QCD effects in B-decays: Lecture 1

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Outline of these lectures

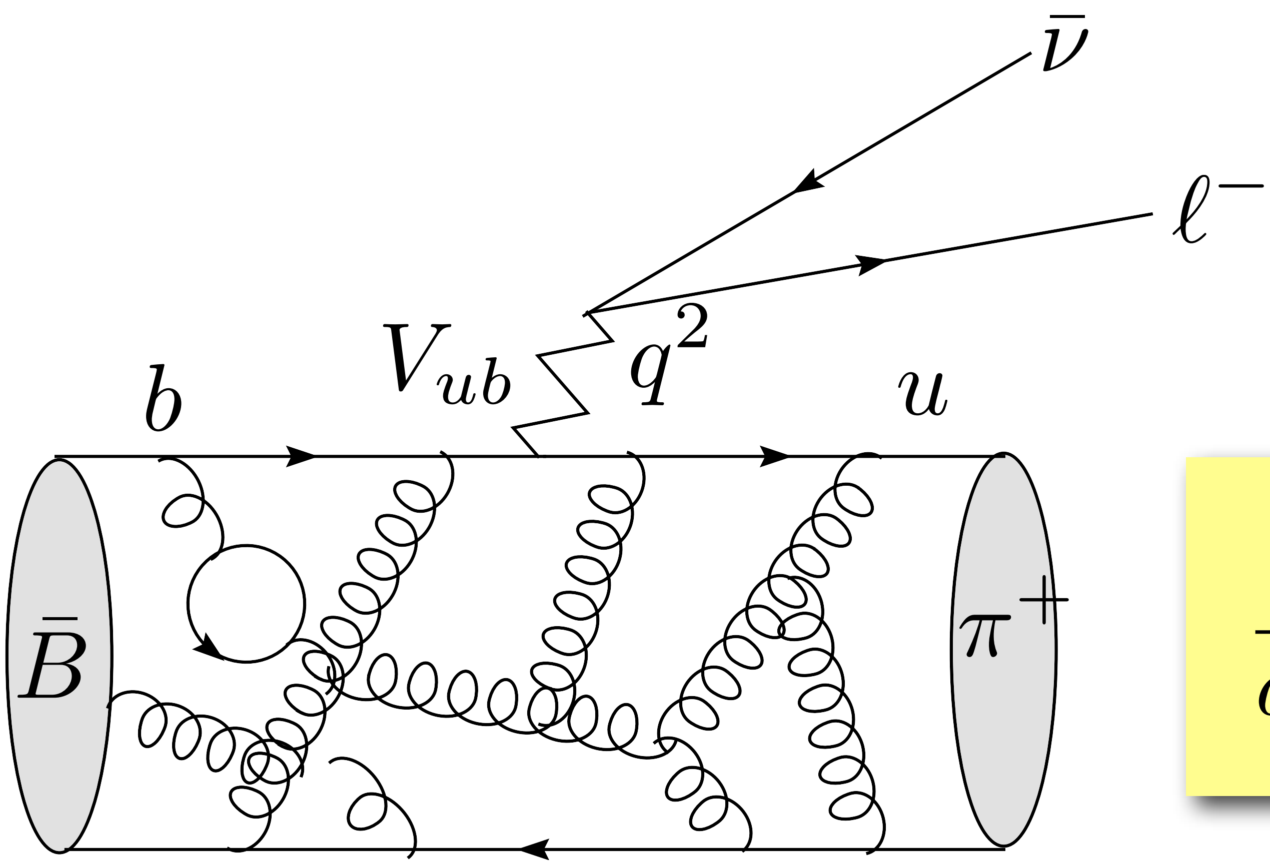
1. Heavy quark physics
   - Heavy-quark spin and flavor symmetry
     - Spectroscopic implications
   - Heavy Quark Effective Theory
     - $|V_{\mathrm{cb}}|$ from exclusive semileptonic decay
2. Inclusive B-decays
   - Operator Product Expansion
   - Determination of $|V_{\mathrm{ub}}|$, $|V_{\mathrm{cb}}|$ from semileptonic decays
   - Radiative decays: test of FCNC interactions
   - Heavy hadron lifetimes
3. Exclusive radiative and hadronic B-decays
   - Factorization, Soft-Collinear Effective Theory

Literature

- A.J. Buras, ``Weak Hamiltonian, CP violation and rare decays,'' hep-ph/9806471.
  - Effective weak Hamiltonian
- M. Neubert,``Heavy quark symmetry,'' Phys. Rept. 245, 259 (1994)
  - HQET, sum rules
- M.B. Wise, ``Heavy quark physics,'' hep-ph/9805468.
  - HQET, inclusive decays
- A.V. Manohar and M.B. Wise, ``Heavy quark physics,'' Camb. Monogr. Part. Phys. 10, 1 (2000)
  - HQET, CHPT, OPE in inclusive decays
- M. Neubert,``Lectures on the theory of non-leptonic B decays,'' hep-ph/0012204
  - Factorization in B-decays. No SCET.

Basic problem

- Want to measure strength and test the structure of flavor changing *quark* interactions, but measure *hadron* decays.



"weak" x "strong"

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |\vec{p}_\pi|^3 |F_+(q^2)|^2$$

- No information on weak interaction without handle on QCD dynamics.

Methods

- Effective theories
  - Scale separation
  - Asymptotic freedom: QCD effects associated large scales are calculable in perturbation theory.
- Symmetries
  - QCD is more symmetric than weak interaction.
  - Same strong interaction physics appears in different weak interaction processes.
- Lattice gauge theory
  - "Simple" QCD matrix elements are calculable

"divide and conquer"*

- After the calculation of QCD effects associated with higher scales, the low energy parts
  - have additional (approximate) symmetries
  - are simpler to evaluate on the lattice
    - need effective theories to simulate *b*-quarks on the lattice

* For many more applications of latin in flavor physics, see I. Bigi, hep-ph/0701273, ...

Symmetries of QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^A_{\mu\nu} G_A^{\mu\nu} + \sum_{\rm flavors} \bar{q}\left(i \not{D} - m_q\right) q$$

- SU(3) gauge symmetry
- C, P, T
  - CP: $A^{\rm QCD}(i \to f) = A^{\rm QCD}(\bar{i} \to \bar{f})$
    - Crucial for extraction of sin(2β)
- Flavor symmetric except for mass term
  - $m_{\rm u}$, $m_{\rm d}$, $m_{\rm s} \ll \Lambda_{\rm QCD}$: isospin, flavor SU(3)
  - $m_{\rm b}$, $m_{\rm c}$, $m_{\rm t} \gg \Lambda_{\rm QCD}$: heavy quark symmetry
- Chiral (“left-right”) symmetry for $m_{\rm q}$=0

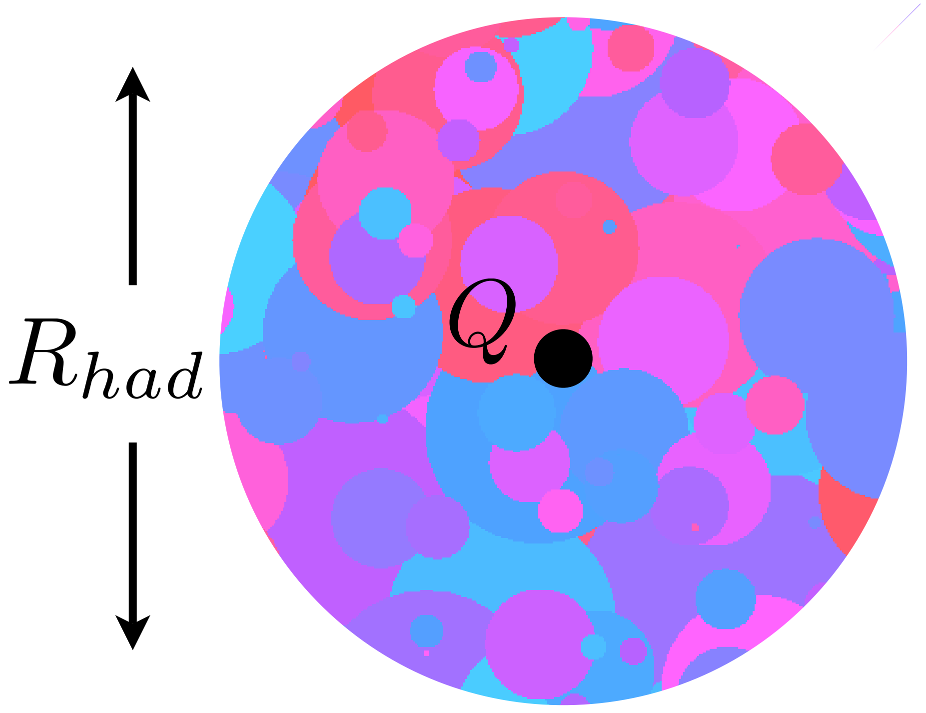
Side remark: Λ_{QCD}

- The "typical QCD scale" Λ_{QCD} stands for any scale which stays finite when we send
  - $m_q \to 0$ for the light quarks
  - $m_Q \to \infty$ for the heavy quarks
- Examples
  - Not $O(\Lambda_{QCD})$: M_π, M_B, ...
  - $O(\Lambda_{QCD})$: M_p, M_ρ, F_π, $M_\pi^2/(m_u+m_d)$...

