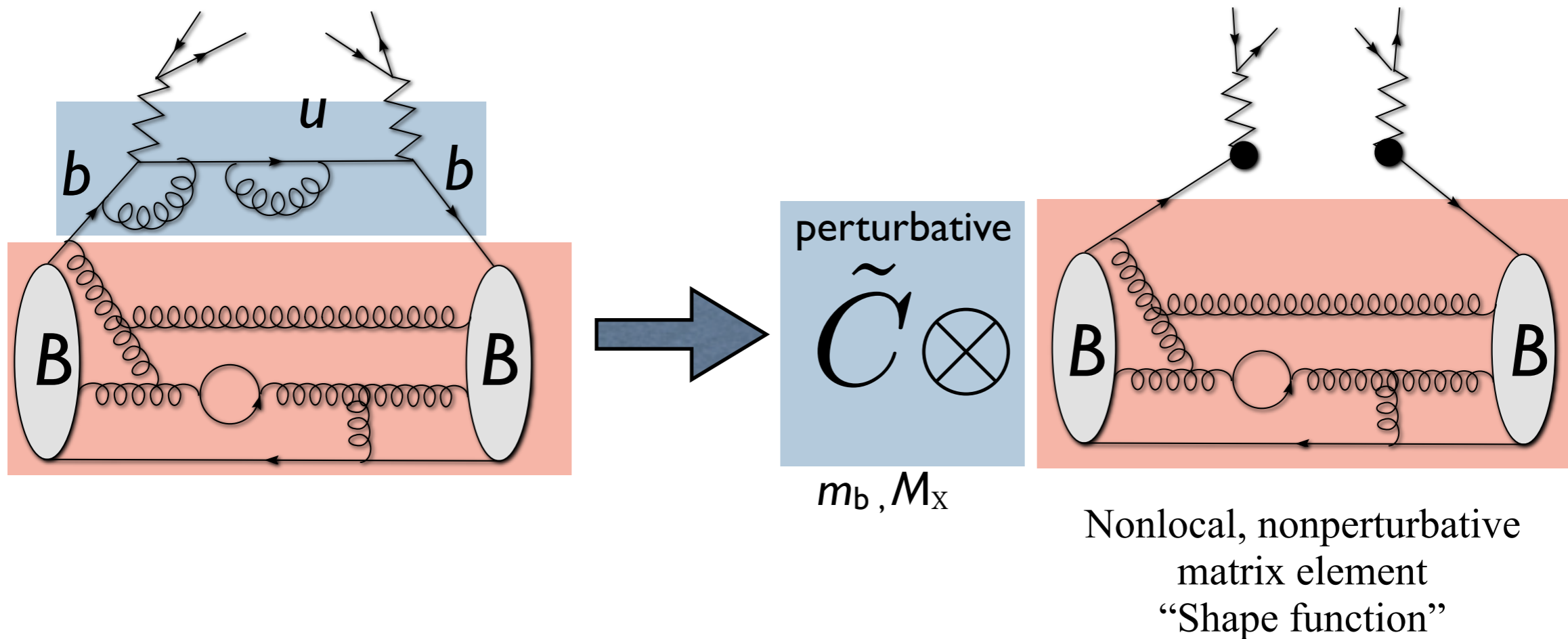


# Total rate (or mild cuts)



Two relevant scales:  $m_b \gg \Lambda_{\text{QCD}}$