    90 MeV 3GeV
- In EFT's we perform expansions in Λ_{QCD}/m_Q (HQETs), m_q/Λ_{QCD} (CHPT)
  - Fortunately, F_π appears as $4\pi F_\pi \sim 1\mathrm{GeV}$
  - Unfortunately, there are $M_\pi^2/(m_b(m_u+m_d))$ "corrections" to hadronic penguin B-decays.

Heavy-Quark Symmetry

Heavy-light meson



- Heavy quark carries almost all four-momentum of the meson.
  - In meson rest frame heavy quark Q is (almost) at rest.
    - Q acts as a static color source.
- As $m_Q \to \infty$

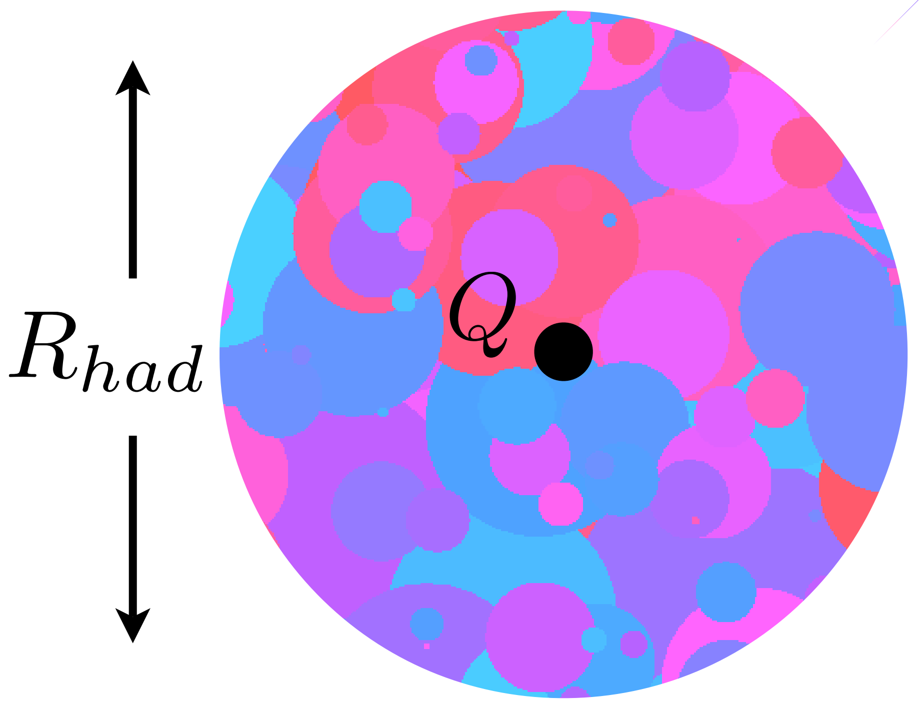
$$v_\mu = \gamma(1, \vec{v}) = \frac{P_H^\mu}{M_H} \to \frac{P_Q^\mu}{m_Q}$$

Simplifications:

$$R_{had} \sim \frac{1}{\Lambda_{\rm QCD}} \gg \frac{1}{m_Q} = \lambda_Q$$

$$\alpha_s(m_Q) \ll 1$$

Heavy-light meson



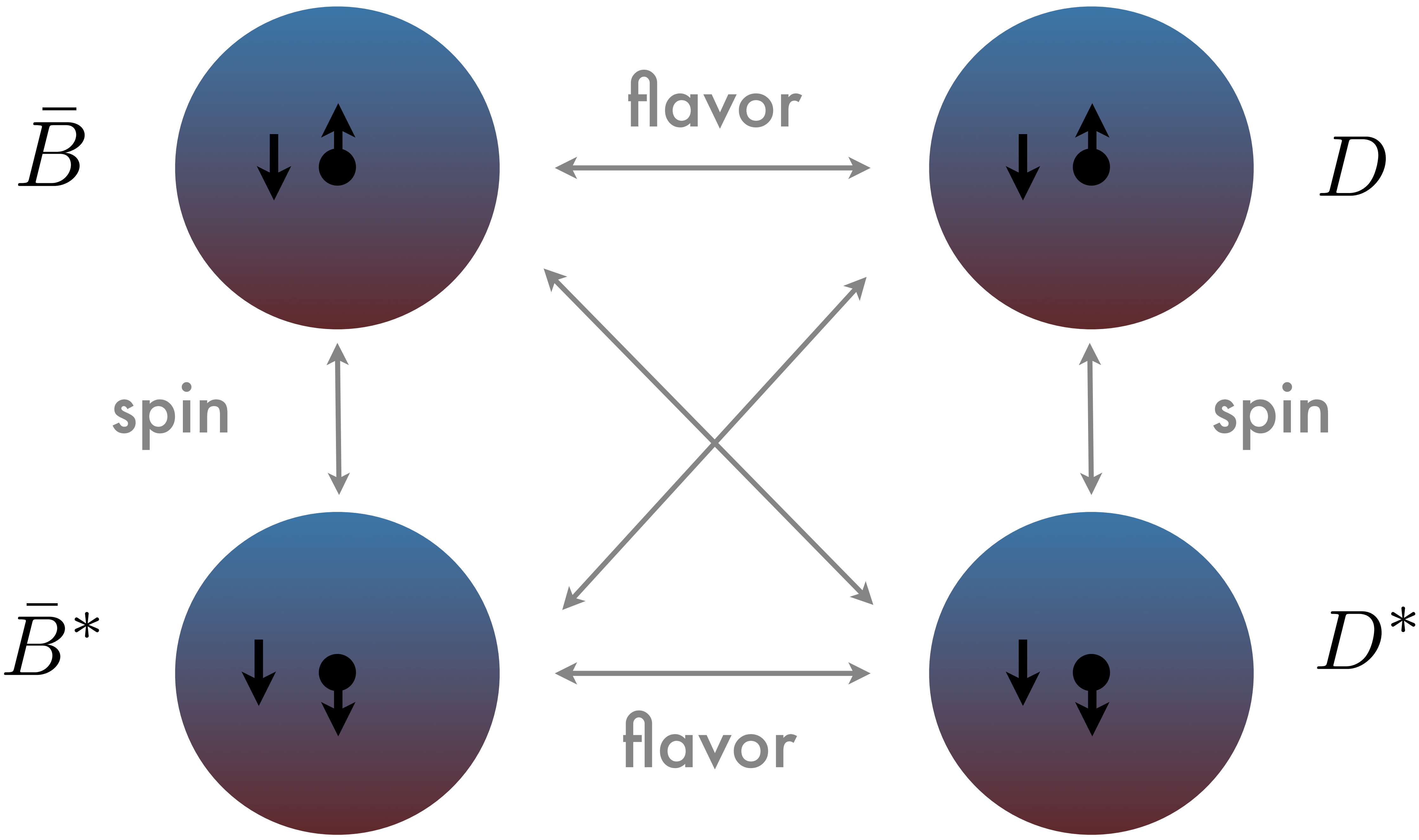
Simplifications:

$$R_{had} \sim \frac{1}{\Lambda_{\mathrm{QCD}}} \gg \frac{1}{m_Q} = \lambda_Q$$

$$\alpha_s(m_Q) \ll 1$$

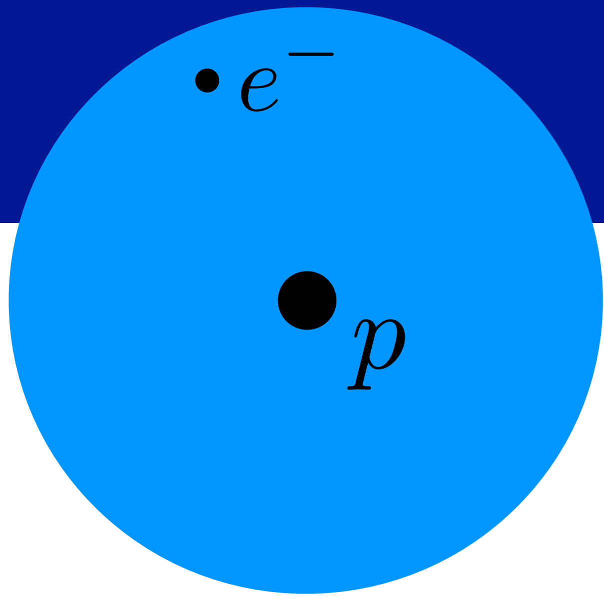
- Cloud of light degrees of freedom ("antiquark" in a meson) does not feel the mass of the heavy quark as $m_Q \to \infty$:
  - "flavor symmetry"
- Magnetic moment $\mu_Q \sim 1/m_Q$. Heavy quark spin decouples:
  - "spin symmetry"

Spin-flavor symmetry



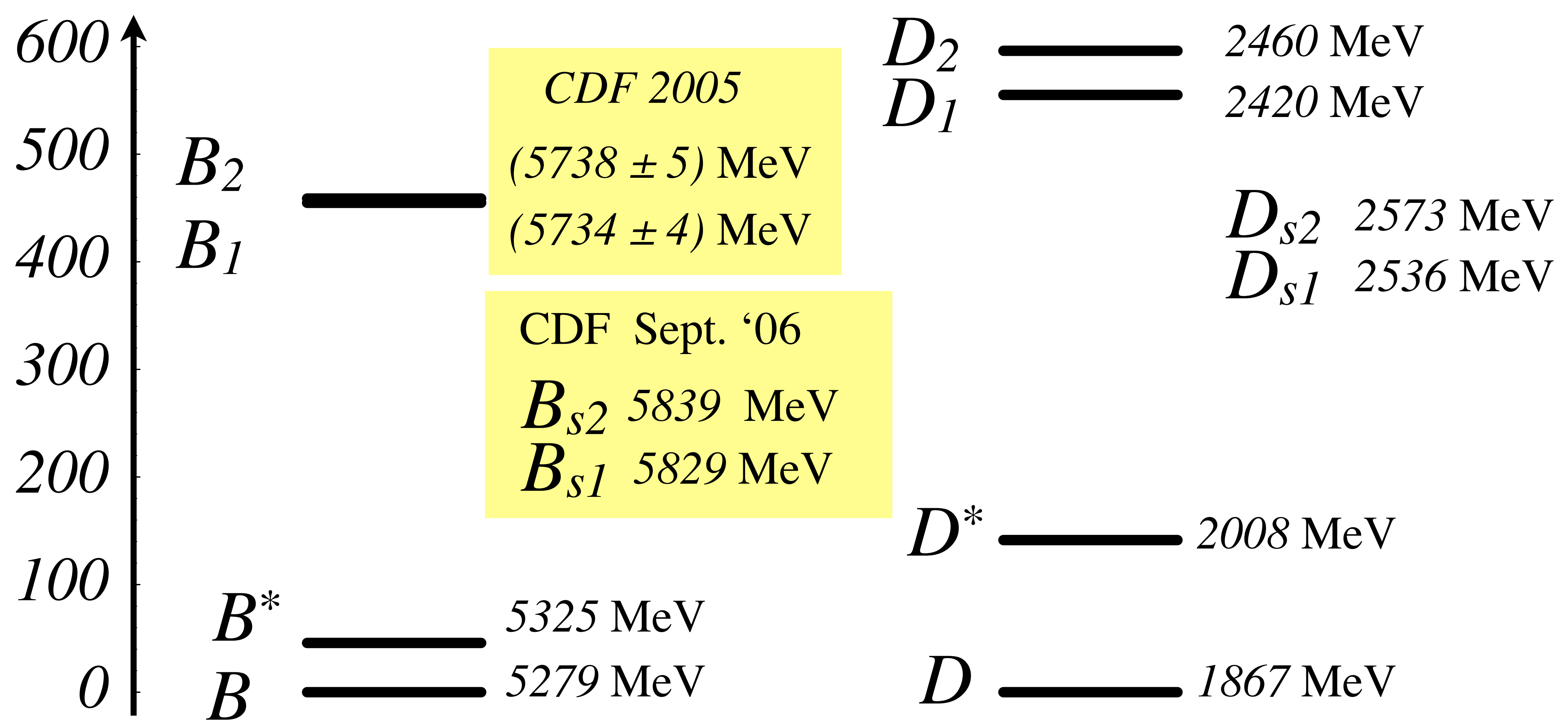
Q: How about baryons, such as Λ_b?

Compare to *H*-atom



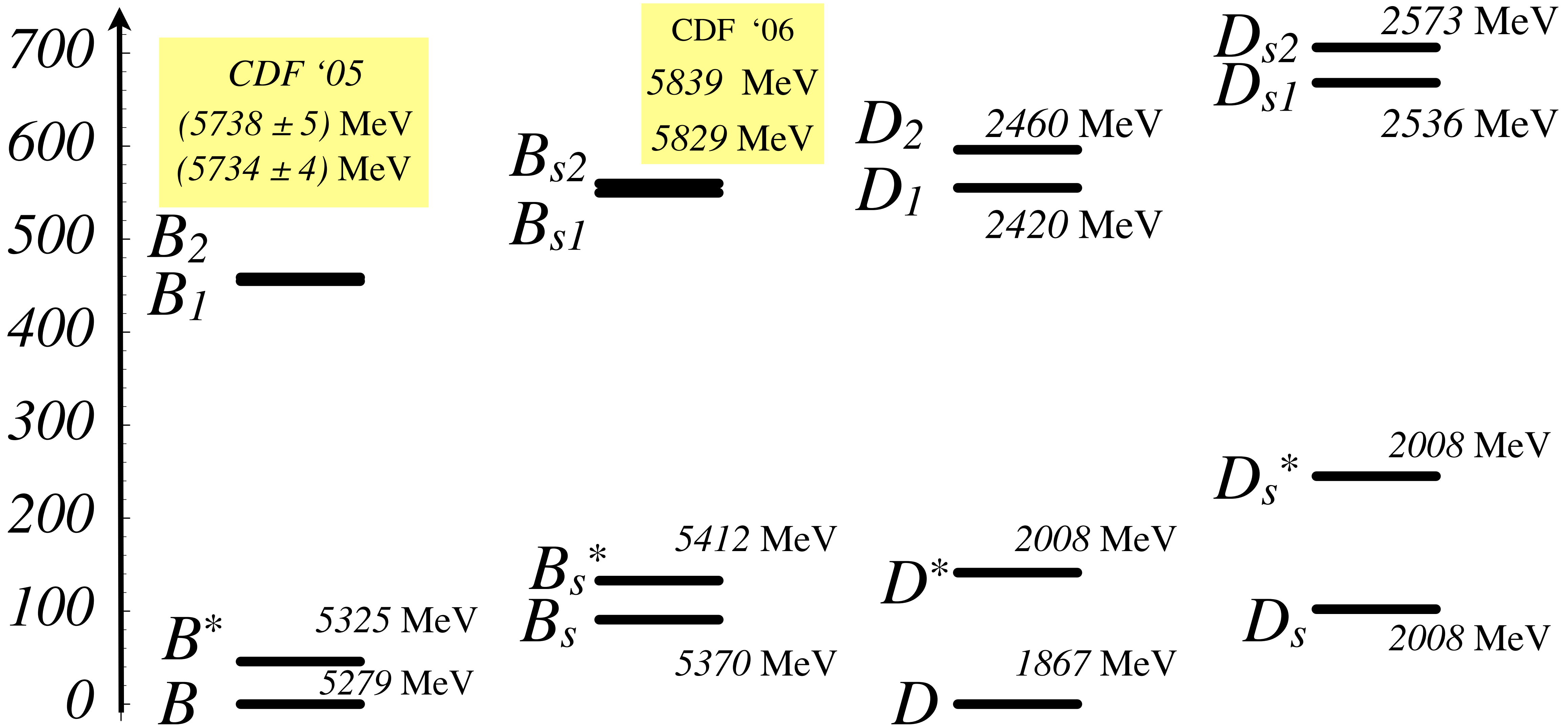
- Proton acts as a static electric source
  - energy levels are indepenent of m_{p}.
- Proton spin effects are suppressed by $1/m_{p}$.
- *However:* dynamics of light degrees in atom is much simpler than Qq meson
  - In contrast, $\bar{\mathrm{Q}}\mathrm{Q}$ meson becomes perturbative as m_{Q} and is described by a Schrödinger-type equation.