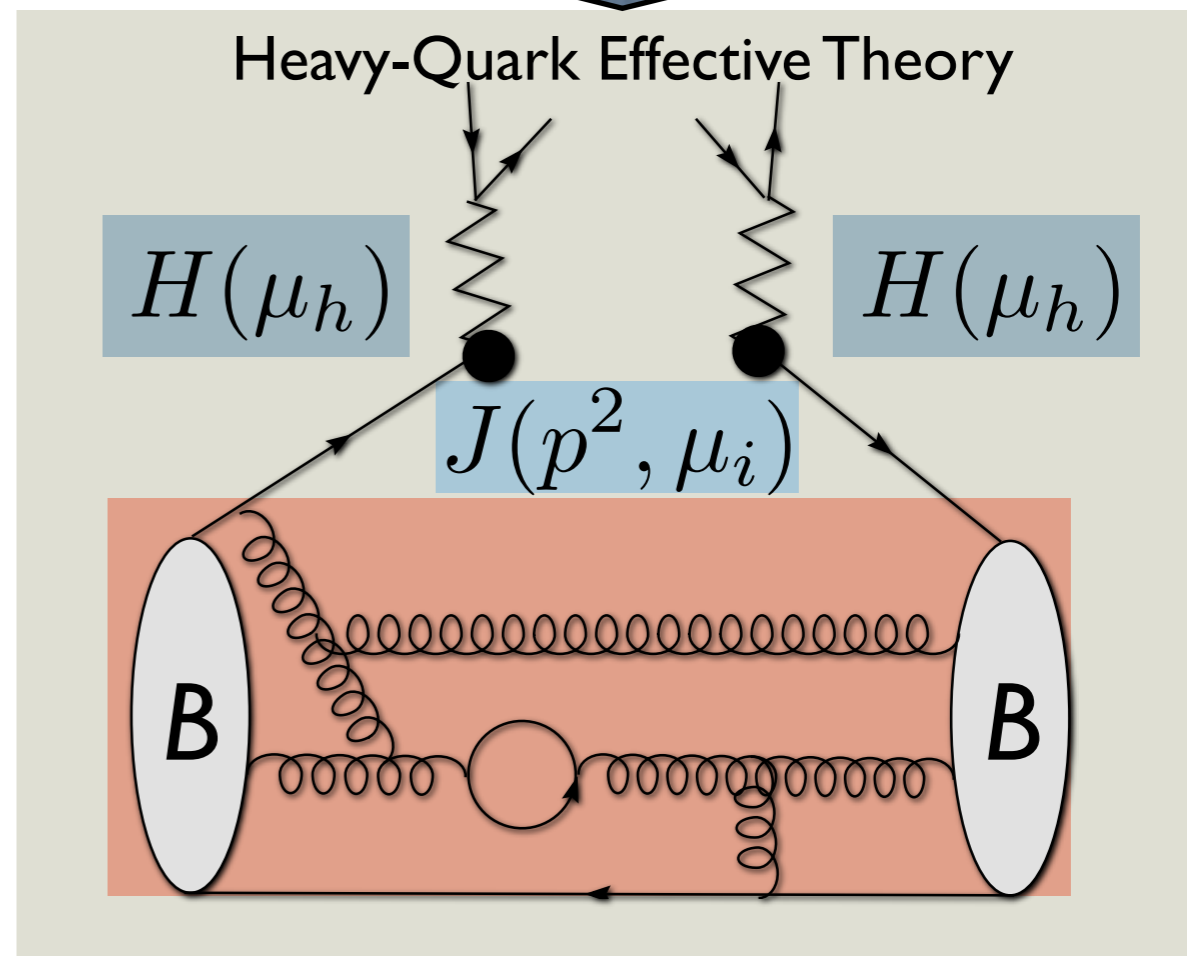
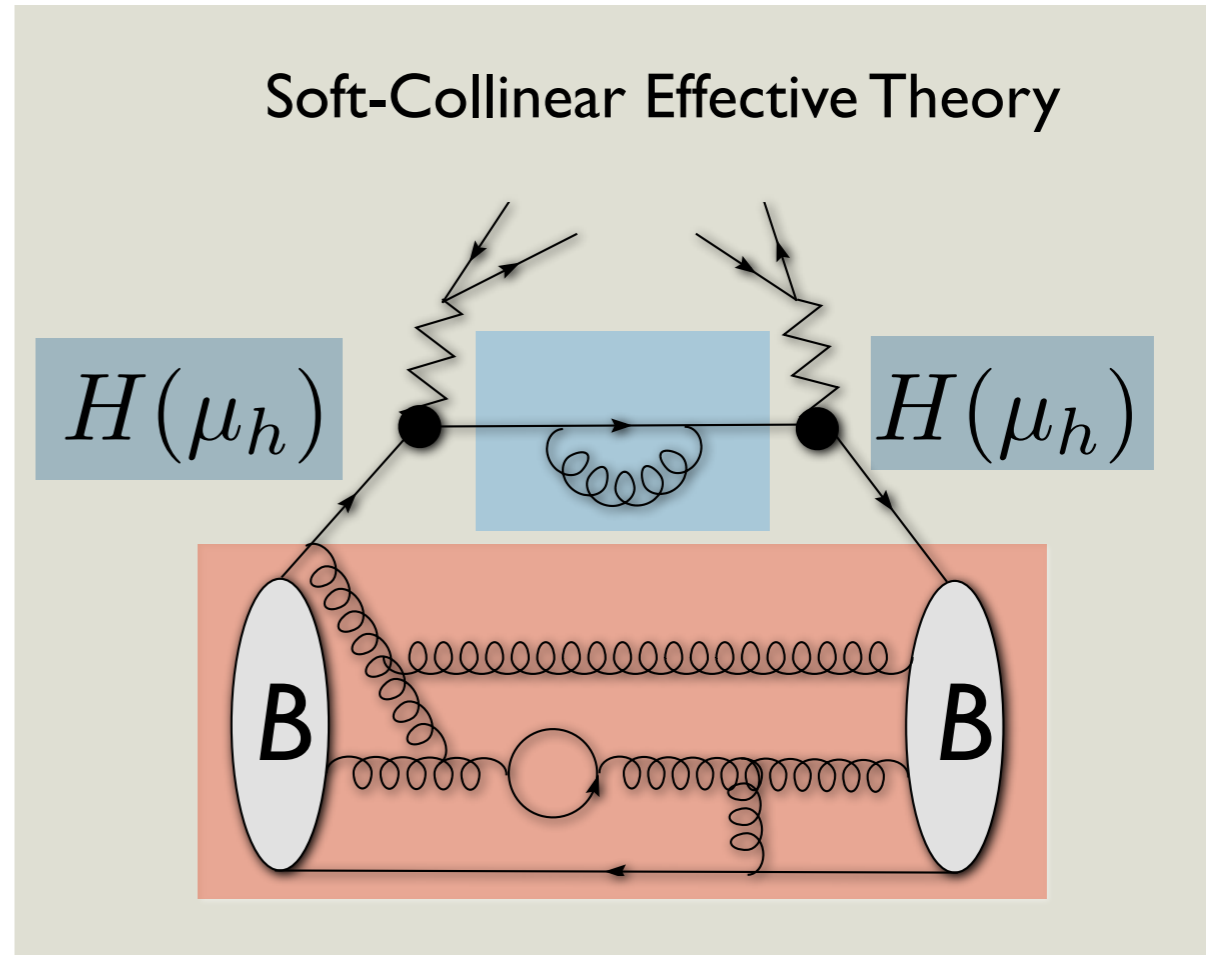