Heavy-light meson spectrum



- To leading power, bottom and charm spectra are simply shifted by constant amount m_b-m_c=3.4GeV.
  - M_{B1}-M_B = (455±5) MeV, M_{D1}-M_D = (553±1)MeV
- "Spin doublets" almost degenerate:
  - e.g. M_{B^*} - M_B = 46 MeV

Heavy-light meson spectrum



- To leading power, bottom and charm spectra are simply shifted by constant amount m_b-m_c=3.4GeV.
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- "Spin doublets" almost degenerate:
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Heavy meson masses

Spin dependence: $d_{J=0} = -3,\ d_{J=1} = 1$

$$M_H = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} + \frac{d_J \lambda_2}{2m_Q}$$

"binding energy"   correction to heavy-quark limit

- Works numerically well:
  - $(M_{B^*} - M_B) / (M_{D^*} - M_D) = 0.32 \approx m_c / m_b$
  - Power corrections: $M_{H^*}^2 - M_H^2 = 4\lambda_2$.
    - $M_{B^*}^2 - M_B^2 = 0.49\ \mathrm{GeV}^2$,
    - $M_{D^*}^2 - M_D^2 = 0.55\ \mathrm{GeV}^2$.
- Note: different parameters Λ, λ_1, λ_2 for ground state (B, B^*) and excited mesons (B_{s1}, B_{s2}), etc.

Heavy baryons

$$M_\Lambda = m_Q + \bar{\Lambda}^{\text{baryon}} - \frac{\lambda_1^{\text{baryon}}}{2m_Q} + \frac{d_J \lambda_2}{2m_Q}$$

Spin of the light degrees of freedom

- No λ_2-term for Λ-baryons:
  - $d_J = 2\{j(j+1) - s_Q(s_Q+1) - s_l(s_l+1)\} = 0$ for s_l=0
- Meson baryon mass difference

$$M_{\Lambda_b} - \frac{1}{4}(3M_{B^*} + M_B) = (306 \pm 2)\text{MeV}$$

$$M_{\Lambda_c} - \frac{1}{4}(3M_{D^*} + M_D) = 314\text{MeV}$$

Spin averaged meson mass (no λ_2-term)

- End of last year CDF has discovered Σ_b's: Σ_b^*-Σ_b= 21MeV
  - Note s_l= 1 for Σ_b
  - $M_{\Sigma b}$-$M_{\Lambda c}$=192MeV, $M_{\Sigma c}$-$M_{\Lambda c}$=169MeV

Heavy-quark effective theory (HQET)

Goal

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\infty + \mathcal{L}_{1/m_Q} + \ldots$$

- Construct low-energy effective theory that describes interactions of heavy quark with light degrees of freedom
  - recover spin-flavor symmetry for $m_Q \to \infty$
  - obtain systematic framework to study corrections to that limit

Dirac equation

- QCD Lagrangian

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{gluons+light\ quarks}} + \bar{\psi}_Q(i\not{D} - m_Q)\psi_Q}$$

$$\gamma_\mu \, (i\underbrace{\frac{\partial}{\partial x_\mu}}_{\sim\, m_Q v_\mu} + \underbrace{g_s\, A^A_\mu\, T_A}_{\sim\, \Lambda_{\mathrm{QCD}}})$$

- Dirac equation

$$\boxed{(i\not{D} - m_Q)\,\psi_Q(x) = 0}$$

- Warm up: solve free Dirac equation for a heavy quark at rest

$$\psi_Q(x) = e^{-im_Q t}\,\psi_Q(0) \longrightarrow m_Q(\gamma_0 - 1)\psi_Q(0) = 0$$

Dirac Matrices

- The γ-matrices fulfill the algebra

$$\boxed{\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\,\mathbf{1}}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \qquad (\gamma^5)^2 = 1 \qquad \{\gamma^5, \gamma^\mu\} = 0$$

- Chirality

$$P_L = \tfrac{1}{2}(1-\gamma^5) \qquad P_R = \tfrac{1}{2}(1+\gamma^5)$$

- Feynman's slash notation: $\not{a} = a_\mu\gamma^\mu$
- Dirac representation

Pauli matrices

$$\gamma^0 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}.$$

Projection operators

$$\left[\begin{pmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{pmatrix}-\begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}\right]\psi_Q(0)\leftrightarrow\psi_Q(0)=\begin{pmatrix}\psi_1\\\psi_2\\0\\0\end{pmatrix}$$

- Only upper components. Two component spinor is sufficient.
- To work in arbitrary frame: (v_μ : meson 4-velocity)

$$P_+=\frac{1+\not{v}}{2}\qquad P_-=\frac{1-\not{v}}{2}\quad\xrightarrow{\text{rest frame}}\quad P_+=\begin{pmatrix}1&&&\\&1&&\\&&0&\\&&&0\end{pmatrix}\quad P_-=\begin{pmatrix}0&&&\\&0&&\\&&1&\\&&&1\end{pmatrix}$$

- These are projection operators. ($\not{v}\,\not{v}=v^2=1$)

$$P_\pm^2=P_\pm\,,\qquad\qquad P_+P_-=P_-P_+=0$$

Derivation of HQET: step 1

- Redefine Quark field

Weak x-dependence from interaction with soft gluons

$$\begin{aligned}\psi_Q(x) &= e^{-im_Q v\cdot x}\ \tilde{\psi}_Q(x) \\ &= e^{-im_Q v\cdot x}\left[P_+\,\tilde{\psi}_Q(x) + P_-\,\tilde{\psi}_Q(x)\right] \\ &= e^{-im_Q v\cdot x}\left[h_v(x) + H_v(x)\right]\end{aligned}$$

- Plug into Dirac equation

$$\left\{m_Q \not{v} + i\not{D} - m_Q\right\}\left[h_v(x) + H_v(x)\right] = 0$$

$$\leftrightarrow\ i\not{D} h_v(x) + (i\not{D} - 2m_Q)H_v(x) = 0$$

$\not{v} h_v = h_v\ ,\ \not{v} H_v = -H_v$

Derivation of HQET: step 2

- Muliply eqn. by P_+ and P_-, use that

$$\begin{aligned} P_+ \not{a} &= \not{a}_\perp P_- + v\cdot a P_+ \\ P_- \not{a} &= \not{a}_\perp P_+ - v\cdot a P_- \end{aligned}$$

$$\text{with } a_\perp^\mu = a^\mu - v\cdot a\, v^\mu$$

- Obtain two equations

$$\boxed{\begin{aligned} iv\cdot D\, h_v(x) + i\not{D}_\perp\, H_v &= 0 \\ i\not{D}_\perp\, h_v(x) - (iv\cdot D + 2m_Q) H_v(x) &= 0 \end{aligned}}$$

$$\mathcal{O}(\Lambda_{\mathrm{QCD}})$$

$$H_v(x) \approx \frac{1}{2m_Q} i\not{D}_\perp\, h_v(x)$$

Derivation of HQET: step 3

- Rewrite

$$
\begin{aligned}
i\not{D}_\perp \, i\not{D}_\perp &= iD_\perp^\mu \, iD_\perp^\nu \left(\frac{1}{2}\{\gamma_\mu, \gamma_\nu\} + \frac{1}{2}[\gamma_\mu, \gamma_\nu] \right) \\
&= iD_\perp^\mu \, iD_\perp^\nu \, (g_{\mu\nu} - i\sigma_{\mu\nu}) \\
&= (iD_\perp)^2 + \frac{i}{2}[D_\perp^\mu, D_\perp^\nu]\sigma_{\mu\nu} \\
&= (iD_\perp)^2 + \frac{g_s}{2}\sigma_{\mu\nu}G_\perp^{\mu\nu}
\end{aligned}
$$

- The EOM follows from the Lagrangian

$$
\mathcal{L}_{\text{eff}} = \bar{h}_v \, iv \cdot D \, h_v + \frac{1}{2m_Q}\bar{h}_v (iD_\perp)^2 h_v + \frac{g_s}{4m_Q}\bar{h}_v \sigma_{\mu\nu} G_\perp^{\mu\nu} h_v + \mathcal{O}(\Lambda/m_Q^2)
$$

Spin and flavor symmetric!

power corrections

Rest frame $v^\mu=(1,0,0,0)$

$$\mathcal{L}_{\text{eff}} = \bar{h}\, iD_t\, h - \frac{1}{2m_Q}\bar{h}\,(i\vec{D})^2\, h - \frac{g_s}{2m_Q}\bar{h}\,\vec{\sigma}\cdot\vec{B}_c\, h + \mathcal{O}(\Lambda/m_Q^2)$$

kinetic energy operator violates flavor symm.

chromo-magnetic operator violates flavor and spin symm.

$$iD_t\, h = \frac{1}{2m_Q}(i\vec{D})^2\, h + \frac{g_s}{2m_Q}\vec{\sigma}\cdot\vec{B}_c\, h + \mathcal{O}(\Lambda/m_Q^2)$$

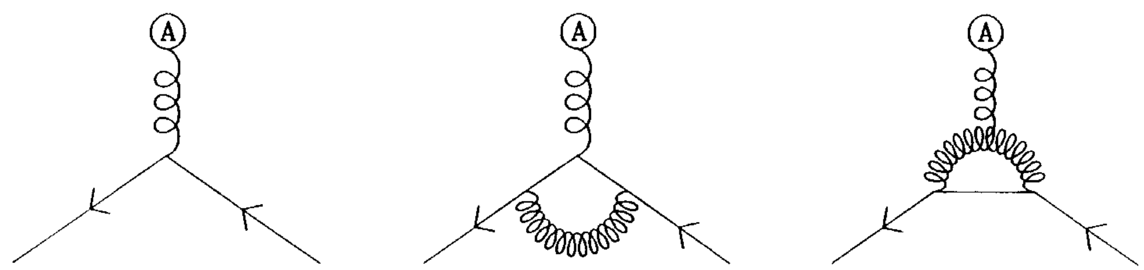
- “Schrödinger-Pauli” equation for non-relativistic particle in background gluon field.

Short-distance corrections

- Heavy quark symmetry is broken by hard exchange.
  - $\propto \alpha_s(m_Q)$. Can be calculated in perturbation theory.
  - Renormalizes coefficients of operators in effective Lagrangian.
  - No renormalization of kinetic energy operator due to Lorentz invariance ("reparameterization invariance").
  - However, perturbative corrections to coefficient of chromomagnetic operator. "Anomalous chromomagnetic moment."
    - Coefficient can be obtained from a matching calculation.

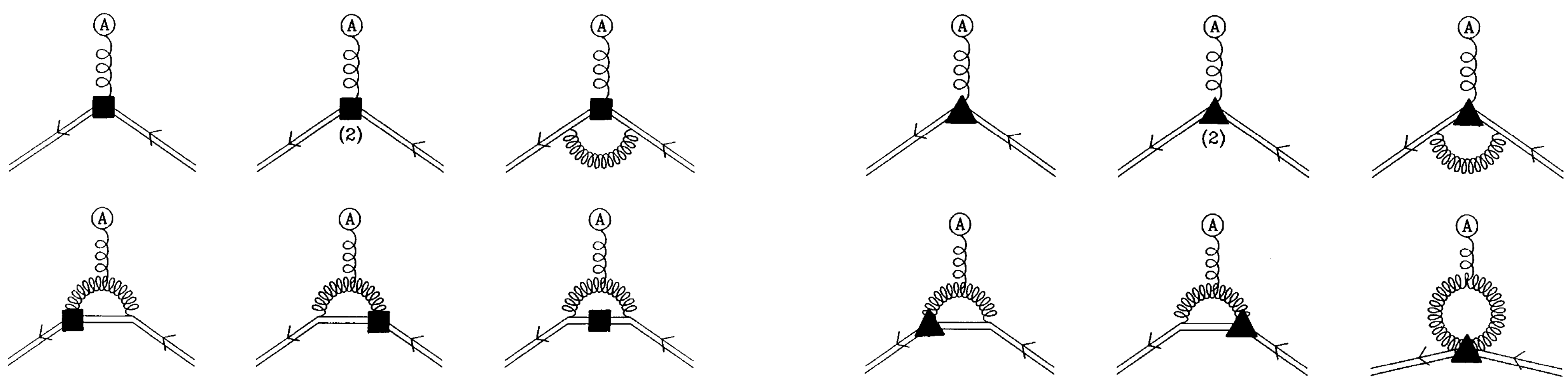
Matching

- Calculate QCD diagrams, expand in $1/m_Q$



- Calculate HQET diagrams

Eichten & Hill '90

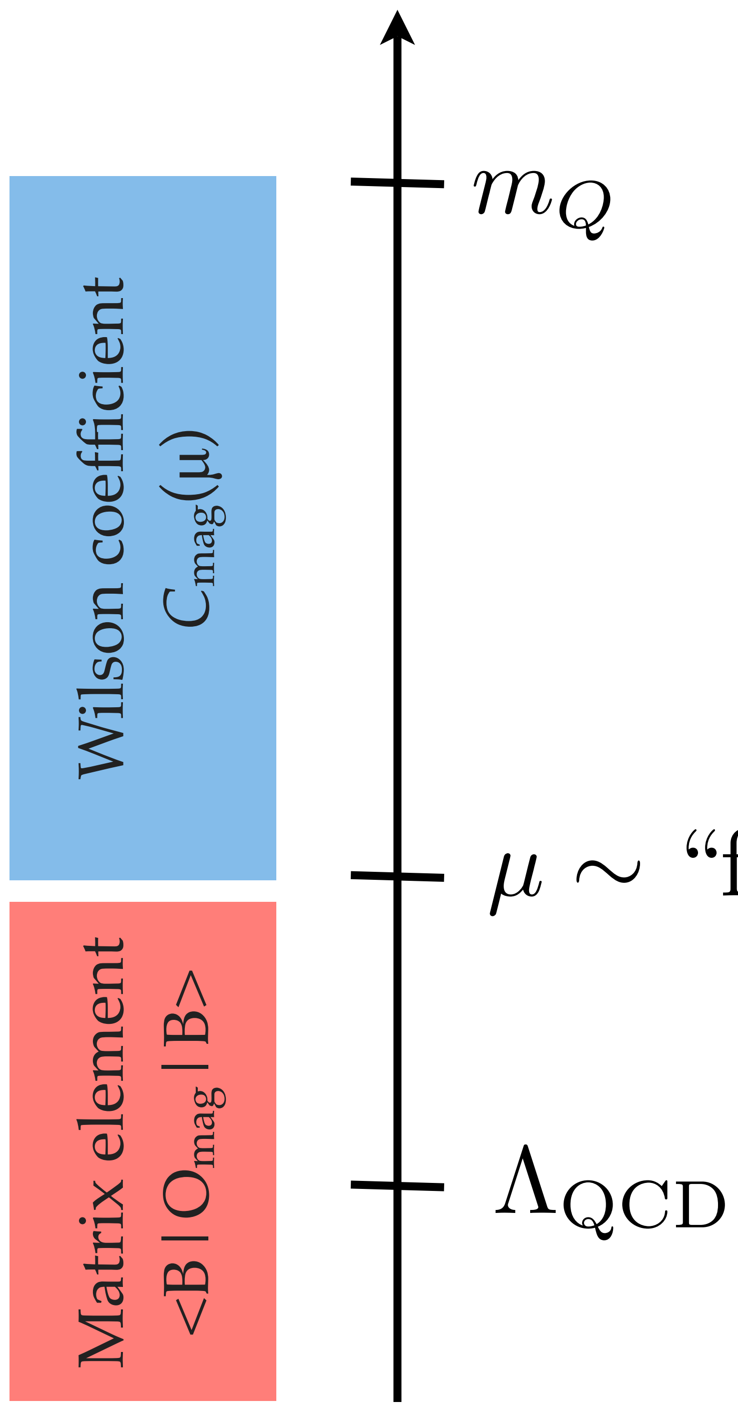


  - Adjust coefficient C_{mag} of chromo-magnetic operator such that the two contributions match

$$C_{\text{mag}}(\mu) = 1 - \frac{3\alpha_s(\mu)}{2\pi}\left(\ln\frac{m_Q}{\mu} - \frac{13}{9}\right)$$

  - Depends on renormalization scale μ.