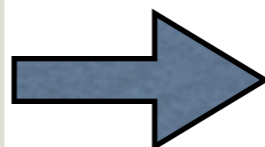
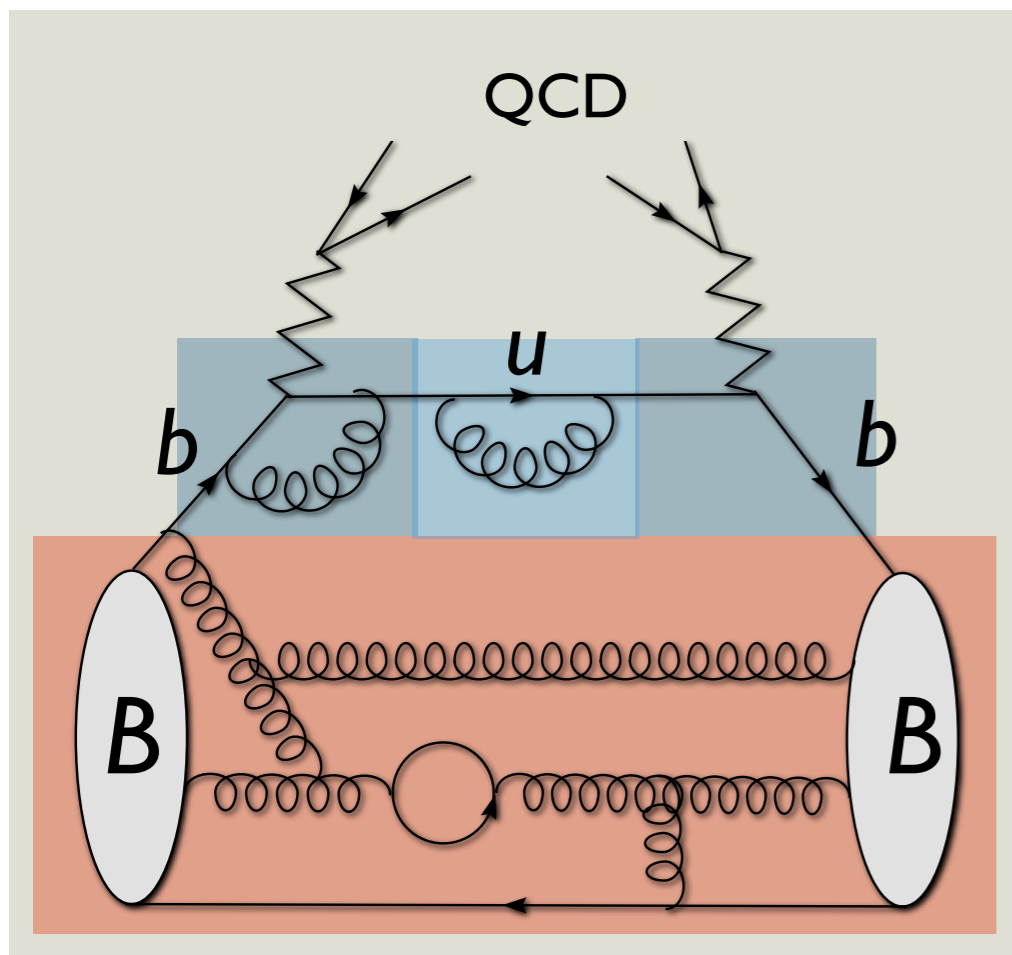
# Small $M_X$ : "shape function region"