Renormalization scale



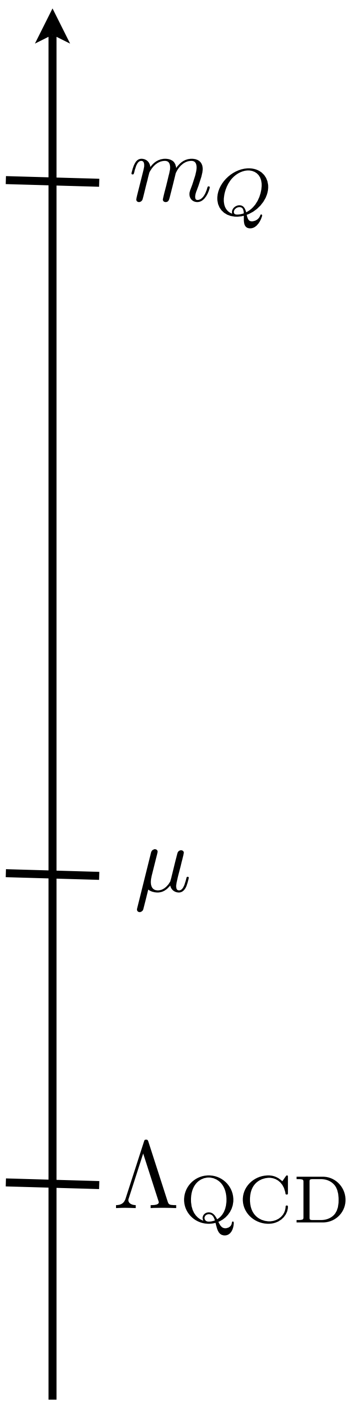
- Physical quantities $C_{\mathrm{mag}}(\mu)$ x $< O_{\mathrm{mag}}(\mu) >$ are independent of scale μ.
- Leads to renormalization group (RG) equation

$$\mu \frac{d}{d\mu} C_{\mathrm{mag}}(\mu) = \gamma_{\mathrm{mag}} C_{\mathrm{mag}}(\mu)$$

"anomalous dimension"

$$\gamma_{\mathrm{mag}} = \frac{C_A \alpha_s}{2\pi} \left[1 + \left(\frac{17}{18} C_A - \frac{13}{18} T_F n_f \right) \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right]$$

Renormalization group evolution



- For $m_Q \gg \mu$ fixed order expansion starts to fail:

$$C_{\rm mag}(\mu) = 1 - \frac{3\alpha_s(\mu)}{2\pi}\left(\ln\frac{m_Q}{\mu} - \frac{13}{9}\right)$$

large log's $\alpha_s^n \ln^n \frac{m_Q}{\mu}$

- Resum log's by solving RG equation:

$$\mu\frac{d}{d\mu}C_{\rm mag}(\mu) = \gamma_{\rm mag}\, C_{\rm mag}(\mu)$$

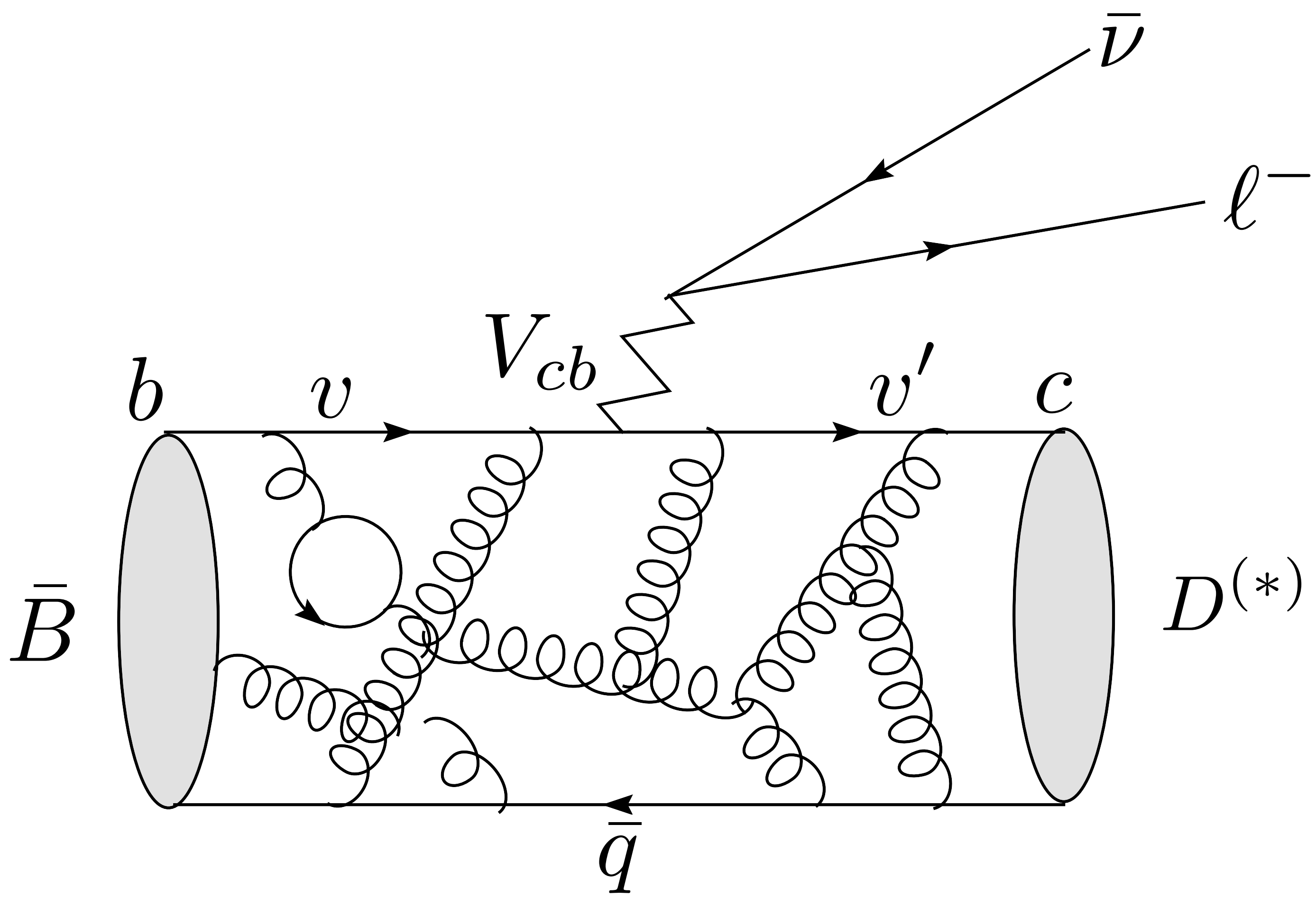
- Match at high scale to avoid log's, then evolve down

$$C_{\rm mag}(\mu) = U(m_Q,\mu)C_{\rm mag}(m_Q)$$

- Solution (for n_f=3 light quarks):

$$C_{\rm mag}(\mu) = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{1/3}\left[1 + \frac{\alpha_s(m_Q)}{4\pi}\frac{26}{3} - \frac{\alpha_s(m_Q)-\alpha_s(\mu)}{4\pi}\frac{1}{27}\right]$$

Exclusive semi-leptonic decays



- Using heavy-quark symmetries, we will calculate the rate at maximum momentum transfer to the lepton pair, where v=v' ("zero recoil point").

Vector current in HQET

- QCD

$$\langle \bar{B}(v')|\, \bar{b}\, \gamma^\mu b\, |\bar{B}(v)\rangle = F_{\rm el}(q^2)\, (p+p')^\mu$$

- HQET

"Isgur-Wise function"

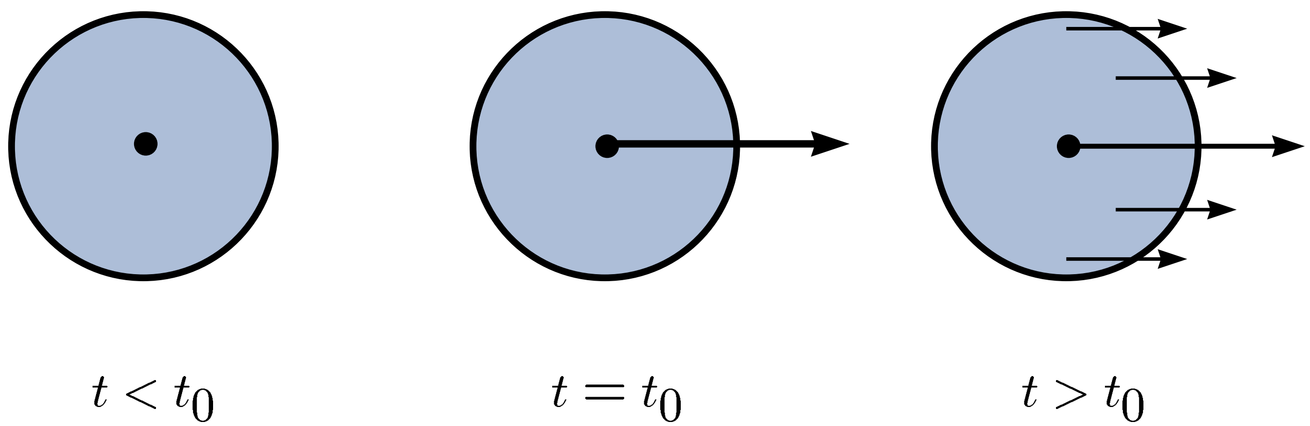
$$\frac{1}{m_B}\, \langle \bar{B}(v')|\, \bar{b}_{v'} \gamma^\mu b_v\, |\bar{B}(v)\rangle = \xi(v\cdot v')\, (v+v')^\mu + \mathcal{O}(1/m_b)$$

  - Factor $1/m_B$ on lhs compensates for normalization $\langle \bar{B}(p')|\bar{B}(p)\rangle = 2m_B v^0\, (2\pi)^3\, \delta^3(\vec{p}-\vec{p}\,')$

- Comparison gives

$$F_{\rm el}(q^2) = \xi(v\cdot v')\,, \qquad q^2 = -2m_B^2(v\cdot v'-1) \quad , \quad F_{\rm el}(0) = 1 \quad \leftrightarrow \quad \xi(1) = 1$$

Physical picture



- At time $t=t_0$ current changes heavy quark with velocity v_μ into heavy quark with velocity v'_μ
  - For $v'_\mu = v_\mu$ nothing happens: $\xi(1)=1$
- For $t>t_0$ light degrees of freedom rearrange themselves to fly along with heavy quark.
  - During this "shake-up" light hadrons (such as π's) can get radiated off: $\xi(v \cdot v'<1)<1$

Heavy-quark symmetry

- Use heavy-quark symmetry to replace b→c

$$\frac{1}{m_B}\langle\bar{B}(v')|\,\bar{b}_{v'}\gamma^\mu b_v\,|\bar{B}(v)\rangle = \xi(v\cdot v')\,(v+v')^\mu$$

$$\boxed{\frac{1}{\sqrt{m_B m_D}}\langle D(v')|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}(v)\rangle = \xi(v\cdot v')\,(v+v')^\mu}$$

- In general there are two form factors:

$$\langle D(v')|\,\bar{c}\,\gamma^\mu b\,|\bar{B}(v)\rangle = f_+(q^2)\,(p+p')^\mu - f_-(q^2)\,(p-p')^\mu$$

- Heavy quark symmetry gives relation

$$\boxed{f_\pm(q^2) = \frac{m_B \pm m_D}{2\sqrt{m_B m_D}}\,\xi(v\cdot v')\,,} \qquad q^2 = m_B^2 + m_D^2 - 2m_B m_D\, v\cdot v'\,.$$

Spin symmetry

- Can use heavy quark spin symmetry to relate B→D and B→D* form factors

$$\frac{1}{\sqrt{m_B m_{D^*}}}\langle D^*(v',\varepsilon)|\,\bar{c}_{v'}\gamma^\mu b_v\,|\bar{B}(v)\rangle = i\epsilon^{\mu\nu\alpha\beta}\,\varepsilon^*_\nu\,v'_\alpha v_\beta\,\xi(v\cdot v')\,,$$

$$\frac{1}{\sqrt{m_B m_{D^*}}}\langle D^*(v',\varepsilon)|\,\bar{c}_{v'}\gamma^\mu\gamma_5\,b_v\,|\bar{B}(v)\rangle = \Big[\varepsilon^{*\mu}\,(v\cdot v'+1) - v'^\mu\,\varepsilon^*\cdot v\Big]\xi(v\cdot v')$$

- In this case, there are in general 4 form factors

$$\frac{m_B+m_{D^*}}{2\sqrt{m_B m_{D^*}}}\,\xi(v\cdot v') = V(q^2) = A_0(q^2) = A_2(q^2)$$

$$= \left[1-\frac{q^2}{(m_B+m_D)^2}\right]^{-1} A_1(q^2)\,,$$

$$q^2 = m_B^2 + m_{D^*}^2 - 2m_B m_{D^*}\,v\cdot v'\,.$$

Semileptonic B→Dlν and B→D*lν decay

- Eff. Hamiltonian

$$\mathcal{H}_{\rm SL} = \frac{G_F}{\sqrt{2}} V_{cb}\ \bar{c}\,\gamma_\mu(1-\gamma_5)\,b\ \bar{e}\,\gamma_\mu(1-\gamma_5)\,\nu_e$$

- Amplitude

Form factors given in terms of $\xi(\mathrm{v}\cdot\mathrm{v}')$

$$\mathcal{A}(\bar{B}\to D^{(*)}e\bar{\nu}_e) \quad = \quad \frac{G_F}{\sqrt{2}} V_{cb}\, \langle D^{(*)}(v')|\bar{c}\,\gamma_\mu(1-\gamma_5)\,b|B(v)\rangle$$

$$\bar{u}_e(p_e)\,\gamma_\mu(1-\gamma_5)\,v_{\nu_e}(p_\nu)$$

- Rate
  - Square amplitude, sum over spins
  - Integrate over phase space

Decay rates

- For $B \to Dlv$ and $B \to D^*lv$ decay rate in terms of single form factor $\xi(w)$, $w = v \cdot v'$ with known normalization $\xi(1)=1$.

$$\frac{\mathrm{d}\Gamma(\bar{B} \to D\,\ell\,\bar{\nu})}{\mathrm{d}w} = \frac{G_F^2}{48\pi^3}\,|V_{cb}|^2\,(m_B + m_D)^2\,m_D^3\,(w^2-1)^{3/2}\,\xi^2(w)\,,$$

$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^*\ell\,\bar{\nu})}{\mathrm{d}w} = \frac{G_F^2}{48\pi^3}\,|V_{cb}|^2\,(m_B - m_{D^*})^2\,m_{D^*}^3\,\sqrt{w^2-1}\,(w+1)^2$$