Three different scales:  $m_b \gg M_X \gg \Lambda_{\text{QCD}}$

Double expansion:  $M_X/m_b$  and  $\Lambda_{\text{QCD}}/M_X$

Can use soft-collinear effective theory (SCET) to perform expansion in a systematic way



## Factorization theorem

$$\Gamma \sim \underbrace{H^2}_{\text{hard}} \otimes \underbrace{J}_{\text{jet}} \otimes \underbrace{S}_{\text{soft shape function}}$$

# Shape function $S(\omega)$

- At this point things look bleak: Even in the limit  $m_b \rightarrow \infty$ , we need a nonperturbative shape *function*  $S(\omega)$  as input!
- Similar to hadron collider physics, where we need non-perturbative parton distributions to make predictions.
- Know a few properties: once we integrate over the spectrum, we obtain usual OPE expression.
- Moments are given in terms of HQET parameters, such as  $m_b$ ,  $\lambda_1$ .

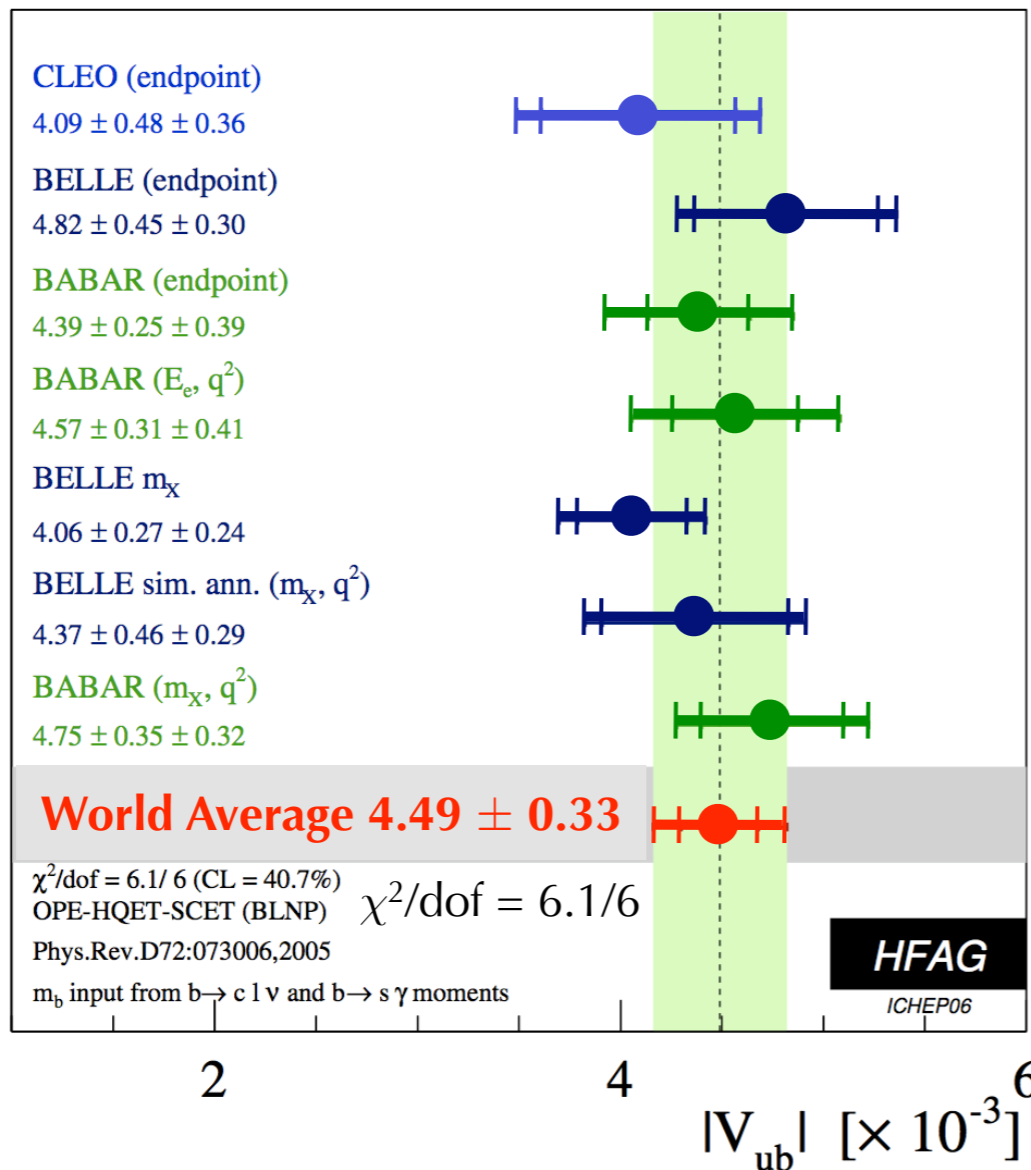
# $B \rightarrow X_s \gamma$ to the rescue

- Fortunately, the *same* shape function  $S(\omega)$  appears in the calculation of the  $B \rightarrow X_s \gamma$  photon energy spectrum.

$$\frac{d\Gamma}{dE_\gamma} = H_\gamma J \otimes S$$

- Only hard function differs from  $B \rightarrow X_u l \nu$ .
- Two strategies to extract  $|V_{ub}|$ :
  - Make ansatz for  $S(\omega)$ , depending on a number of parameters. Constrain with  $B \rightarrow X_s \gamma$  spectrum and  $B \rightarrow X_c l \nu$  moments.
  - Use relations between the  $B \rightarrow X_s \gamma$  and  $B \rightarrow X_u l \nu$  spectra in which shape function drops out.

# $V_{ub}$ using shape function



■  $|V_{ub}|$  determined to  $\pm 7.3\%$

Statistical	$\pm 2.2\%$
Expt. syst.	$\pm 2.8\%$
$b \rightarrow c l \nu$ model	$\pm 1.9\%$
$b \rightarrow u l \nu$ model	$\pm 1.6\%$
SF params.	$\pm 4.2\%$
Theory	$\pm 4.2\%$