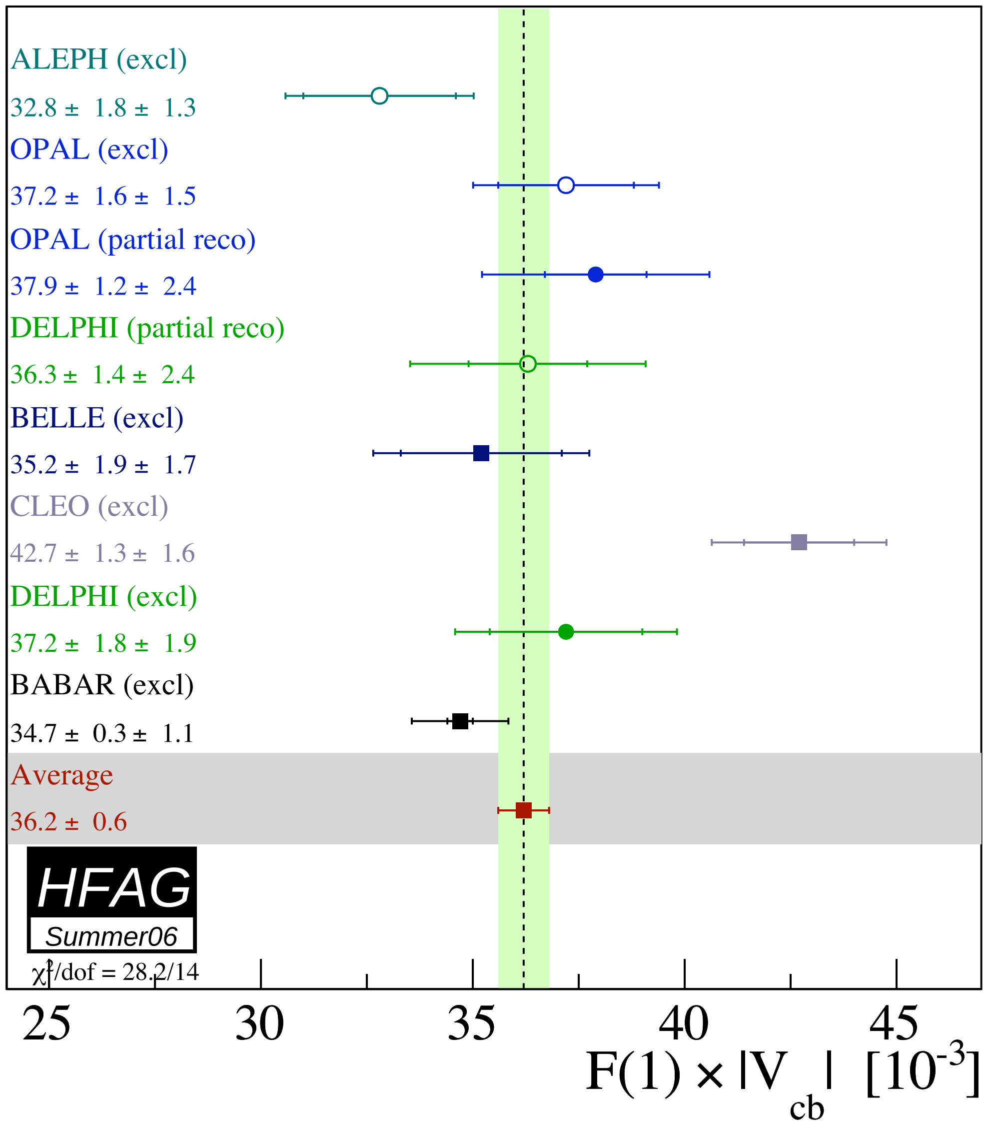
$$\times\left[1 + \frac{4w}{w+1}\,\frac{m_B^2 - 2w\,m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right]\xi^2(w)\,.$$

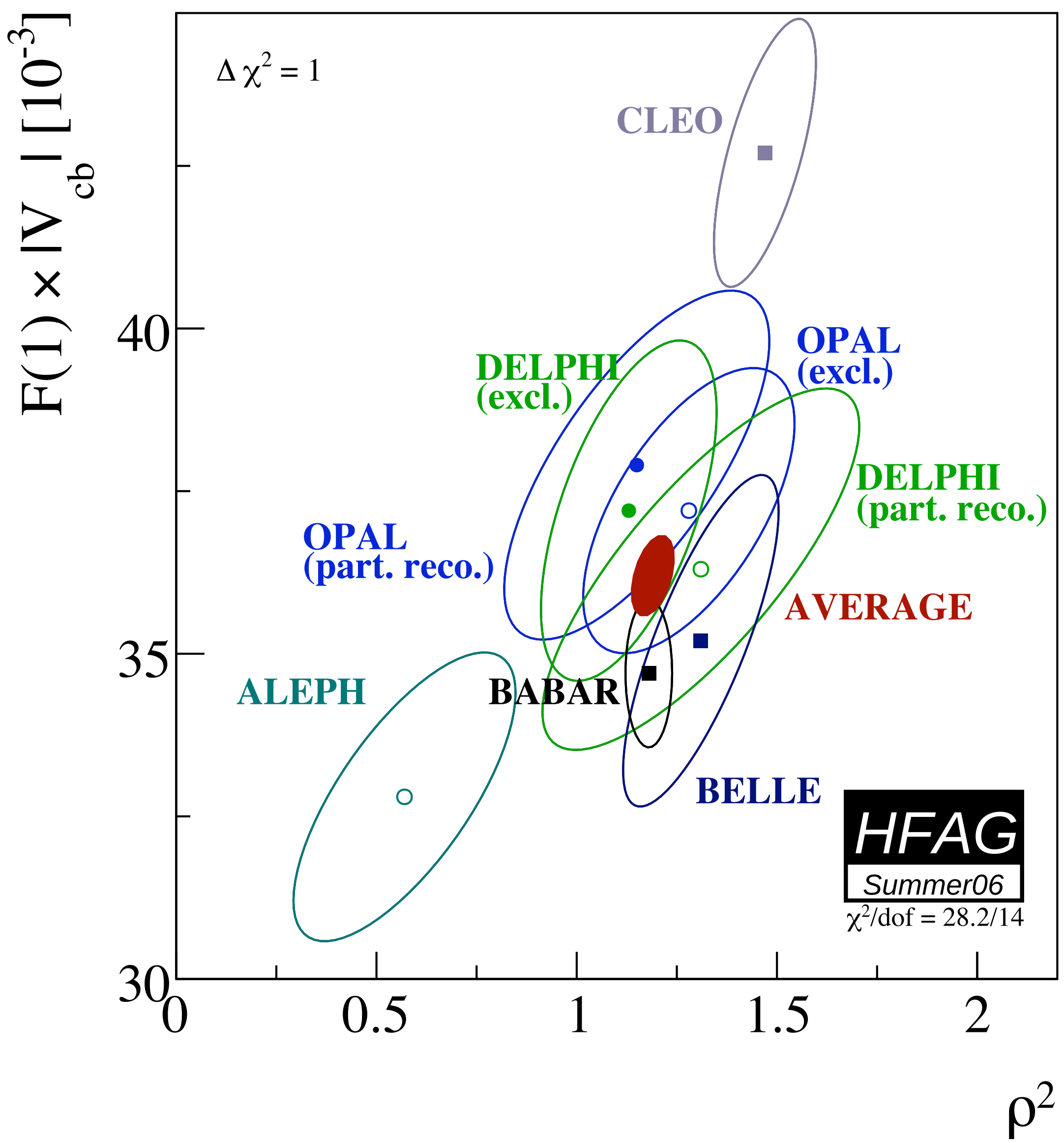
- Corrections:
  - $\alpha_s(m_Q)$: known to 2 loops.
  - Λ/m_Q: lattice QCD ($\to$ Andreas's lectures)

Extraction of V_{cb}

- For $B \to D^* l\nu$, the first order power corrections vanish at the zero-recoil point (Luke's theorem).
  - Measure the rate as a function of w, extrapolate to the zero-recoil point $w=1$.
  - Calculate second order power correction using LQCD.
- There are strong constraints on the shape of the form factor (analyticity).
  - Once these are taken into account a single parameter ρ is sufficient to parameterize the shape.

Experimental results





FERMILAB-PUB-01/317-T :

$$\mathcal{F}_{B\to D^*}(1) \;=\; 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}{}^{+0.000}_{-0.016}{}^{+0.006}_{-0.014}$$

statistics   latt. matching   cont. limit m_q extr.   quenched appr.

pert: -(4.0±0.7)% , power: -(5.5±2.5)%