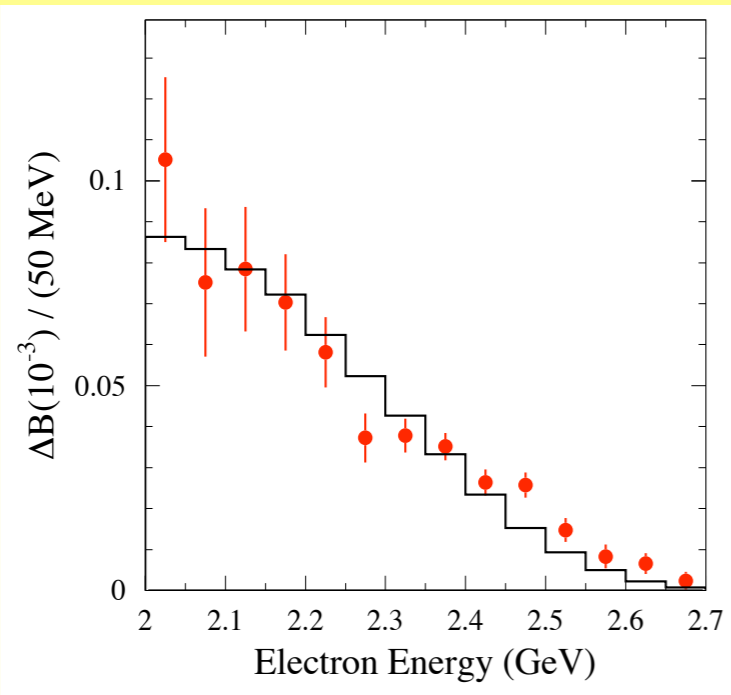
← subleading SF, PT, WA

HFAG number is based on  
 Lange, Neubert and Paz  
 Phys.Rev.D72:073006,2005

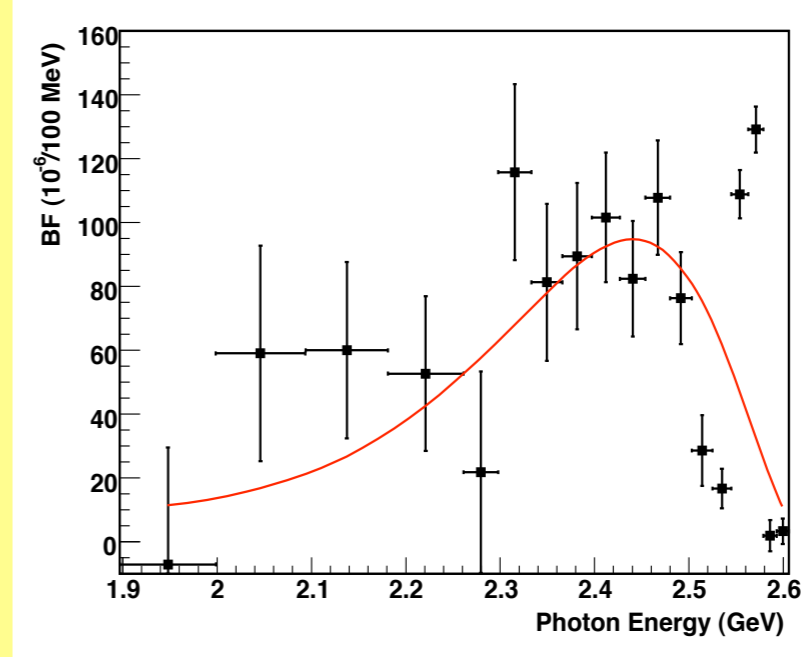
- Analysis also includes subleading shape functions.
- Uses  $B \rightarrow X_s \gamma$  and  $B \rightarrow X_c l \nu$  to constrain shape functions.
- Uses different parameterizations to estimate dependence on functional form.

# $V_{ub}$ without shape function

$B \rightarrow X_u e \nu$  electron energy spectrum



$B \rightarrow X_s \gamma$  photon energy spectrum



- Babar  $|V_{ub}|$  values from weighted integrals over the two spectra appeared very recently in hep-ph/0702072.
- They use 3 different theoretical evaluations of the weight function.

Method	$ V_{ub}  \cdot 10^3$
LLR [3, 4]	$4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28$
Neubert [6]	$4.01 \pm 0.27 \pm 0.29 \pm 0.32 \pm 0.27$
BLNP [7, 8]	$4.40 \pm 0.30 \pm 0.41 \pm 0.23$

Uncertainties:  $b \rightarrow u$ ,  $b \rightarrow s$ , theory,  $V_{ts}$

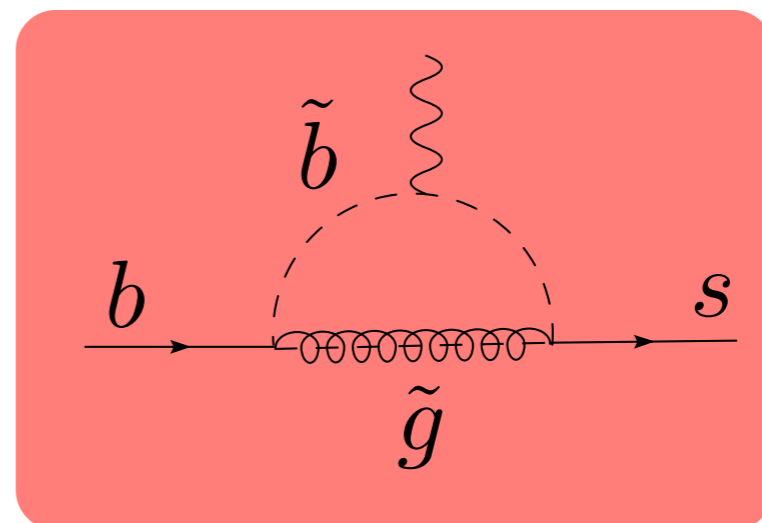
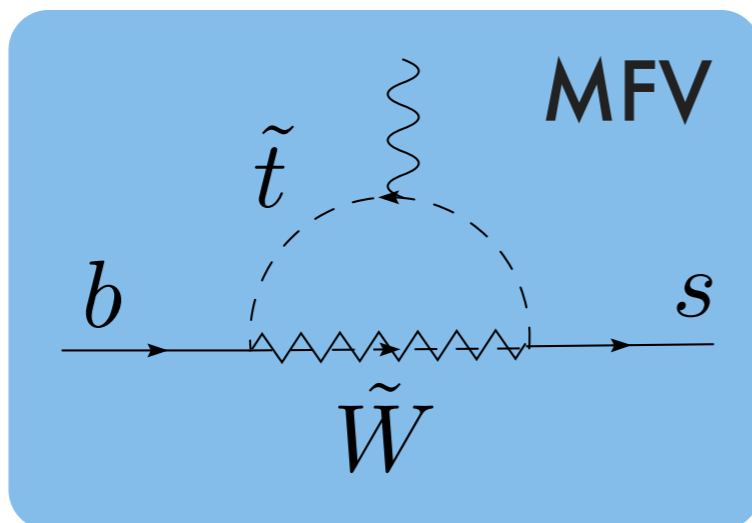
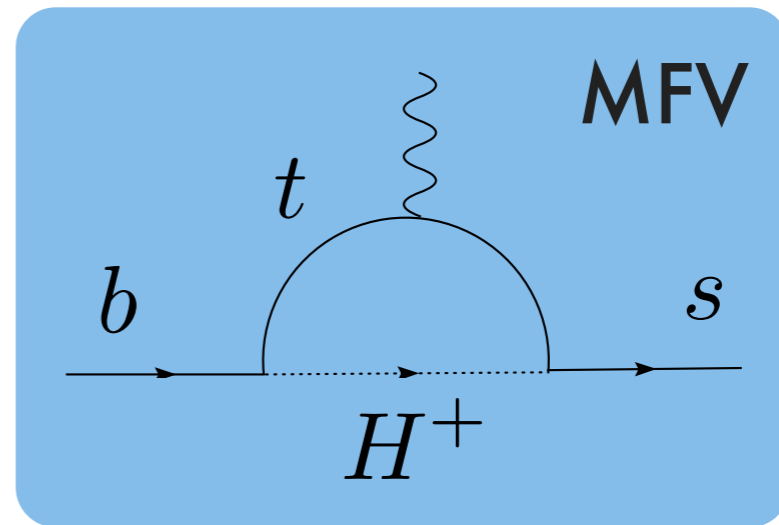
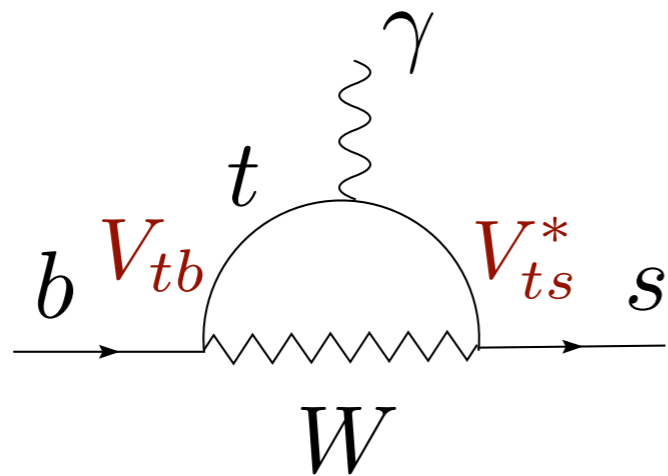
$$B \rightarrow X_s \gamma$$

Chasing New Physics with 4-loops



# A sensitive probe of New Physics

- FCNC process. Loop suppressed in the SM
- e.g. strong constraint on the MSSM

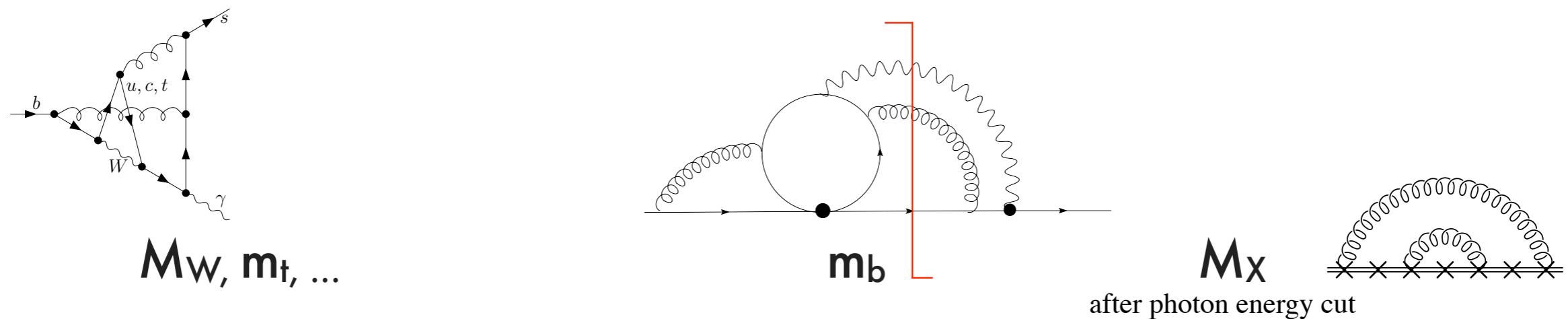


# Elements of the NNLO calculation

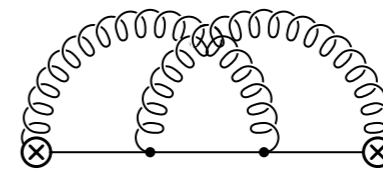
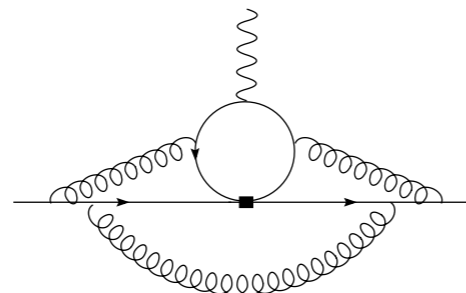
- After large theory effort over the last years we have obtained the rate at NNLO level.
- $O(20)$  papers with necessary calculations.
- Needs all of the following at NNLO:
  1. Effective weak Hamiltonian
    - a. Matching at high scale (2- and 3-loop)
    - b. RG evolution to low scale (3- and 4-loop)
  2. Calculation of rate at NNLO
    - a. OPE for total rate ( ← so far only estimate)
    - b. Effect of the photon energy cut  
 $E_\gamma > E_0 \approx 1.9 \text{ GeV}$

# Match and run, match and run...

- Many energy scales. Use different effective theories to treat each of them in turn.
- $O(10^5)$  diagrams along the way...



**SM**  $\longrightarrow$  **Fermi Theory**  $\longrightarrow$  **SCET**  $\longrightarrow$  **HQET**



# NNLO result

- Experimental average (HFAG)

$$\text{Br}(\bar{B} \rightarrow X_s \gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4}$$

- for cut  $E_\gamma > E_0 = 1.6 \text{ GeV}$
- stat.+syst., extrapolation to low  $E_0$ ,  $b \rightarrow \gamma d$  subtr.
- Theory @ NNLO (hep-ph/0610067 with hep-ph/0609232)

$$\text{Br}(\bar{B} \rightarrow X_s \gamma) = (2.98 \pm 0.26) \cdot 10^{-4}$$

- $^{+4}_{-6}$  % perturbative, 4% parametric, 5% power corrections, 3% interpolation in  $m_c$ .
- $1.4\sigma$  below exp. value.  $1-2\sigma$  below NLO value. (Gambino Misiak '01 found  $\text{BR} = (3.6 \pm 0.3) \times 10^{-4}$  at NLO.)